A Suite of Models for Dynare

Description of Models

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Version 1.0

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1 A real Business Cycle Model

The problem of the household is

$$\max \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\zeta_{c,t+j}^{\frac{1}{\sigma}} \frac{c_{t+j}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{\psi}{1+\nu} \zeta_{h,t+j}^{-\nu} h_{t+j}^{1+\nu} \right) \right]$$

$$c_t + i_t = w_t h_t + z_t k_t - \tau_t$$
$$k_{t+1} = i_t + (1-\delta) k_t$$

The last two conditions can be combined to give

$$k_{t+1} = w_t h_t + (z_t + 1 - \delta) k_t - c_t - \tau_t$$

Then the first order conditions are given by

$$\zeta_{c,t}^{\frac{1}{\sigma}} c_t^{-\frac{1}{\sigma}} = \lambda_t \tag{1}$$
$$\psi \zeta_{h,t}^{-\nu} h_t^{\nu} = \lambda_t w_t \tag{2}$$
$$\lambda_t = \beta \mathbb{E}_t \left(\lambda_{t+1} (z_{t+1} + 1 - \delta) \right) \tag{3}$$

The problem of the firm is

$$\max a_t k_t^{\alpha} h_t^{1-\alpha} - w_t h_t - z_t k_t$$

which leads to the first order conditions

$$w_t = (1-\alpha) \frac{y_t}{h_t} \text{ and } z_t = \alpha \frac{y_t}{k_t}$$

The taxes, τ_t, finance an exogenous stream of government expenditures, g_t, such that $\tau_t = g_t$.

All shocks follow AR(1) processes of the type

$$\log(a_{t+1}) = \rho_a \log(a_t) + \varepsilon_{a,t+1}$$
$$\log(g_{t+1}) = \rho_g \log(g_t) + (1-\rho_g) \log(\overline{g}) + \varepsilon_{g,t+1}$$
$$\log(\zeta_{c,t+1}) = \rho_c \log(\zeta_{c,t}) + \varepsilon_{c,t+1}$$
$$\log(\zeta_{h,t+1}) = \rho_h \log(\zeta_{h,t}) + \varepsilon_{h,t+1}$$

The general equilibrium is therefore represented by the following set of equations

$$\zeta_{c,t}^{\frac{1}{\sigma}} c_t^{-\frac{1}{\sigma}} = \lambda_t$$
$$\psi \zeta_{h,t}^{-\nu} h_t^{\nu} = \lambda_t (1-\alpha) \frac{y_t}{h_t}$$
$$\lambda_t = \beta \mathbb{E}_t \left(\lambda_{t+1} (\alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta) \right)$$
$$k_{t+1} = i_t + (1-\delta) k_t$$
$$y_t = a_t k_t^{\alpha} h_t^{1-\alpha}$$
$$y_t = c_t + i_t + g_t$$

and the definition of the shocks.

2 A Nominal Model with Price Adjustment Costs

In all what follows, we will assume zero inflation in the steady state.

The problem of the household is

$$\max \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\zeta_{c,t+j}^{\frac{1}{\sigma}} \frac{c_{t+j}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{\psi}{1+\nu} \zeta_{h,t+j}^{-\nu} h_{t+j}^{1+\nu} \right) \right]$$

$$B_t + P_t c_t + P_t i_t = R_{t-1} B_{t-1} P_t w_t h_t + P_t z_t k_t - P_t \tau_t$$

$$k_{t+1} = i_t + (1-\delta) k_t$$

The last two conditions can be combined to give

$$B_t + P_t c_t + P_t k_{t+1} = R_{t-1} B_{t-1} P_t w_t h_t + P_t (z_t + 1 - \delta) k_t - P_t \tau_t$$

Then the first order conditions are given by

$$\zeta_{c,t}^{\frac{1}{\sigma}} c_t^{-\frac{1}{\sigma}} = \Lambda_t P_t \tag{4}$$

$$\psi \zeta_{h,t}^{-\nu} h_t^{\nu} = \Lambda_t P_t w_t \tag{5}$$

$$\Lambda_t P_t = \beta \mathbb{E}_t \left(\Lambda_{t+1} P_{t+1} (z_{t+1} + 1 - \delta) \right) \tag{6}$$

$$\Lambda_t = \beta R_t \mathbb{E}_t \Lambda_{t+1} \tag{7}$$

The economy is comprised of many sectors. The first sector —the final good sector— combines intermediate goods to form a final good in quantity y_t:

$$y_t = \left(\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} \mathrm{d}i \right)^{\frac{\theta}{\theta-1}} \tag{8}$$

The problem of the final good firm is then

$$\max_{\{y_t(i); i \in (0,1)\}} P_t y_t - \int_0^1 P_t(i) y_t(i) \mathrm{d}i$$

subject to (8), which rewrites

$$\max_{\{y_t(i); i \in (0,1)\}} P_t \left(\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} \mathrm{d}i \right)^{\frac{\theta}{\theta-1}} - \int_0^1 P_t(i) y_t(i) \mathrm{d}i$$

which gives rise to the demand function

$$y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} y_t \tag{9}$$

These intermediate goods are produced by intermediaries, each of which has a local monopoly power. Each intermediate firm i, $i \in (0,1)$, uses a constant returns to scale technology

$$y_t(i) = a_t k_t(i)^{\alpha} h_t(i)^{1-\alpha} \tag{10}$$

where $k_t(i)$ and $h_t(i)$ denote capital and labor. The firm minimizes its real cost subject to (10). Minimized real total costs are then given by $s_t x_t(i)$ where the real marginal cost, s_t, is given by

$$s_t = \frac{z_t^\alpha w_t^{1-\alpha}}{\varsigma a_t}$$

with $\varsigma = \alpha^\alpha(1-\alpha)^{1-\alpha}$.

Intermediate goods producers are monopolistically competitive, and therefore set prices for the good they produce. However, it incurs a cost whenever it changes its price relatively to the earlier period. This cost is given by

$$\frac{\varphi_p}{2}\left(\frac{P_t(i)}{P_{t-1}(i} - 1\right)^2 y_t$$

The problem of the firm is then to maximize the profit function

$$\mathbb{E}_t\left[\sum_{j=0}^{\infty}\Phi_{t,t+j}\left(P_{t+j}(i)y_{t+j}(i) - P_{t+j}s_{t+j}y_{t+j}(i) - P_{t+j}\frac{\varphi_p}{2}\left(\frac{P_{t+j}(i)}{P_{t+j-1}(i} - 1\right)^2 y_{t+j}\right)\right] \tag{11}$$

where $\Phi_{t,t+j}$ is an appropriate discount factor derived from the household's optimality conditions, and proportional to $\beta^j\frac{\Lambda_{t+j}}{\Lambda_t}$. The first order condition of the problem is given by

$$\begin{aligned}(1-\theta)\left(\frac{P_t(i)}{P_t}\right)^{-\theta} y_t + \theta\frac{P_t}{P_t(i)}s_t\left(\frac{P_t(i)}{P_t}\right)^{-\theta} y_t - \frac{P_t}{P_{t-1}(i)}\varphi_p\left(\frac{P_t(i)}{P_{t-1}(i} - 1\right)y_t \\ + \beta\mathbb{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\frac{P_{t+1}P_{t+1}(i)}{P_t(i)^2}\varphi_p\left(\frac{P_{t+1}(i)}{P_t(i} - 1\right)y_{t+1}\right] = 0\end{aligned}$$

Using Sheppard's lemma we get the demand for each input

$$\begin{aligned} w_t &= (1-\alpha)s_t\frac{y_t(i)}{h_t(i)} \\ z_t &= \alpha s_t\frac{y_t(i)}{k_t(i)}\end{aligned}$$

The taxes, τ_t, finance an exogenous stream of government expenditures, g_t, such that

$$\tau_t = g_t$$

In order to close the model, we add a Taylor rule that determines the nominal interest rate

$$\log(R_t) = \rho_r\log(R_{t-1}) + (1-\rho)\left(\log(\overline{R}) + \gamma_\pi(\log(\pi_t) - \log(\overline{\pi})) + \gamma_y(\log(y_t) - \log(\overline{y}))\right)$$

where $\pi_t = P_t/P_{t-1}$ denotes aggregate inflation.

All shocks follow AR(1) processes of the type

$$\begin{aligned}\log(a_{t+1}) &= \rho_a\log(a_t) + \varepsilon_{a,t+1} \\ \log(g_{t+1}) &= \rho_g\log(g_t) + (1-\rho_g)\log(\overline{g}) + \varepsilon_{g,t+1} \\ \log(\zeta_{c,t+1}) &= \rho_c\log(\zeta_{c,t}) + \varepsilon_{c,t+1} \\ \log(\zeta_{h,t+1}) &= \rho_h\log(\zeta_{h,t}) + \varepsilon_{h,t+1}\end{aligned}$$

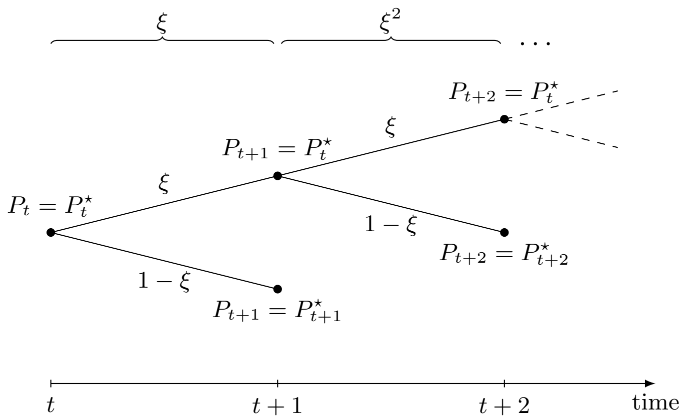
The symmetric general equilibrium is therefore represented by the following set of equations

$$
\begin{aligned}
\zeta_{c,t}^{\frac{1}{\sigma}} c_t^{-\frac{1}{\sigma}} &= \lambda_t \\
\psi \zeta_{h,t}^{-\nu} h_t^{\nu} &= \lambda_t (1-\alpha) s_t \frac{y_t}{h_t} \\
\lambda_t &= \beta \mathbb{E}_t \left(\lambda_{t+1} \left(\alpha s_{t+1} \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) \right] \\
\lambda_t &= \beta R_t \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right) \\
0 &= (1-\theta) y_t + \theta s_t y_t - \pi_t \varphi_p \left(\pi_t - 1 \right) y_t + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1} \varphi_p \left(\pi_{t+1} - 1 \right) y_{t+1} \right] \\
k_{t+1} &= i_t + (1-\delta) k_t \\
y_t &= a_t k_t^{\alpha} h_t^{1-\alpha} \\
y_t &= c_t + i_t + g_t + \frac{\varphi_p}{2} (\pi_t - 1)^2 y_t \\
\log(R_t) &= \rho_r \log(R_{t-1}) + (1-\rho) \left(\log(\overline{R}) + \gamma_\pi (\log(\pi_t) - \log(\overline{\pi})) + \gamma_y (\log(y_t) - \log(\overline{y})) \right)
\end{aligned}
$$

and the definition of the shocks.

3 A Nominal Model with Staggered Price Contracts

The main difference between this model and the previous one lies in the specification of the nominal rigidities. Intermediate goods producers are monopolistically competitive, and therefore set prices for the good they produce. We follow Calvo [1983] in assuming that firms set their prices for a stochastic number of periods. In each and every period, a firm either gets the chance to adjust its price (an event occurring with probability $1-\xi$) or it does not. This is illustrated in the following figure.



When the firm does not reset its price, it just applies the price it charged in the last period such that $P_t(i) = P_{t-1}(i)$. When it gets a chance to do it, firm i resets its price, $P_t^\star(i)$, in period t in order to maximize its expected discounted profit flow this new price will generate. In period t, the profit is given by $\Pi(P_t^\star(i))$. In period $t+1$, either the firm resets its price, such that it will get

$\Pi(P^\star_{t+1}(i))$ with probability q, or it does not and its $t+1$ profit will be $\Pi(P^\star_t(i))$ with probability ξ. Likewise in $t+2$. The expected profit flow generated by setting $P^\star_t(i)$ in period t writes

$$\max_{P^\star_t(i)} E_t \sum_{j=0}^{\infty} \Phi_{t,t+j}\xi^j \Pi(P^\star_t(i))$$

subject to the total demand it faces:

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} y_t$$

and where $\Pi(P^\star_t(i)) = (P^\star_t(i) - P_{t+j}s_{t+j})\, y_{t+j}(i)$. $\Phi_{t,t+j}$ is an appropriate discount factor related to the way the household value future as opposed to current consumption, such that

$$\Phi_{t,t+j} \propto \beta^j \frac{\Lambda_{t+j}}{\Lambda_t}$$

This leads to the price setting equation

$$E_t\left[\sum_{j=0}^{\infty}(\beta\xi)^j \frac{\Lambda_{t+j}}{\Lambda_t}\left((1-\theta)\left(\frac{P^\star_t(i)}{P_{t+j}}\right)^{-\theta} y_{t+j} + \theta\frac{P_{t+j}}{P_{t+j}(i)}\left(\frac{P^\star_t(i)}{P_{t+j}}\right)^{-\theta} s_{t+j}y_{t+j}\right)\right] = 0$$

from which it shall be clear that all firms that reset their price in period t set it at the same level ($P^\star_t(i) = P^\star_t$, for all $i \in (0,1)$). This implies that

$$P^\star_t = \frac{P^{\mathrm{N}}_t}{P^{\mathrm{D}}_t} \tag{12}$$

where

$$P^n_t = E_t\left[\sum_{j=0}^{\infty}(\beta\xi)^j \Lambda_{t+j}\frac{\theta}{\theta-1}P^{1+\theta}_{t+j}s_{t+j}y_{t+j}\right]$$

and

$$P^d_t = E_t\left[\sum_{j=0}^{\infty}(\beta\xi)^j \Lambda_{t+j}P^{\theta}_{t+j}y_{t+j}\right]$$

Fortunately, both P^{N}_t and P^{D}_t admit a recursive representation, such that

$$P^{\mathrm{N}}_t = \frac{\theta}{\theta-1}\Lambda_t P^{1+\theta}_t s_t y_t + \beta\xi\mathbb{E}_t[P^n_{t+1}] \tag{13}$$

$$P^{\mathrm{D}}_t = \Lambda_t P^{\theta}_t y_t + \beta\xi\mathbb{E}_t[P^d_{t+1}] \tag{14}$$

Recall now that the price index is given by

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta}\mathrm{d}i\right)^{\frac{1}{1-\theta}}$$

In fact it is composed of surviving contracts and newly set prices. Given that in each an every period a price contract has a probability $1-\xi$ of ending, the probability that a contract signed in period $t-j$ survives until period t and ends at the end of period t is given by $(1-\xi)\xi^j$. Therefore, the aggregate price level may be expressed as the average of all surviving contracts

$$P_t = \left(\sum_{j=0}^{\infty}(1-\xi)\xi^j {P^{\star}_{t-j}}^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

which can be expressed recursively as

$$P_t = \left((1-\xi)P_t^{\star 1-\theta} + \xi P_{t-1}^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{15}$$

Note that since the wage rate is common to all firms, the capital labor ratio is the same for any firm:

$$\frac{k_t(i)}{h_t(i)} = \frac{k_t(j)}{h_t(j)} = \frac{k_t}{h_t}$$

we therefore have

$$y_t(i) = a_t\left(\frac{k_t}{h_t}\right)^{\alpha} h_t(i)$$

integrating across firms, we obtain

$$\int_0^1 y_t(i)\mathrm{d}i = a_t\left(\frac{k_t}{h_t}\right)^{\alpha}\int_0^1 h_t(i)\mathrm{d}i$$

denoting $h_t = \int_0^1 h_t(i)\mathrm{d}i$, and making use of the demand for $y_t(i)$, we have

$$\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta}\mathrm{d}i y_t = a_t k_t^{\alpha} h_t^{1-\alpha}$$

Denote

$$\begin{aligned}
\Delta_t &= \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta}\mathrm{d}i \\
&= \sum_{j=0}^{\infty}(1-\xi)\xi^j\left(\frac{P^{\star}_{t-j}}{P_t}\right)^{-\theta} \\
&= (1-\xi)\left(\frac{P^{\star}_t}{P_t}\right)^{-\theta} + \sum_{j=1}^{\infty}(1-\xi)\xi^j\left(\frac{P^{\star}_{t-j}}{P_t}\right)^{-\theta} \\
&= (1-\xi)\left(\frac{P^{\star}_t}{P_t}\right)^{-\theta} + \sum_{\ell=0}^{\infty}(1-\xi)\xi^{\ell+1}\left(\frac{P^{\star}_{t-\ell-1}}{P_t}\right)^{-\theta} \\
&= (1-\xi)\left(\frac{P^{\star}_t}{P_t}\right)^{-\theta} + \xi\left(\frac{P_{t-1}}{P_t}\right)^{-\theta}\sum_{\ell=0}^{\infty}(1-\xi)\xi^{\ell}\left(\frac{P^{\star}_{t-\ell-1}}{P_{t-1}}\right)^{-\theta} \\
&= (1-\xi)\left(\frac{P^{\star}_t}{P_t}\right)^{-\theta} + \xi\left(\frac{P_{t-1}}{P_t}\right)^{-\theta}\Delta_{t-1} \\
\Delta_t &= (1-\xi)\left(\frac{P^{\star}_t}{P_t}\right)^{-\theta} + \xi\pi_t^{\theta}\Delta_{t-1}
\end{aligned}$$

Hence aggregate output is given by

$$\Delta_t y_t = a_t k_t^{\alpha} h_t^{1-\alpha}$$

Hence the set of equations defining the general equilibrium is given by, where $p_t^n = P_t^n / P_t^\theta$, $p_t^d = P_t^d / P_t^{\theta-1}$, $\lambda_t = \Lambda_t P_t$ and $\pi_t = P_t / P_{t-1}$.

$$
\begin{aligned}
\zeta_{c,t}^{\frac{1}{\sigma}} c_t^{-\frac{1}{\sigma}} &= \lambda_t \\
\psi \zeta_{h,t}^{-\nu} h_t^{\nu} &= \lambda_t (1-\alpha) s_t \frac{y_t}{h_t} \\
\lambda_t &= \beta \mathbb{E}_t \left[\lambda_{t+1} \left(\alpha s_{t+1} \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) \right] \\
\lambda_t &= \beta R_t \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right) \\
p_t^{\mathrm{N}} &= \frac{\theta}{\theta - 1} \lambda_t s_t y_t + \beta \xi \mathbb{E}_t [\pi_{t+1}^{\theta} p_{t+1}^{n}] \\
p_t^{\mathrm{D}} &= \lambda_t y_t + \beta \xi \mathbb{E}_t [\pi_{t+1}^{\theta-1} p_{t+1}^{d}] \\
1 &= (1-\xi) \left(\frac{p_t^n}{p_t^d} \right)^{1-\theta} + \xi \pi_t^{\theta-1} \\
\Delta_t &= (1-\xi) \left(\frac{p_t^n}{p_t^d} \right)^{-\theta} + \xi \pi^{\theta} \Delta_{t-1} \\
k_{t+1} &= i_t + (1-\delta) k_t \\
\Delta_t y_t &= a_t k_t^{\alpha} h_t^{1-\alpha} \\
y_t &= c_t + i_t + g_t \\
\log(R_t) &= \rho_r \log(R_{t-1}) + (1-\rho) \big(\log(\overline{R}) + \gamma_\pi (\log(\pi_t) - \log(\overline{\pi})) + \gamma_y (\log(y_t) - \log(\overline{y})) \big)
\end{aligned}
$$

and the definition of the shocks.

4 A Small Open Economy Model with Staggered Prices

The problem of the domestic household is

$$\max \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\zeta_{c,t+j}^{\frac{1}{\sigma}} \frac{c_{t+j}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{\psi}{1+\nu} \zeta_{h,t+j}^{-\nu} h_{t+j}^{1+\nu} \right) \right]$$

$$B_t^h + e_t B_t^f + P_t c_t + P_t i_t = R_{t-1} B_{t-1} + e_t R_{t-1}^{\star} B_{t-1}^f + P_t w_t h_t + P_t z_t k_t - P_t \tau_t - P_t \frac{\chi}{2} (e_t B_t^f)^2$$

$$k_{t+1} = i_t \left(1 - \frac{\varphi_k}{2} \left(\frac{i_t}{k_t} - \delta \right)^2 \right) + (1-\delta) k_t$$

Then the first order conditions are given by

$$\zeta_{c,t}^{\frac{1}{\sigma}} c_t^{-\frac{1}{\sigma}} = \Lambda_t P_t \tag{16}$$

$$\psi \zeta_{h,t}^{-\nu} h_t^{\nu} = \Lambda_t P_t w_t \tag{17}$$

$$\Lambda_t P_t = Q_t \left(1 - \varphi_k \left(\frac{i_t}{k_t} - \delta \right) \right) \tag{18}$$

$$\Lambda_t = \beta R_t \mathbb{E}_t \Lambda_{t+1} \tag{19}$$

$$\Lambda_t (1 + \chi e_t B_t^f) = \beta R_t^{\star} \mathbb{E}_t \frac{e_{t+1}}{e_t} \Lambda_{t+1} \tag{20}$$

$$Q_t = \beta \mathbb{E}_t \left[\Lambda_{t+1} P_{t+1} z_{t+1} + Q_{t+1} \left(1 - \delta + \frac{\varphi_k}{2} \left(\left(\frac{i_{t+1}}{k_{t+1}} \right)^2 - \delta^2 \right) \right) \right] \tag{21}$$

where Λ_t and Q_t denote respectively the Lagrange multiplier of the first and second constraint.

The retailer firm combines foreign and domestic goods to produce a non–tradable final good. It determines its optimal production plans by maximizing its profit

$$\max_{\{x_t^d, x_t^f\}} P_t Y_t - P_{x,t} x_t^d - e_t P_{x,t}^{\star} x_t^f$$

where $P_{x,t}$ and $P_{xt}^{\star}$ denote the price of the domestic and foreign good respectively, denominated in terms of the currency of the *seller*. The final good production function is described by the following CES function

$$y_t = \left(\omega^{\frac{1}{1-\rho}} x_t^{d\rho} + (1-\omega)^{\frac{1}{1-\rho}} x_t^{f\rho} \right)^{\frac{1}{\rho}} \tag{22}$$

where $\omega \in (0,1)$ and $\rho \in (-\infty, 1)$. Optimal behavior of the retailer gives rise to the demand for the domestic and foreign goods

$$x_t^d = \left(\frac{P_{x,t}}{P_t} \right)^{\frac{1}{\rho-1}} \omega y_t \text{ and } x_t^f = \left(\frac{e_t P_{x,t}^{\star}}{P_t} \right)^{\frac{1}{\rho-1}} (1-\omega) y_t \tag{23}$$

x^d and x^f are themselves combinations of the domestic and foreign intermediate goods according to

$$x_t^d = \left(\int_0^1 x_t^d(i)^{\frac{\theta-1}{\theta}} \mathrm{d}i \right)^{\frac{\theta}{\theta-1}} \text{ and } x_t^f = \left(\int_0^1 x_t^f(i)^{\frac{\theta-1}{\theta}} \mathrm{d}i \right)^{\frac{\theta}{\theta-1}} \tag{24}$$

where $\theta \in (-\infty, 1)$.

Profit maximization yields demand functions of the form:

$$x_t^d(i) = \left(\frac{P_{xt}(i)}{P_{xt}}\right)^{-\theta} x_t^d,\; x_t^f(i) = \left(\frac{P^\star_{xt}(i)}{P^\star_{xt}}\right)^{-\theta} x_t^f$$

At this stage, we need to take a stand on the behavior of the foreign firms in order to determine the demand for the domestic good by foreign agents. We assume that their behavior is symmetrical to the one observed in the domestic economy, such

$$x_t^{d\star}(i) = \left(\frac{P_{xt}(i)}{e_t P^\star_{xt}}\right)^{-\theta} x_t^{d\star}$$

Plugging these demand functions in profits and using free entry in the final good sector, we get the following general price indexes

$$P_{xt} = \left(\int_0^1 P_{xt}(i)^{1-\theta}\mathrm{d}i\right)^{\frac{1}{1-\theta}} \tag{25}$$

$$P_t = \left(\omega P_{xt}^{\frac{\rho}{\rho-1}} + (1-\omega)(e_t P^\star_{xt})^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}} \tag{26}$$

$$\tag{27}$$

The intermediate goods are produced by intermediaries, each of which has a local monopoly power. Each intermediate firm i, $i \in (0,1)$, uses a constant returns to scale technology

$$x_t(i) = a_t k_t(i)^\alpha h_t(i)^{1-\alpha} \tag{28}$$

where $k_t(i)$ and $h_t(i)$ denote capital and labor. The firm minimizes its real cost subject to (28). Minimized real total costs are then given by $s_t x_t(i)$ where the real marginal cost, s_t, is given by

$$s_t = \frac{z_t^\alpha w_t^{1-\alpha}}{\varsigma a_t}$$

with $\varsigma = \alpha^\alpha(1-\alpha)^{1-\alpha}$.

The price setting behavior is essentially the same as in a closed economy. The expected profit flow generated by setting $\widetilde{P}_t(i)$ in period t writes

$$\max_{\widetilde{P}_{x,t}(i)} E_t \sum_{j=0}^{\infty} \Phi_{t,t+j}\xi^j \Pi(\widetilde{P}_{x,t}(i))$$

subject to the total demand it faces:

$$x_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} x_t \text{ with } x_t = x_t^d + x_t^{d\star}$$

and where $\Pi(\widetilde{P}_{x,t}(i)) = \left(\widetilde{P}_{x,t}(i) - P_{t+j}s_{t+j}\right) x_{t+j}(i)$. $\Phi_{t,t+j}$ is an appropriate discount factor related to the way the household value future as opposed to current consumption, such that

$$\Phi_{t,t+j} \propto \beta^j \frac{\Lambda_{t+j}}{\Lambda_t}$$

This leads to the price setting equation

$$E_t\left[\sum_{j=0}^{\infty}(\beta\xi)^j\frac{\Lambda_{t+j}}{\Lambda_t}\left((1-\theta)\left(\frac{\widetilde{P}_{x,t}(i)}{P_{x,t+j}}\right)^{-\theta}y_{t+j}+\theta\frac{P_{t+j}}{P_{x,t+j}(i)}\left(\frac{\widetilde{P}_{x,t}(i)}{P_{x,t+j}}\right)^{-\theta}s_{t+j}x_{t+j}\right)\right]=0$$

from which it shall be clear that all firms that reset their price in period t set it at the same level ($\widetilde{P}_t(i)=\widetilde{P}_t$, for all $i\in(0,1)$). This implies that

$$\widetilde{P}_{x,t}=\frac{P^{\mathrm{N}}_{x,t}}{P^{\mathrm{D}}_{x,t}} \tag{29}$$

where

$$P^n_{x,t}=E_t\left[\sum_{j=0}^{\infty}(\beta\xi)^j\Lambda_{t+j}\frac{\theta}{\theta-1}P_{t+j}P^{\theta}_{x,t+j}s_{t+j}x_{t+j}\right]$$

and

$$P^d_{x,t}=E_t\left[\sum_{j=0}^{\infty}(\beta\xi)^j\Lambda_{t+j}P^{\theta}_{x,t+j}x_{t+j}\right]$$

Fortunately, both $P^{\mathrm{N}}_{x,t}$ and $P^{\mathrm{D}}_{x,t}$ admit a recursive representation, such that

$$P^n_{x,t} = \frac{\theta}{\theta-1}\Lambda_t P_t P^{\theta}_{x,t}s_t x_t+\beta\xi\mathbb{E}_t[P^n_{x,t+1}] \tag{30}$$

$$P^d_{x,t} = \Lambda_t P^{\theta}_{x,t}x_t+\beta\xi\mathbb{E}_t[P^d_{x,t+1}] \tag{31}$$

Recall now that the price index is given by

$$P_{x,t}=\left(\int_0^1 P_{x,t}(i)^{1-\theta}\mathrm{d}i\right)^{\frac{1}{1-\theta}}$$

In fact it is composed of surviving contracts and newly set prices. Given that in each an every period a price contract has a probability $1-\xi$ of ending, the probability that a contract signed in period $t-j$ survives until period t and ends at the end of period t is given by $(1-\xi)\xi^j$. Therefore, the aggregate price level may be expressed as the average of all surviving contracts

$$P_{x,t}=\left(\sum_{j=0}^{\infty}(1-\xi)\xi^j\widetilde{P}^{1-\theta}_{x,t-j}\right)^{\frac{1}{1-\theta}}$$

which can be expressed recursively as

$$P_{x,t}=\left((1-\xi)\widetilde{P}^{1-\theta}_{x,t}+\xi P^{1-\theta}_{x,t-1}\right)^{\frac{1}{1-\theta}} \tag{32}$$

Note that since the wage rate is common to all firms, the capital labor ratio is the same for any firm:

$$\frac{k_t(i)}{h_t(i)}=\frac{k_t(j)}{h_t(j)}=\frac{k_t}{h_t}$$

we therefore have

$$x_t(i)=a_t\left(\frac{k_t}{h_t}\right)^{\alpha}h_t(i)$$

integrating across firms, we obtain

$$\int_0^1 x_t(i)\mathrm{d}i = a_t \left(\frac{k_t}{h_t}\right)^{\alpha} \int_0^1 h_t(i)\mathrm{d}i$$

denoting $h_t = \int_0^1 h_t(i)\mathrm{d}i$, and making use of the demand for $y_t(i)$, we have

$$\int_0^1 \left(\frac{P_{x,t}(i)}{P_{x,t}}\right)^{-\theta} \mathrm{d}i y_t = a_t k_t^{\alpha} h_t^{1-\alpha}$$

Denote

$$\begin{aligned}
\Delta_t &= \int_0^1 \left(\frac{P_{x,t}(i)}{P_{x,t}}\right)^{-\theta} \mathrm{d}i \\
&= \sum_{j=0}^{\infty} (1-\xi)\xi^j \left(\frac{\widetilde{P}_{x,t-j}}{P_{x,t}}\right)^{-\theta} \\
&= (1-\xi)\left(\frac{\widetilde{P}_{x,t}}{P_{x,t}}\right)^{-\theta} + \sum_{j=1}^{\infty} (1-\xi)\xi^j \left(\frac{\widetilde{P}_{x,t-j}}{P_{x,t}}\right)^{-\theta} \\
&= (1-\xi)\left(\frac{\widetilde{P}_{x,t}}{P_{x,t}}\right)^{-\theta} + \sum_{\ell=0}^{\infty} (1-\xi)\xi^{\ell+1} \left(\frac{\widetilde{P}_{x,t-\ell-1}}{P_{x,t}}\right)^{-\theta} \\
&= (1-\xi)\left(\frac{\widetilde{P}_{x,t}}{P_{x,t}}\right)^{-\theta} + \xi \left(\frac{P_{x,t-1}}{P_{x,t}}\right)^{-\theta} \sum_{\ell=0}^{\infty} (1-\xi)\xi^{\ell} \left(\frac{\widetilde{P}_{x,t-\ell-1}}{P_{x,t-1}}\right)^{-\theta} \\
&= (1-\xi)\left(\frac{\widetilde{P}_{x,t}}{P_{x,t}}\right)^{-\theta} + \xi \left(\frac{P_{x,t-1}}{P_{x,t}}\right)^{-\theta} \Delta_{t-1} \\
\Delta_t &= (1-\xi)\left(\frac{\widetilde{P}_{x,t}}{P_{x,t}}\right)^{-\theta} + \xi \pi_{x,t}^{\theta} \Delta_{t-1}
\end{aligned}$$

Hence aggregate output is given by

$$\Delta_t x_t = a_t k_t^{\alpha} h_t^{1-\alpha}$$

The behavior of the foreign economy is assumed to be very similar to the one observed domestically. We however assumethat foreign output and prices are exogenously given and modelled as AR(1) processes. We further assume that $P^{\star}_{x,t} = P^{\star}_t$. The foreign household's saving behavior is the same as in the domestic economy (absent preference shocks), such that the foreign nomina interest rate is given by

$$y_t^{\star -\frac{1}{\sigma}} = \beta R_t^{\star} \mathbb{E}_t y_{t+1}^{\star}{}^{-\frac{1}{\sigma}} \frac{P_t^{\star}}{P_{t+1}^{\star}}$$

We assume that foreign households do not buy domestic bonds, which implies that in equilibrium $B_t^d = 0$.

The general equilibrium is then given by

$$\zeta_{c,t}^{\frac{1}{\sigma}} c_t^{-\frac{1}{\sigma}} = \lambda_t \tag{33}$$

$$\psi \zeta_{h,t}^{-\nu} h_t^{\nu} = \lambda_t (1-\alpha) s_t \frac{p_{x,t} x_t}{h_t} \tag{34}$$

$$\lambda_t = q_t \left(1 - \varphi_k \left(\frac{i_t}{k_t} - \delta\right)\right) \tag{35}$$

$$y_t = c_t + i_t + g_t + \frac{\chi}{2} b_t^{f2} \tag{36}$$

$$\Delta_t x_t = a_t k_t^{\alpha} h_t^{1-\alpha} \tag{37}$$

$$x_t^d = p_{x,t}^{\frac{1}{\rho-1}} \omega y_t \tag{38}$$

$$x_t^{d\star} = \left(\frac{p_{x,t}}{rer_t}\right)^{\frac{1}{\rho-1}} (1-\omega) y_t^{\star} \tag{39}$$

$$x_t^f = (rer_t p_{x,t}^{\star})^{\frac{1}{\rho-1}} (1-\omega) y_t \tag{40}$$

$$x_t = x_t^d + x_t^{d\star} \tag{41}$$

$$1 = \omega p_{x,t}^{\frac{\rho}{\rho-1}} + (1-\omega)(rer_t p_t^{\star})^{\frac{\rho}{\rho-1}} \tag{42}$$

$$\lambda_t = \beta R_t \mathbb{E}_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{43}$$

$$\lambda_t (1 + \chi b_t^f) = \beta R_t^{\star} \mathbb{E}_t \frac{\Delta_{t+1}^e}{\pi_{t+1}} \lambda_{t+1} \tag{44}$$

$$q_t = \beta \mathbb{E}_t \left[\lambda_{t+1} \alpha s_{t+1} \frac{p_{x,t+1} x_{t+1}}{k_{t+1}} + q_{t+1} \left(1 - \delta + \frac{\varphi_k}{2} \left(\left(\frac{i_{t+1}}{k_{t+1}}\right)^2 - \delta^2\right)\right)\right] \tag{45}$$

$$b_t^f = \frac{\Delta_t^e}{\pi_t} R_{t-1}^{\star} b_{t-1}^f + p_{x,t} x_t - y_t \tag{46}$$

$$k_{t+1} = i_t \left(1 - \frac{\varphi_k}{2} \left(\frac{i_t}{k_t} - \delta\right)^2\right) + (1-\delta) k_t \tag{47}$$

$$rer_t = \frac{\Delta_t^e p_t^{\star}}{\pi_t p_{t-1}^{\star}} rer_{t-1} \tag{48}$$

$$p_{x,t} = \frac{\pi_{x,t}}{\pi_t} p_{x,t-1} \tag{49}$$

$$\log(R_t) = \rho_r \log(R_{t-1}) + (1-\rho) \left(\log(\overline{R}) + \gamma_\pi (\log(\pi_t) - \log(\overline{\pi})) + \gamma_y (\log(y_t) - \log(\overline{y}))\right) \tag{50}$$

$$p_{x,t}^n = \frac{\theta}{\theta - 1} \lambda_t p_{x,t}^{\theta} s_t x_t + \beta \xi \mathbb{E}_t [p_{x,t+1}^n \pi_{t+1}^{\theta}] \tag{51}$$

$$p_{x,t}^d = \lambda_t p_{x,t}^{\theta} x_t + \beta \xi \mathbb{E}_t [p_{x,t+1}^d \pi_{t+1}^{\theta-1}] \tag{52}$$

$$p_{x,t} = \left((1-\xi) \left(\frac{p_{x,t}^n}{p_{x,t}^d}\right)^{1-\theta} + \xi \left(\frac{p_{x,t-1}}{\pi_t}\right)^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{53}$$

$$\Delta_t = (1-\xi) \left(\frac{p_{x,t}^n}{p_{x,t} p_{x,t}^d}\right)^{-\theta} + \xi \Delta_{t-1} \pi_{x,t}^{\theta} \tag{54}$$

$$y_t^{\star -\frac{1}{\sigma}} = \beta R^{\star} \mathbb{E}_t y_{t+1}^{\star}{}^{-\frac{1}{\sigma}} \frac{p_t^{\star}}{p_{t+1}^{\star}} \tag{55}$$

where $\lambda_t = \Lambda_t P_t$, $rer_t = e_t P_t^{\star}/P_t$, $p_{x,t} = P_{x,t}/P_t$, $p_t^n = P_t^n/P_t^{\theta}$, $p_t^d = P_t^d/P_t^{\theta-1}$, $\pi_t = P_t/P_{t-1}$,

$\pi_{x,t} = P_{x,t}/P_{x,t-1}$, $\Delta_t^e = e_t/e_{t-1}$.

All shocks follow AR(1) processes of the type

$$\begin{aligned}
\log(a_{t+1}) &= \rho_a \log(a_t) + \varepsilon_{a,t+1} \\
\log(g_{t+1}) &= \rho_g \log(g_t) + (1-\rho_g)\log(\overline{g}) + \varepsilon_{g,t+1} \\
\log(\zeta_{c,t+1}) &= \rho_c \log(\zeta_{c,t}) + \varepsilon_{c,t+1} \\
\log(\zeta_{h,t+1}) &= \rho_h \log(\zeta_{h,t}) + \varepsilon_{h,t+1} \\
\log(y^\star_{t+1}) &= \rho_y \log(y^\star_t) + (1-\rho_y)\log(\overline{y}) + \varepsilon^\star_{y,t+1} \\
\log(p^\star_{t+1}) &= \rho_p \log(p^\star_t) + (1-\rho_p)\log(\overline{p}) + \varepsilon^\star_{p,t+1}
\end{aligned}$$

5 A Nominal Small Open Economy Model with Price Adjustment Costs

When price contracts are replaced with price adjustment costs, equations (36)–(37) become

$$\begin{aligned}
y_t &= c_t + i_t + g_t + \frac{\chi}{2} b_t^{f2} + \frac{\varphi_p}{2}(\pi_{x,t}-1)^2 y_t \\
x_t &= a_t k_t^\alpha h_t^{1-\alpha}
\end{aligned}$$

and equations (51)–(54) are replaced with

$$(1-\theta)p_{x,t}x_t + \theta s_t x_t - \varphi_p \pi_{x,t}(\pi_{x,t}-1)y_t + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\pi_{x,t+1}\varphi_p(\pi_{x,t+1}-1)y_{t+1}\right] = 0$$

6 A Real Small Open Economy Model

All nominal aspects disappear, such that the general equilibrium becomes

$$\zeta_{c,t}^{\frac{1}{\sigma}} c_t^{-\frac{1}{\sigma}} = \lambda_t \tag{56}$$

$$\psi \zeta_{h,t}^{-\nu} h_t^\nu = \lambda_t(1-\alpha)\frac{p_{x,t}x_t}{h_t} \tag{57}$$

$$\lambda_t = q_t\left(1 - \varphi_k\left(\frac{i_t}{k_t} - \delta\right)\right) \tag{58}$$

$$y_t = c_t + i_t + g_t + \frac{\chi}{2} b_t^{f2} \tag{59}$$

$$x_t = a_t k_t^\alpha h_t^{1-\alpha} \tag{60}$$

$$x_t^d = p_{x,t}^{\frac{1}{\rho-1}} \omega y_t \tag{61}$$

$$x_t^{d\star} = \left(\frac{p_{x,t}}{rer_t}\right)^{\frac{1}{\rho-1}} (1-\omega) y_t^\star \tag{62}$$

$$x_t^f = (rer_t p^\star_{x,t})^{\frac{1}{\rho-1}}(1-\omega)y_t \tag{63}$$

$$x_t = x_t^d + x_t^{d\star} \tag{64}$$

$$1 = \omega p_{x,t}^{\frac{\rho}{\rho-1}} + (1-\omega)(rer_t p^\star_t)^{\frac{\rho}{\rho-1}} \tag{65}$$

$$\lambda_t = \beta R_t \mathbb{E}_t \lambda_{t+1} \tag{66}$$

$$\lambda_t(1+\chi b_t^f) = \beta R_t^\star \mathbb{E}_t \lambda_{t+1} \tag{67}$$

$$q_t = \beta \mathbb{E}_t \left[\lambda_{t+1} \alpha \frac{p_{x,t+1} x_{t+1}}{k_{t+1}} + q_{t+1} \left(1 - \delta + \frac{\varphi_k}{2} \left(\left(\frac{i_{t+1}}{k_{t+1}} \right)^2 - \delta^2 \right) \right) \right] \tag{68}$$

$$b_t^f = R_{t-1}^\star b_{t-1}^f + p_{x,t} x_t - y_t \tag{69}$$

$$k_{t+1} = i_t \left(1 - \frac{\varphi_k}{2} \left(\frac{i_t}{k_t} - \delta \right)^2 \right) + (1-\delta) k_t \tag{70}$$

$${y_t^\star}^{-\frac{1}{\sigma}} = \beta R_t^\star \mathbb{E}_t {y_{t+1}^\star}^{-\frac{1}{\sigma}} \tag{71}$$

together with the shocks.

7 A Nominal 2–Country Model with Staggered Prices

The problem of the domestic household is

$$\max \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\zeta_{c,t+j}^{\frac{1}{\sigma}} \frac{c_{t+j}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{\psi}{1+\nu} \zeta_{h,t+j}^{-\nu} h_{t+j}^{1+\nu} \right) \right]$$

$$B_t^h + e_t B_t^f + P_t c_t + P_t i_t = R_{t-1} B_{t-1} + e_t R_{t-1}^{\star} B_{t-1}^f + P_t w_t h_t + P_t z_t k_t - P_t \tau_t$$

$$k_{t+1} = i_t \left(1 - \frac{\varphi_k}{2} \left(\frac{i_t}{k_t} - \delta \right)^2 \right) + (1-\delta) k_t$$

Then the first order conditions are given by

$$\zeta_{c,t}^{\frac{1}{\sigma}} c_t^{-\frac{1}{\sigma}} = \Lambda_t P_t \tag{72}$$

$$\psi \zeta_{h,t}^{-\nu} h_t^{\nu} = \Lambda_t P_t w_t \tag{73}$$

$$\Lambda_t P_t = Q_t \left(1 - \varphi_k \left(\frac{i_t}{k_t} - \delta \right) \right) \tag{74}$$

$$\Lambda_t = \beta R_t \mathbb{E}_t \Lambda_{t+1} \tag{75}$$

$$\Lambda_t = \beta R_t^{\star} \mathbb{E}_t \frac{e_{t+1}}{e_t} \Lambda_{t+1} \tag{76}$$

$$Q_t = \beta \mathbb{E}_t \left[\Lambda_{t+1} P_{t+1} z_{t+1} + Q_{t+1} \left(1 - \delta + \frac{\varphi_k}{2} \left(\left(\frac{i_{t+1}}{k_{t+1}} \right)^2 - \delta^2 \right) \right) \right] \tag{77}$$

where Λ_t and Q_t denote respectively the Lagrange multiplier of the first and second constraint. The behavior of the foreign household, hereafter denoted by a $\star$, is totally symmetrical. We further have the following risk sharing condition

$$\Lambda_t = \frac{\Lambda_t^{\star}}{e_t}$$

The retailer firm combines foreign and domestic goods to produce a non–tradable final good. It determines its optimal production plans by maximizing its profit

$$\max_{\{x_t^d, x_t^f\}} P_t Y_t - P_{x,t} x_t^d - e_t P_{x,t}^{\star} x_t^f$$

where $P_{x,t}$ and $P_{xt}^{\star}$ denote the price of the domestic and foreign good respectively, denominated in terms of the currency of the *seller*. The final good production function is described by the following CES function

$$y_t = \left(\omega^{\frac{1}{1-\rho}} x_t^{d\rho} + (1-\omega)^{\frac{1}{1-\rho}} x_t^{f\rho} \right)^{\frac{1}{\rho}} \tag{78}$$

where $\omega \in (0,1)$ and $\rho \in (-\infty, 1)$. Optimal behavior of the retailer gives rise to the demand for the domestic and foreign goods

$$x_t^d = \left(\frac{P_{x,t}}{P_t} \right)^{\frac{1}{\rho-1}} \omega y_t \text{ and } x_t^f = \left(\frac{e_t P_{x,t}^{\star}}{P_t} \right)^{\frac{1}{\rho-1}} (1-\omega) y_t \tag{79}$$

Abroad, the behavior is symmetrical, such that

$$y_t^{\star} = \left(\omega^{\frac{1}{1-\rho}} x_t^{f\star\rho} + (1-\omega)^{\frac{1}{1-\rho}} x_t^{d\star\rho} \right)^{\frac{1}{\rho}} \tag{80}$$

and

$$x_t^{d\star} = \left(\frac{P_{x,t}}{e_t P_t^\star}\right)^{\frac{1}{\rho-1}} (1-\omega) y_t^\star \text{ and } x_t^{f\star} = \left(\frac{P_{x,t}^\star}{P_t^\star}\right)^{\frac{1}{\rho-1}} \omega y_t^\star \tag{81}$$

x^d and x^f are themselves combinations of the domestic and foreign intermediate goods according to

$$x_t^d = \left(\int_0^1 x_t^d(i)^{\frac{\theta-1}{\theta}} \mathrm{d}i\right)^{\frac{\theta}{\theta-1}} \text{ and } x_t^f = \left(\int_0^1 x_t^f(i)^{\frac{\theta-1}{\theta}} \mathrm{d}i\right)^{\frac{\theta}{\theta-1}} \tag{82}$$

where $\theta \in (-\infty, 1)$. Likewise abroad

$$x_t^{d\star} = \left(\int_0^1 x_t^{d\star}(i)^{\frac{\theta-1}{\theta}} \mathrm{d}i\right)^{\frac{\theta}{\theta-1}} \text{ and } x_t^{f\star} = \left(\int_0^1 x_t^{f\star}(i)^{\frac{\theta-1}{\theta}} \mathrm{d}i\right)^{\frac{\theta}{\theta-1}} \tag{83}$$

Profit maximization yields demand functions of the form:

$$x_t^d(i) = \left(\frac{P_{xt}(i)}{P_{xt}}\right)^{-\theta} x_t^d,\ x_t^f(i) = \left(\frac{P_{xt}^\star(i)}{P_{xt}^\star}\right)^{-\theta} x_t^f,$$

similarly abroad

$$x_t^{d\star}(i) = \left(\frac{P_{xt}(i)}{P_{xt}}\right)^{-\theta} x_t^{d\star},\ x_t^{f\star}(i) = \left(\frac{P_{xt}^\star(i)}{P_{xt}^\star}\right)^{-\theta} x_t^{f\star}$$

Plugging these demand functions in profits and using free entry in the final good sector, we get the following general price indexes

$$P_{xt} = \left(\int_0^1 P_{xt}(i)^{1-\theta} \mathrm{d}i\right)^{\frac{1}{1-\theta}},\ P_{xt}^\star = \left(\int_0^1 P_{xt}^\star(i)^{1-\theta} \mathrm{d}i\right)^{\frac{1}{1-\theta}} \tag{84}$$

$$P_t = \left(\omega P_{xt}^{\frac{\rho}{\rho-1}} + (1-\omega)(e_t P_{xt}^\star)^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}} \tag{85}$$

$$P_t^\star = \left(\omega \left(\frac{P_{xt}}{e_t}\right)^{\frac{\rho}{\rho-1}} + (1-\omega) P_{xt}^{\star\,\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}} \tag{86}$$

The intermediate goods are produced by intermediaries, each of which has a local monopoly power. Each intermediate firm i, $i \in (0,1)$, uses a constant returns to scale technology

$$x_t(i) = a_t k_t(i)^\alpha h_t(i)^{1-\alpha} \tag{87}$$

where $k_t(i)$ and $h_t(i)$ denote capital and labor. The firm minimizes its real cost subject to (87). Minimized real total costs are then given by $s_t x_t(i)$ where the real marginal cost, s_t, is given by

$$s_t = \frac{z_t^\alpha w_t^{1-\alpha}}{\varsigma a_t}$$

with $\varsigma = \alpha^\alpha (1-\alpha)^{1-\alpha}$. Similarly abroad

$$x_t^\star(i) = a_t^\star k_t^\star(i)^\alpha h_t^\star(i)^{1-\alpha} \tag{88}$$

where $k_t^\star(i)$ and $h_t^\star(i)$ denote capital and labor. The firm minimizes its real cost subject to (87). Minimized real total costs are then given by $s_t^\star x_t^\star(i)$ where the real marginal cost, $s_t^\star$, is given by

$$s_t^\star = \frac{z_t^{\star\alpha} w_t^{\star 1-\alpha}}{\varsigma a_t^\star}$$

with $\varsigma = \alpha^\alpha(1-\alpha)^{1-\alpha}$.

The price setting behavior is essentially the same as in a closed economy. The expected profit flow generated by setting $\widetilde{P}_t(i)$ in period t writes

$$\max_{\widetilde{P}_{x,t}(i)} E_t \sum_{j=0}^{\infty} \Phi_{t,t+j}\xi^j \Pi(\widetilde{P}_{x,t}(i))$$

subject to the total demand it faces:

$$x_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} x_t \text{ with } x_t = x_t^d + x_t^{d\star}$$

and where $\Pi(\widetilde{P}_{x,t}(i)) = \left(\widetilde{P}_{x,t}(i) - P_{t+j}s_{t+j}\right) x_{t+j}(i)$. $\Phi_{t,t+j}$ is an appropriate discount factor related to the way the household value future as opposed to current consumption, such that

$$\Phi_{t,t+j} \propto \beta^j \frac{\Lambda_{t+j}}{\Lambda_t}$$

This leads to the price setting equation

$$E_t\left[\sum_{j=0}^{\infty}(\beta\xi)^j \frac{\Lambda_{t+j}}{\Lambda_t}\left((1-\theta)\left(\frac{\widetilde{P}_{x,t}(i)}{P_{x,t+j}}\right)^{-\theta} y_{t+j} + \theta \frac{P_{t+j}}{P_{x,t+j}(i)}\left(\frac{\widetilde{P}_{x,t}(i)}{P_{x,t+j}}\right)^{-\theta} s_{t+j}x_{t+j}\right)\right] = 0$$

from which it shall be clear that all firms that reset their price in period t set it at the same level ($\widetilde{P}_t(i) = \widetilde{P}_t$, for all $i \in (0,1)$). This implies that

$$\widetilde{P}_{x,t} = \frac{P_{x,t}^{\mathrm{N}}}{P_{x,t}^{\mathrm{D}}} \tag{89}$$

where

$$P_{x,t}^n = E_t\left[\sum_{j=0}^{\infty}(\beta\xi)^j \Lambda_{t+j}\frac{\theta}{\theta-1}P_{t+j}P_{x,t+j}^\theta s_{t+j}x_{t+j}\right]$$

and

$$P_{x,t}^d = E_t\left[\sum_{j=0}^{\infty}(\beta\xi)^j \Lambda_{t+j}P_{x,t+j}^\theta x_{t+j}\right]$$

Fortunately, both $P_{x,t}^{\mathrm{N}}$ and $P_{x,t}^{\mathrm{D}}$ admit a recursive representation, such that

$$P_{x,t}^n = \frac{\theta}{\theta-1}\Lambda_t P_t P_{x,t}^\theta s_t x_t + \beta\xi\mathbb{E}_t[P_{x,t+1}^n] \tag{90}$$

$$P_{x,t}^d = \Lambda_t P_{x,t}^\theta x_t + \beta\xi\mathbb{E}_t[P_{x,t+1}^d] \tag{91}$$

Likewise abroad,

$$P_{x,t}^{n\star} = \frac{\theta}{\theta-1}\Lambda_t^\star P_t^\star {P_{x,t}^\star}^\theta s_t^\star x_t^\star + \beta\xi\mathbb{E}_t[P_{x,t+1}^{n\star}] \quad (92)$$

$$P_{x,t}^{d\star} = \Lambda_t^\star {P_{x,t}^\star}^\theta x_t^\star + \beta\xi\mathbb{E}_t[P_{x,t+1}^{d\star}] \quad (93)$$

Recall now that the price index is given by

$$P_{x,t} = \left(\int_0^1 P_{x,t}(i)^{1-\theta}\mathrm{d}i\right)^{\frac{1}{1-\theta}}$$

In fact it is composed of surviving contracts and newly set prices. Given that in each an every period a price contract has a probability $1-\xi$ of ending, the probability that a contract signed in period $t-j$ survives until period t and ends at the end of period t is given by $(1-\xi)\xi^j$. Therefore, the aggregate price level may be expressed as the average of all surviving contracts

$$P_{x,t} = \left(\sum_{j=0}^{\infty}(1-\xi)\xi^j \widetilde{P}_{x,t-j}^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

which can be expressed recursively as

$$P_{x,t} = \left((1-\xi)\widetilde{P}_{x,t}^{1-\theta} + \xi P_{x,t-1}^{1-\theta}\right)^{\frac{1}{1-\theta}} \quad (94)$$

Note that since the wage rate is common to all firms, the capital labor ratio is the same for any firm:

$$\frac{k_t(i)}{h_t(i)} = \frac{k_t(j)}{h_t(j)} = \frac{k_t}{h_t}$$

we therefore have

$$x_t(i) = a_t\left(\frac{k_t}{h_t}\right)^\alpha h_t(i)$$

integrating across firms, we obtain

$$\int_0^1 x_t(i)\mathrm{d}i = a_t\left(\frac{k_t}{h_t}\right)^\alpha \int_0^1 h_t(i)\mathrm{d}i$$

denoting $h_t = \int_0^1 h_t(i)\mathrm{d}i$, and making use of the demand for $y_t(i)$, we have

$$\int_0^1 \left(\frac{P_{x,t}(i)}{P_{x,t}}\right)^{-\theta}\mathrm{d}i y_t = a_t k_t^\alpha h_t^{1-\alpha}$$

Denote

$$\begin{aligned}
\Delta_t &= \int_0^1 \left(\frac{P_{x,t}(i)}{P_{x,t}}\right)^{-\theta} \mathrm{d}i \\
&= \sum_{j=0}^{\infty}(1-\xi)\xi^j \left(\frac{\widetilde{P}_{x,t-j}}{P_{x,t}}\right)^{-\theta} \\
&= (1-\xi)\left(\frac{\widetilde{P}_{x,t}}{P_{x,t}}\right)^{-\theta} + \sum_{j=1}^{\infty}(1-\xi)\xi^j \left(\frac{\widetilde{P}_{x,t-j}}{P_{x,t}}\right)^{-\theta} \\
&= (1-\xi)\left(\frac{\widetilde{P}_{x,t}}{P_{x,t}}\right)^{-\theta} + \sum_{\ell=0}^{\infty}(1-\xi)\xi^{\ell+1} \left(\frac{\widetilde{P}_{x,t-\ell-1}}{P_{x,t}}\right)^{-\theta} \\
&= (1-\xi)\left(\frac{\widetilde{P}_{x,t}}{P_{x,t}}\right)^{-\theta} + \xi\left(\frac{P_{x,t-1}}{P_{x,t}}\right)^{-\theta} \sum_{\ell=0}^{\infty}(1-\xi)\xi^{\ell} \left(\frac{\widetilde{P}_{x,t-\ell-1}}{P_{x,t-1}}\right)^{-\theta} \\
&= (1-\xi)\left(\frac{\widetilde{P}_{x,t}}{P_{x,t}}\right)^{-\theta} + \xi\left(\frac{P_{x,t-1}}{P_{x,t}}\right)^{-\theta} \Delta_{t-1}
\end{aligned}$$

therefore

$$\Delta_t = (1-\xi)\left(\frac{\widetilde{P}_{x,t}}{P_{x,t}}\right)^{-\theta} + \xi\pi_{x,t}^{\theta}\Delta_{t-1}$$

Hence aggregate output is given by

$$\Delta_t x_t = a_t k_t^{\alpha} h_t^{1-\alpha}$$

In a general equilibrium, we will have $B_t^d + B_t^{d\star} = 0$ and $B_t^f + B_t^{f\star} = 0$. The general equilibrium is then given by

$$\zeta_{c,t}^{\frac{1}{\sigma}} c_t^{-\frac{1}{\sigma}} = \lambda_t \tag{95}$$

$$\zeta_{c,t}^{\star\,\frac{1}{\sigma}} c_t^{\star-\frac{1}{\sigma}} = \lambda_t^\star \tag{96}$$

$$\psi\zeta_{h,t}^{-\nu} h_t^{\nu} = \lambda_t(1-\alpha)p_{x,t}s_t\frac{x_t}{h_t} \tag{97}$$

$$\psi\zeta_{h,t}^{\star\,-\nu} h_t^{\star\nu} = \lambda_t^\star(1-\alpha)p_{x,t}^\star s_t^\star\frac{x_t^\star}{h_t^\star} \tag{98}$$

$$\lambda_t = q_t\left(1-\varphi_k\left(\frac{i_t}{k_t}-\delta\right)\right) \tag{99}$$

$$\lambda_t^\star = q_t^\star\left(1-\varphi_k\left(\frac{i_t^\star}{k_t^\star}-\delta\right)\right) \tag{100}$$

$$y_t = c_t + i_t + g_t \tag{101}$$

$$y_t^\star = c_t^\star + i_t^\star + g_t^\star \tag{102}$$

$$\Delta_t x_t = a_t k_t^{\alpha} h_t^{1-\alpha} \tag{103}$$

$$\Delta_t^\star x_t^\star = a_t^\star k_t^{\star\alpha} h_t^{\star 1-\alpha} \tag{104}$$

$$x_t^d = p_{x,t}^{\frac{1}{\rho-1}} \omega y_t \tag{105}$$

$$x_t^{d\star} = \left(\frac{p_{x,t}}{rer_t}\right)^{\frac{1}{\rho-1}} (1-\omega) y_t^\star \tag{106}$$

$$x_t^f = (rer_t p_{x,t}^\star)^{\frac{1}{\rho-1}} (1-\omega) y_t \tag{107}$$

$$x_t^{f\star} = {p_{x,t}^\star}^{\frac{1}{\rho-1}} \omega y_t^\star \tag{108}$$

$$x_t = x_t^d + x_t^{d\star} \tag{109}$$

$$x_t^\star = x_t^f + x_t^{f\star} \tag{110}$$

$$1 = \omega p_{x,t}^{\frac{\rho}{\rho-1}} + (1-\omega)(rer_t p_{x,t}^\star)^{\frac{\rho}{\rho-1}} \tag{111}$$

$$1 = \omega {p_{x,t}^\star}^{\frac{\rho}{\rho-1}} + (1-\omega) \left(\frac{p_{x,t}^\star}{rer_t}\right)^{\frac{\rho}{\rho-1}} \tag{112}$$

$$\lambda_t = \frac{\lambda_t^\star}{rer_t} \tag{113}$$

$$\lambda_t = \beta R_t \mathbb{E}_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{114}$$

$$q_t = \beta \mathbb{E}_t \left[\lambda_{t+1} \alpha s_{t+1} \frac{p_{x,t+1} x_{t+1}}{k_{t+1}} + q_{t+1} \left(1 - \delta + \frac{\varphi_k}{2} \left(\left(\frac{i_{t+1}}{k_{t+1}}\right)^2 - \delta^2\right)\right)\right] \tag{115}$$

$$q_t^\star = \beta \mathbb{E}_t \left[\lambda_{t+1}^\star \alpha s_{t+1}^\star \frac{p_{x,t+1}^\star x_{t+1}^\star}{k_{t+1}^\star} + q_{t+1}^\star \left(1 - \delta + \frac{\varphi_k}{2} \left(\left(\frac{i_{t+1}^\star}{k_{t+1}^\star}\right)^2 - \delta^2\right)\right)\right] \tag{116}$$

$$k_{t+1} = i_t \left(1 - \frac{\varphi_k}{2} \left(\frac{i_t}{k_t} - \delta\right)^2\right) + (1-\delta) k_t \tag{117}$$

$$k_{t+1}^\star = i_t^\star \left(1 - \frac{\varphi_k}{2} \left(\frac{i_t^\star}{k_t^\star} - \delta\right)^2\right) + (1-\delta) k_t^\star \tag{118}$$

$$R_t = R_t^\star \mathbb{E}_t \Delta_{t+1}^e \tag{119}$$

$$rer_t = \frac{\Delta_t^e \pi_t^\star}{\pi_t} rer_{t-1} \tag{120}$$

$$p_{x,t} = \frac{\pi_{x,t}}{\pi_t} p_{x,t-1} \tag{121}$$

$$p_{x,t}^\star = \frac{\pi_{x,t}^\star}{\pi_t^\star} p_{x,t-1}^\star \tag{122}$$

$$\log(R_t) = \rho_r \log(R_{t-1}) + (1-\rho)\left(\log(\overline{R}) + \gamma_\pi(\log(\pi_t) - \log(\overline{\pi})) + \gamma_y(\log(y_t) - \log(\overline{y}))\right) \tag{123}$$

$$\log(R_t^\star) = \rho_r \log(R_{t-1}^\star) + (1-\rho)\left(\log(\overline{R}) + \gamma_\pi(\log(\pi_t^\star) - \log(\overline{\pi})) + \gamma_y(\log(y_t^\star) - \log(\overline{y}))\right) \tag{124}$$

$$p_{x,t}^n = \frac{\theta}{\theta-1} \lambda_t p_{x,t}^\theta s_t x_t + \beta\xi \mathbb{E}_t[p_{x,t+1}^n \pi_{t+1}^\theta] \tag{125}$$

$$p_{x,t}^{n\star} = \frac{\theta}{\theta-1} \lambda_t^\star {p_{x,t}^\star}^\theta s_t^\star x_t^\star + \beta\xi \mathbb{E}_t[p_{x,t+1}^{n\star} \pi_{t+1}^{\star\theta}] \tag{126}$$

$$p_{x,t}^d = \lambda_t p_{x,t}^\theta x_t + \beta\xi \mathbb{E}_t[p_{x,t+1}^d \pi_{t+1}^{\theta-1}] \tag{127}$$

$$p_{x,t}^{d\star} = \lambda_t^\star {p_{x,t}^\star}^\theta x_t^\star + \beta\xi \mathbb{E}_t[p_{x,t+1}^{d\star} \pi_{x,t+1}^{\star\theta-1}] \tag{128}$$

$$p_{x,t} = \left((1-\xi) \left(\frac{p^n_{x,t}}{p^d_{x,t}} \right)^{1-\theta} + \xi \left(\frac{p_{x,t-1}}{\pi_t} \right)^{1-\theta} \right)^{\frac{1}{1-\theta}} \tag{129}$$

$$p^\star_{x,t} = \left((1-\xi) \left(\frac{p^{n\star}_{x,t}}{p^{d\star}_{x,t}} \right)^{1-\theta} + \xi \left(\frac{p^\star_{x,t-1}}{\pi^\star_t} \right)^{1-\theta} \right)^{\frac{1}{1-\theta}} \tag{130}$$

$$\Delta_t = (1-\xi) \left(\frac{p^n_{x,t}}{p_{x,t} p^d_{x,t}} \right)^{-\theta} + \xi \Delta_{t-1} \pi^\theta_{x,t} \tag{131}$$

$$\Delta^\star_t = (1-\xi) \left(\frac{p^{n\star}_{x,t}}{p^\star_{x,t} p^{d\star}_{x,t}} \right)^{-\theta} + \xi \Delta^\star_{t-1} {\pi^\star_{x,t}}^\theta \tag{132}$$

where $\lambda_t = \Lambda_t P_t$, $\lambda^\star_t = \Lambda^\star_t P^\star_t$, $rer_t = e_t P^\star_t / P_t$, $p_{x,t} = P_{x,t}/P_t$, $p^\star_{x,t} = P^\star_{x,t}/P^\star_t$, $p^n_t = P^n_t / P^\theta_t$, $p^d_t = P^d_t / P^{\theta-1}_t$, $p^{n\star}_t = P^{n\star}_t / P^{\star\theta}_t$, $p^{d\star}_t = P^{d\star}_t / P^{\star\theta-1}_t$, $\pi_t = P_t/P_{t-1}$, $\pi^\star_t = P^\star_t / P^\star_{t-1}$, $\pi_{x,t} = P_{x,t}/P_{x,t-1}$, $\pi^\star_{x,t} = P^\star_{x,t}/P^\star_{x,t-1}$, $\Delta^e_t = e_t/e_{t-1}$.

All shocks follow AR(1) processes of the type

$$\begin{aligned}
\log(a_{t+1}) &= \rho_a \log(a_t) + \rho^\star_a \log(a^\star_t) + \varepsilon_{a,t+1} \\
\log(a^\star_{t+1}) &= \rho^\star_a \log(a_t) + \rho_a \log(a^\star_t) + \varepsilon^\star_{a,t+1} \\
\log(g_{t+1}) &= \rho_g \log(g_t) + (1-\rho_g) \log(\overline{g}) + \varepsilon_{g,t+1} \\
\log(g^\star_{t+1}) &= \rho_g \log(g^\star_t) + (1-\rho_g) \log(\overline{g}) + \varepsilon^\star_{g,t+1} \\
\log(\zeta_{c,t+1}) &= \rho_c \log(\zeta_{c,t}) + \varepsilon_{c,t+1} \\
\log(\zeta^\star_{c,t+1}) &= \rho_c \log(\zeta^\star_{c,t}) + \varepsilon^\star_{c,t+1} \\
\log(\zeta_{h,t+1}) &= \rho_h \log(\zeta_{h,t}) + \varepsilon_{h,t+1} \\
\log(\zeta^\star_{h,t+1}) &= \rho_h \log(\zeta^\star_{h,t}) + \varepsilon^\star_{h,t+1}
\end{aligned}$$

8 A Nominal 2–Country Model with Price Adjustment Costs

When price contracts are replaced with price adjustment costs, equations (101)–(104) become

$$\begin{aligned}
y_t &= c_t + i_t + g_t + \frac{\varphi_p}{2} (\pi_{x,t} - 1)^2 y_t \\
y^\star_t &= c^\star_t + i^\star_t + g^\star_t + \frac{\varphi_p}{2} (\pi^\star_{x,t} - 1)^2 y^\star_t \\
x_t &= a_t k^\alpha_t h^{1-\alpha}_t \\
x^\star_t &= a^\star_t {k^\star_t}^\alpha {h^\star_t}^{1-\alpha}
\end{aligned}$$

and equations (125)–(132) are replaced with

$$\begin{aligned}
0 &= (1-\theta) p_{x,t} x_t + \theta s_t x_t - \varphi_p \pi_{x,t} (\pi_{x,t} - 1) y_t + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \pi_{x,t+1} \varphi_p (\pi_{x,t+1} - 1) y_{t+1} \right] \\
0 &= (1-\theta) p^\star_{x,t} x^\star_t + \theta s^\star_t x^\star_t - \varphi_p \pi^\star_{x,t} (\pi^\star_{x,t} - 1) y^\star_t + \beta \mathbb{E}_t \left[\frac{\lambda^\star_{t+1}}{\lambda^\star_t} \pi^\star_{x,t+1} \varphi_p (\pi^\star_{x,t+1} - 1) y^\star_{t+1} \right]
\end{aligned}$$

9 A Real 2–Country Model

All nominal aspects disappear, such that the general equilibrium becomes

$$\zeta_{c,t}^{\frac{1}{\sigma}} c_t^{-\frac{1}{\sigma}} = \lambda_t \tag{133}$$
$$\zeta_{c,t}^{\star\,\frac{1}{\sigma}} c_t^{\star-\frac{1}{\sigma}} = \lambda_t^\star \tag{134}$$
$$\psi \zeta_{h,t}^{-\nu} h_t^{\nu} = \lambda_t (1-\alpha) p_{x,t} \frac{x_t}{h_t} \tag{135}$$
$$\psi \zeta_{h,t}^{\star\,-\nu} h_t^{\star\nu} = \lambda_t^\star (1-\alpha) p_{x,t}^\star \frac{x_t^\star}{h_t^\star} \tag{136}$$
$$\lambda_t = q_t \left(1 - \varphi_k \left(\frac{i_t}{k_t} - \delta\right)\right) \tag{137}$$
$$\lambda_t^\star = q_t^\star \left(1 - \varphi_k \left(\frac{i_t^\star}{k_t^\star} - \delta\right)\right) \tag{138}$$
$$y_t = c_t + i_t + g_t \tag{139}$$
$$y_t^\star = c_t^\star + i_t^\star + g_t^\star \tag{140}$$
$$x_t^d = p_{x,t}^{\frac{1}{\rho-1}} \omega y_t \tag{141}$$
$$x_t^{d\star} = \left(\frac{p_{x,t}}{rer_t}\right)^{\frac{1}{\rho-1}} (1-\omega) y_t^\star \tag{142}$$
$$x_t^f = (rer_t p_{x,t}^\star)^{\frac{1}{\rho-1}} (1-\omega) y_t \tag{143}$$
$$x_t^{f\star} = p_{x,t}^{\star\,\frac{1}{\rho-1}} \omega y_t^\star \tag{144}$$
$$x_t = x_t^d + x_t^{d\star} \tag{145}$$
$$x_t^\star = x_t^f + x_t^{f\star} \tag{146}$$
$$x_t = a_t k_t^\alpha h_t^{1-\alpha} \tag{147}$$
$$x_t^\star = a_t^\star k_t^{\star\alpha} h_t^{\star 1-\alpha} \tag{148}$$
$$1 = \omega p_{x,t}^{\frac{\rho}{\rho-1}} + (1-\omega)(rer_t p_{x,t}^\star)^{\frac{\rho}{\rho-1}} \tag{149}$$
$$1 = \omega p_{x,t}^{\star\,\frac{\rho}{\rho-1}} + (1-\omega)\left(\frac{p_{x,t}^\star}{rer_t}\right)^{\frac{\rho}{\rho-1}} \tag{150}$$
$$\lambda_t = \lambda_t^\star \tag{151}$$
$$q_t = \beta \mathbb{E}_t \left[\lambda_{t+1} \alpha \frac{p_{x,t+1} x_{t+1}}{k_{t+1}} + q_{t+1} \left(1 - \delta + \frac{\varphi_k}{2} \left(\left(\frac{i_{t+1}}{k_{t+1}}\right)^2 - \delta^2\right)\right)\right] \tag{152}$$
$$q_t^\star = \beta \mathbb{E}_t \left[\lambda_{t+1}^\star \alpha \frac{p_{x,t+1}^\star x_{t+1}^\star}{k_{t+1}^\star} + q_{t+1}^\star \left(1 - \delta + \frac{\varphi_k}{2} \left(\left(\frac{i_{t+1}^\star}{k_{t+1}^\star}\right)^2 - \delta^2\right)\right)\right] \tag{153}$$
$$k_{t+1} = i_t \left(1 - \frac{\varphi_k}{2} \left(\frac{i_t}{k_t} - \delta\right)^2\right) + (1-\delta) k_t \tag{154}$$
$$k_{t+1}^\star = i_t^\star \left(1 - \frac{\varphi_k}{2} \left(\frac{i_t^\star}{k_t^\star} - \delta\right)^2\right) + (1-\delta) k_t^\star \tag{155}$$