Lecture Notes 2

Towards non–linear methods

In the previous lectures, we dealt with linear economies, for which there exist straightforward methods to solve the involved expectational difference equations. However, most of the models we encounter in economics are fundamentally characterized by non–linear dynamical features. We therefore need methods to solve such models. The aim of this chapter is to introduce you to such methods, by first pointing out the possible drawbacks of simple linearization — a method commonly used in the literature. We will present four methods which may be used to solve RE models:

1. Perturbation methods (see Judd (1998), Judd and Gaspard (1997), Collard and Juillard (2001a), Collard and Juillard (2001b) or **?**)), which essentially amount to take higher–order Taylor approximation of the model;

2. Value iteration (see Christiano (1990), Tauchen and Hussey (1991)), which may be simply thought of as finding a fixed point on an operator;

3. Parameterized Expectations Algorithm (PEA) (see Marcet (1988), Den Haan and Marcet (1990) or Marcet and Lorenzoni (1999) among others), which may be thought of as a "generalization" of the method of undetermined coefficients to the higher order relying on simulations;

4. Minimum weighted residual methods (see Judd (1992), McGrattan (1996),

McGrattan (1999)), which, as PEA, may be thought of as a "generalization" of the method of undetermined coefficients to the higher order but exploits some orthogonality conditions rather than relying on simulations;

Each method is illustrated by an economic example, which is intended to show you the potential and simplicity of the method. However, before going to such methods, we shall now see why linearizing many not always be a good idea. The big question is then

What are we missing?

2.1 Risk and the Certainty Equivalence Hypothesis

Taking a either linear or log–linear approximation to the decision rules of an economic model, is actually equivalent to taking a quadratic approximation to the optimization problem that lies behind the optimal behavior of agents. In so doing we encounter the so–called Certainty Equivalence property. In order to understand the certainty equivalence property, let us consider the following problem. Let us consider that x is a random variable with probability density $g(x)$ and let y be a variable decided by a decision maker (this may be consumption for an household, investment or labor for a firm, the price for a monopolist...). This decision maker has an objective function $u(y,x)$ which is concave and twice continuously differentiable. Its y plan is then chosen by solving

$$\max_{y} E(u(y,x)) \equiv \int u(y,x)g(x)\mathrm{d}x$$

The first order condition[1] for choosing y is then given by (applying Leibniz rule)

$$\frac{\partial}{\partial y}\int u(y,x)g(x)\mathrm{d}x = 0 \Longleftrightarrow \int \frac{\partial}{\partial y}u(y,x)g(x)\mathrm{d}x = 0 \tag{2.1}$$

[1]Note that since $u(.)$ is concave and $g(.)$ is positive, this condition is necessary and sufficient.

Now, let us assume that $u(y,x)$ is a second order Taylor expansion of another objective, such that

$$u(y,x) = (y\ x)J + (y\ x)\frac{H}{2}\begin{pmatrix} y \\ x \end{pmatrix}$$

where H is a negative–definite (2×2) matrix, such that

$$\begin{aligned} u(y,x) &= (y\ x)\begin{pmatrix} J_y \\ J_x \end{pmatrix} + \frac{1}{2}(y\ x)\begin{pmatrix} h_{xx} & h_{xy} \\ h_{yx} & h_{yy} \end{pmatrix}\begin{pmatrix} y \\ x \end{pmatrix} \\ &= J_y y + J_x x + \frac{1}{2}\left(h_{xx}x^2 + (h_{xy}+h_{yx})xy + h_{yy}y^2\right) \end{aligned} \tag{2.2}$$

In such a case, (2.1) rewrites

$$\int \left[J_y + (h_{xy}+h_{yx})\frac{x}{2} + h_{yy}y\right] g(x)\mathrm{d}x = 0 \Longleftrightarrow y = -\frac{2J_y + (h_{xy}+h_{yx})Ex}{2h_{yy}}$$

Now let us consider a situation where the objective of the decision maker is

$$\max_y u(y, E(x)) \equiv u\left(y, \int xg(x)\mathrm{d}x\right)$$

Note that in this case, we are not maximizing the expected value of the problem but the value, taking into account the expected value of x. Given the functional form (2.2), the first order condition is now

$$J_y + (h_{xy}+h_{yx})\frac{Ex}{2} + h_{yy}y = 0 \Longleftrightarrow y = -\frac{2J_y + (h_{xy}+h_{yx})Ex}{2h_{yy}}$$

which is exactly the same as before. In other words, we have — for the quadratic formulation

$$\underset{y}{\mathrm{Argmax}}\, E(u(y,x)) = \underset{y}{\mathrm{Argmax}}\, u(y, E(x))$$

This is what is usually called the Certainty equivalence principle: risk does not matter in decision making, the only thing that matters is the average value of the random variable x, not its variability. But this is usually not a general result. Let us consider, for example, the case of Burnside's [1998] asset pricing model.

2.1.1 An asset–pricing example

This model is a standard asset pricing model for which *(i)* the marginal intertemporal rate of substitution is an exponential function of the rate of growth of consumption and *(ii)* the endowment is a Gaussian exogenous process. As shown by Burnside (1998), this setting permits to obtain a closed form solution to the problem. We consider a frictionless pure exchange economy *à la* Mehra and Prescott (1985) and Rietz (1988) with a single household and a unique perishable consumption good produced by a single "tree". The household can hold equity shares to transfer wealth from one period to another. The problem of a single agent is then to choose consumption and equity holdings to maximize her expected discounted stream of utility, given by

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{c_{t+\tau}^{\theta}}{\theta} \text{ with } \theta \in (-\infty, 0) \cup (0, 1] \tag{2.3}$$

subject to the budget constraint

$$p_t e_{t+1} + c_t = (p_t + d_t) e_t \tag{2.4}$$

where $\beta \in (0,1)$ is the agent's subjective discount factor, c_t is household's consumption of a single perishable good at date t, p_t denotes the price of the equity in period t and e_t is the household's equity holdings in period t. Finally, d_t is the tree's dividend in period t. Dividends are assumed to grow at rate x_t such that :

$$d_t = \exp(x_t) d_{t-1} \tag{2.5}$$

where x_t, the rate of growth of dividends, is assumed to be a Gaussian stationary AR(1) process

$$x_t = (1-\rho)\overline{x} + \rho x_{t-1} + \varepsilon_t \tag{2.6}$$

where ε is i.i.d. $\mathcal{N}(0, \sigma^2)$ with $|\rho| < 1$. Market clearing requires that $e_t = 1$ so that $c_t = d_t$ in equilibrium. Like in Burnside (1998), let y_t denote the price–dividend ratio, $y_t = p_t/d_t$. Then, condition for the household's problem can be shown to rewrite as

$$y_t = \beta E_t \left[\exp(\theta x_{t+1})(1 + y_{t+1})\right] \tag{2.7}$$

Burnside (1998) shows that the above equation admits an exact solution of the form[2]

$$y_t = \sum_{i=1}^{\infty} \beta^i \exp\left[a_i + b_i(x_t - \overline{x})\right] \tag{2.8}$$

where

$$a_i = \theta\overline{x}i + \frac{\theta^2\sigma^2}{2(1-\rho)^2}\left[i - \frac{2\rho(1-\rho^i)}{1-\rho} + \frac{\rho^2(1-\rho^{2i})}{1-\rho^2}\right]$$

and

$$b_i = \frac{\theta\rho(1-\rho^i)}{1-\rho}$$

As can be seen from the definition of a_i, σ — the volatility of the shock, directly enters the decision rule,, therefore Burnside's [1998] model does not make the certainty equivalent hypothesis: risk matters for asset holdings decisions.

What happens then, if we now obtain a solution relying on a first order Taylor approximation of the model?

First of all let us determine the deterministic steady state of the economy:

$$\begin{aligned} y^\star &= \beta \exp(\theta x^\star)(1 + y^\star) \\ x^\star &= \rho x^\star + (1-\rho)\overline{x} \end{aligned}$$

such that we get

$$y^\star = \frac{\beta \exp(\theta x^\star)}{1 - \beta \exp(\theta x^\star)} \tag{2.9}$$

$$x^\star = \overline{x} \tag{2.10}$$

The first order Taylor expansion of the Euler equation yields

$$\widehat{y}_t = \beta \exp(\theta x^\star) E_t(\widehat{y}_{t+1}) + \theta\beta \exp(\theta x^\star) E_t(\widehat{x}_{t+1}) \tag{2.11}$$

[2] See appendix **??** for a detailed exposition of the solution.

We actually recognize the simplest RE model we have been dealing with in chapter 2 ($y_t = aE_t y_{t+1} + bx_t$) such that we may use a undetermined coefficient approach and guess a decision rule of the form

$$\widehat{y}_t = \alpha \widehat{x}_t$$

Plugging the guess in (2.11), we get

$$\alpha \widehat{x}_t = \beta \exp(\theta x^\star) E_t(\alpha \widehat{x}_{t+1}) + \theta\beta \exp(\theta x^\star) E_t(\widehat{x}_{t+1})$$

taking expectations and identifying, we obtain

$$\alpha = \frac{\theta\rho\beta \exp(\theta x^\star)}{(1-\beta\exp(\theta x^\star))(1-\rho\beta\exp(\theta x^\star))}$$

such that the approximate decision rule may be written as

$$y_t = y^\star + \alpha(x_t - x^\star)$$

We are now endowed to compute the approximation error we make using linear approximation. As the model admits a closed–form solution, the accuracy of the approximation method can be directly checked against the "true" decision rule. This is undertaken relying on the two following criteria

$$E_1 = 100 \times \frac{1}{N} \sum_{t=1}^{N} \left| \frac{y_t - \widetilde{y}_t}{y_t} \right|$$

and

$$E_\infty = 100 \times \max \left\{ \left| \frac{y_t - \widetilde{y}_t}{y_t} \right| \right\}$$

where y_t denotes the true solution to price–dividend ratio and $\widetilde{y}_t$ is the approximation of the true solution by the method under study. E_1 represents the average relative error an agent makes using the approximation rather than the true solution, while E_∞ is the maximal relative error made using the approximation rather than the true solution. These criteria are evaluated over the interval $x_t \in [\overline{x} - \Delta\sigma_x, \overline{x} + \Delta\sigma_x]$ where Δ is selected such that we explore

99.99% of the distribution of x. Table 2.1 reports E_1 and E_∞ for the different cases. Our benchmark experiment amounts to considering the Mehra and Prescott's [1985] parameterization of the asset pricing model. We therefore set the mean of the rate of growth of dividend to $\overline{x} = 0.0179$, its persistence to $\rho = -0.139$ and the volatility of the innovations to $\sigma = 0.0348$. These values are consistent with the properties of consumption growth in annual data from 1889 to 1979. θ was set to -1.5, the value widely used in the literature, and β to 0.95, which is standard for annual frequency. We then investigate the implications of changes in these parameters in terms of accuracy. In particular, we study the implications of larger and lower impatience, higher volatility, larger curvature of the utility function and more persistence in the rate of growth of dividends.

Table 2.1: Accuracy check

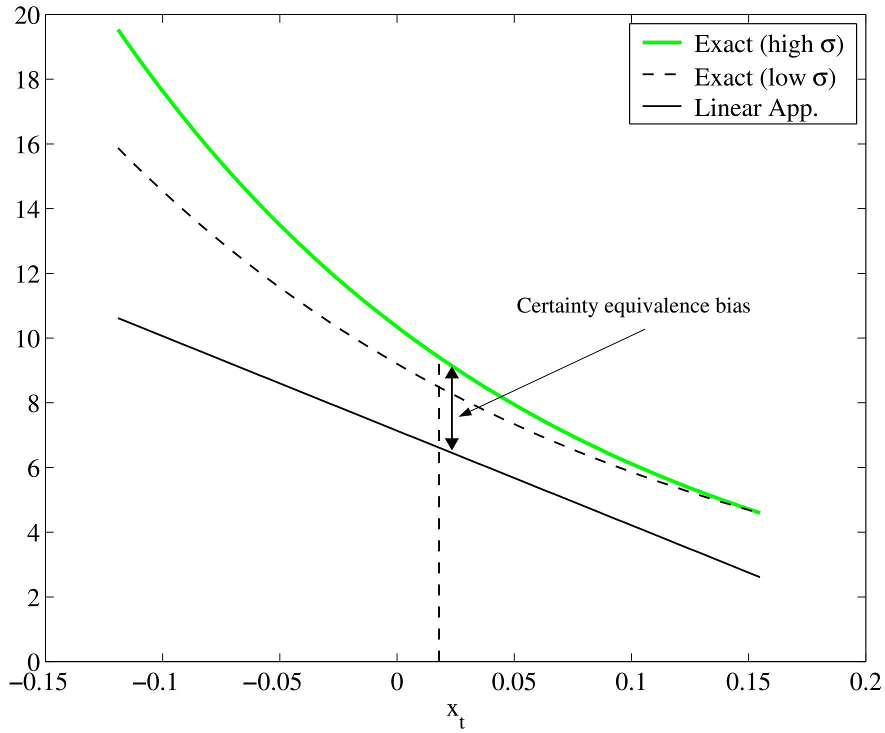
	Benchmark	β=0.5	β=0.99	θ=-10	θ=-5	θ=0
E_1	1.43	0.24	2.92	23.53	8.57	0.50
E_∞	1.46	0.26	2.94	24.47	8.85	0.51
	θ=0.5	σ=0.001	σ=0.1	ρ=0	ρ=0.5	ρ=0.9
E_1	0.29	0.01	11.70	1.57	5.52	37.50
E_∞	0.29	0.03	11.72	1.57	6.76	118.94

Note: The series defining the true solution was truncated after 800 terms, as no significant improvement was found adding additional terms at the machine accuracy. When exploring variations in ρ, the overall volatility of the rate of growth of dividends was maintained to its benchmark level.

At a first glance at table 2.1, it appears that linear approximation can only accommodate situations where the economy does not experiment high volatility or large persistence of the growth of dividends, or where the utility of individuals does not exhibit much curvature. This is for instance the case in the Mehra and Prescott's [1985] parameterization (benchmark) case as both the average and maximal approximation error lie around 1.5%. But, as is nowadays well–known, increases along one of the aforementioned dimension yields lower accuracy of the linear approximation. For instance, increasing the volatility of the innovations of the rate of growth of dividends to σ=0.1 yields approximation errors of almost 12% both in average and at the maximum, thus indicating that the approximation performs particularly badly in this case. This is even worse when the persistence of the exogenous process increases, as ρ=0.9 yields an average approximation error of about 40% and a maximal approximation of about 120%. This is also true for increases in the curvature of the utility function (see row 4 and 5 of table 2.1).

Figure 2.1 sheds light on these results. It reports the exact solution to the problem (grey line) and the linear approximation of the true solution (thin black line). We consider a rather extreme situation where $\theta = -5$, $\rho = 0.5$ and the volatility of the shock is preserved. As can be seen from figure 2.1,

Figure 2.1: Decision rule



Note: This graph was obtained for $\theta = -5$ and $\rho = 0.5$.

using a linear approximation induces two types of error:

1. in terms of curvature,

2. in terms of level.

The first type of error is obvious, as the linear approximation is not intended (as it cannot) to capture any curvature. The second type of error is related to the fact that we are using a approximation about the deterministic steady state. Therefore, the latter source of error is related to the risk component. In fact, this may be understood in light of the a_i terms in the exact solution which include σ — the volatility of the innovations. In order to be sure that this error is related to this component, we also report the exact solution when we cut the overall volatility by 25% (thick dashed line). As can be seen the level error tends to diminish dramatically, which indicates that the risk component plays a major role in this as the average error is cut by 20% then (5% as far as the maximal error is concerned). Hence, this suggests that the linear approximation may only be accurate for low enough variability and curvature, which prevents its use for studying structural breaks.

2.2 Non–Linear Dynamics and Asymmetries

We now consider another situation where the linear approximation may perform poorly. This situation is related to the existence of strong asymmetries in decision rules or strong asymmetries in the objective functions the economic agents have to optimize. In order to illustrate this situation, let us take the problem of a firm that has to decide on employment and which faces asymmetric adjustment costs. Asymmetric adjustment costs may be justified on institutional grounds. We may argue for example that there exist laws in the economy that render firings more costly than hirings.

We consider the case of a firm that has to decide on its level of employment. The firm is infinitely lived and produces a good relying on a decreasing returns to scale technology that essentially uses labor — another way to think of it would be to assume that physical capital is a fixed–factor. This technology is represented by the constant returns–to–scale production function[3]

$$Y_t = An_t \text{ with } A > 0.$$

Using labor incurs two sources of cost

1. The standard payment for labor services: $w_t n_t$ where w_t is the real wage, which positive sequence $\{w_t\}_{t=0}^{\infty}$ is taken as given by the firm and is assumed to evolve as

   $$w_t = \rho w_{t-1} + (1-\rho)\overline{w} + \varepsilon_t$$

   with $\varepsilon_t \rightsquigarrow \mathcal{U}_{[-\sigma_w,\sigma_w]}$ and[4] $\sigma_w < (1-\rho)\overline{w}$.

2. A cost of adjusting labor, $\mathcal{C}(\Delta_t)$ which satisfies

   $$\mathcal{C}(0) = 0, \mathcal{C}'(0) = 0, \mathcal{C}''(\Delta) > 0$$

   but that displays some asymmetries. An example of such a function is depicted in figure (2.2)

Labor is then determined by maximizing the expected intertemporal profit

$$\max_{\{n_\tau,\Delta_\tau\}_{\tau=0}^{\infty}} E_t \sum_{s=0}^{\infty} \left(\frac{1}{1+r}\right)^s (f_0 n_{t+s} - w_{t+s}n_{t+s} - \mathcal{C}(\Delta_{t+s}))$$

subject to

$$n_t = \Delta_t + n_{t-1} \tag{2.12}$$

which yields the two first order conditions

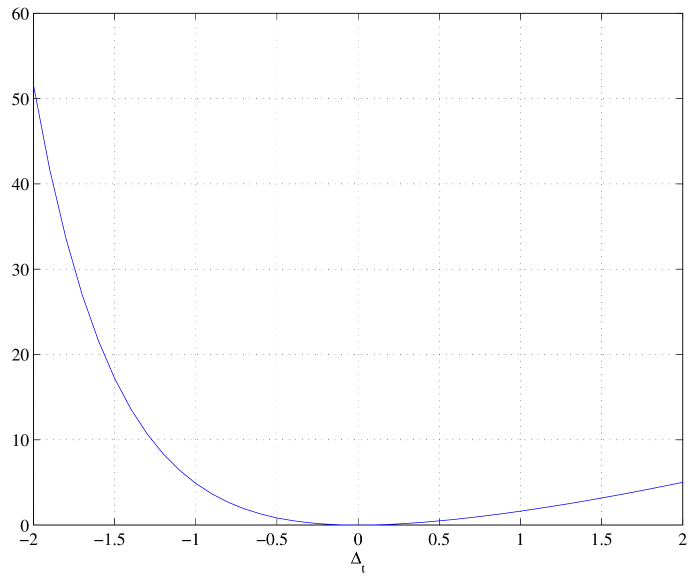
$$\lambda_t = \mathcal{C}'(\Delta_t) \tag{2.13}$$

$$\lambda_t = A - w_t + \frac{1}{1+r}E_t\lambda_{t+1} \tag{2.14}$$

[3]This will enable us to obtain an analytical solution to the problem

[4]This assumption is imposed in order to guaranty the positivity of the real wage. Indeed assume the economy experiences the worst shock in each and every period, then we would have $w_{t+j} = \rho^j w_t + (1-\rho^j)\overline{w} - \sum_{k=0}^{j} \rho^j \sigma$ which in the limit yields $\lim_{j\to\infty} w_{t+j} = \overline{w} - \sigma/(1-\rho)$. The positivity condition then corresponds to what we impose in the main text.

Figure 2.2: Asymmetric adjustment costs



where λ_t is the lagrange multiplier associated to (2.12) The second equation may be simply solved iterating to yield

$$\lambda_t = \sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^i E_t(f_0 - w_{t+i})$$

Note that

$$E_t(w_{t+j}) = \rho^j w_t + (1-\rho^j)\overline{w} + \sum_{i=0}^{j-1} \rho^j \int_{-\sigma}^{\sigma} \varepsilon \frac{\mathrm{d}\varepsilon}{2\sigma} = \rho^j w_t + (1-\rho^j)\overline{w}$$

therefore

$$\lambda_t = \frac{(1+r)A}{r} - \frac{\rho\overline{w}}{1+r-\rho} - \frac{1+r}{1+r-\rho} w_t$$

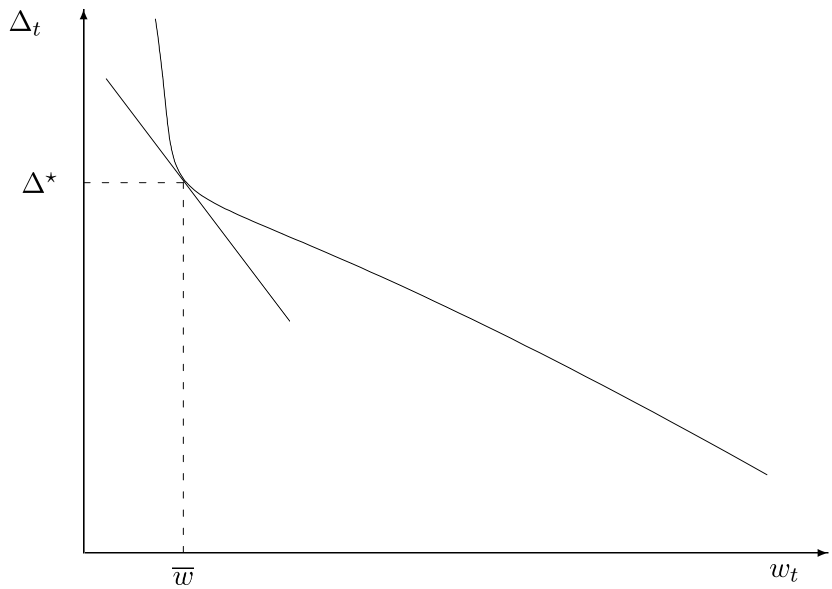
Then, Δ_t is given by

$$\Delta_t = \delta(w_t) \equiv \mathcal{C}'^{-1}\left(\frac{(1+r)A}{r} - \frac{\rho\overline{w}}{1+r-\rho} - \frac{1+r}{1+r-\rho} w_t\right)$$

and we have

$$n_t = \delta(w_t) + n_{t-1}$$

Figure 2.3: Decision rule for Δ_t



Since $\mathcal{C}'(.)$ may exhibit strong asymmetries, the decision rule may be extremely non–linear too to yield a decision rule of the form we depict in figure (2.3). As can be seen from the graph, the linear approximation would do a very poor job, as any departure from the steady state level ($\Delta^\star = 0$) would create a large error. In other words, and as should have been expected, strong non–linearities forbid the use of linear approximations.

Beyond this point that may appear quite peculiar, since such important non–linearities are barely encountered after all, there exists a large class of models for which linear approximation would do a bad job: models with binding constraints that we now investigate.

2.3 Dealing with binding constraints

In this section, we will provide you with an example where linear approximation should not be used because the decision rules are not differentiable. This is the case when the agent faces possibly binding constraints. To illustrate it we will develop a model of a consumer who is constrained on its borrowing in the financial market.

We consider the case of a household who determines her consumption/saving plans in order to maximize her lifetime expected utility, which is characterized by the function: [5]

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left(\frac{c_{t+\tau}^{1-\sigma} - 1}{1-\sigma} \right) \text{ with } \sigma \in (0,1) \cup (1,\infty) \tag{2.15}$$

where $0 < \beta < 1$ is a constant discount factor, c denotes the consumption bundle. In each and every period, the household enters the period with a level of asset a_t carried from the previous period, for which it receives an constant real interest rate r. It also receives an endowment ω_t, which may either be thought of as something totally extrinsic to the economy or as wages. But this is taken to be exogenous as we are not interested by its determination. Therefore, this will be an exogenous stochastic process of the form

$$\log(\omega_t) = \rho \log(\omega_{t-1}) + (1-\rho)\log(\overline{\omega}) + \varepsilon_t \tag{2.16}$$

with $\varepsilon_t \rightsquigarrow \mathcal{N}(0, \sigma_\omega^2)$. These revenus are then used to consume and purchase assets on the financial market. Therefore, the budget constraint in t is given by

$$a_{t+1} = (1+r)a_t + \omega_t - c_t \tag{2.17}$$

In addition, the household is submitted to a borrowing contraint that states that she cannot borrow

$$a_{t+1} \geqslant 0$$

The first order conditions to this model may be obtained forming the

[5] $E_t(.)$ denotes mathematical conditional expectations. Expectations are conditional on information available at the beginning of period t.

Lagrangean to the system:

$$\mathcal{L}_t = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left(\frac{c_{t+\tau}^{1-\sigma} - 1}{1-\sigma} + \lambda_{t+\tau} \left((1+r)a_{t+\tau} + \omega_{t+\tau} - c_{t+\tau} - a_{t+\tau+1} \right) + \mu_{t+\tau} a_{t+\tau+1} \right)$$

where λ_t and μ_t respectively denote the Lagrangean multipliers associated to the budget and the borrowing constraints. The first order conditions associated to the system are then

$$c_t^{-\sigma} = \lambda_t \tag{2.18}$$

$$\lambda_t = \mu_t + \beta(1+r)E_t\lambda_{t+1} \tag{2.19}$$

together with

$$\lambda_t \left((1+r)a_t + \omega_t - c_t - a_{t+1} \right) = 0 \tag{2.20}$$

$$\mu_t a_{t+1} = 0 \tag{2.21}$$

$$\lambda_t \geqslant 0 \tag{2.22}$$

$$\mu_t \geqslant 0 \tag{2.23}$$

manipulating the system, we see that consumption is given by

$$c_t^{-\sigma} = \min \left(((1+r)a_t + \omega_t)^{-\sigma}, \beta(1+r)E_t c_{t+1}^{-\sigma} \right)$$

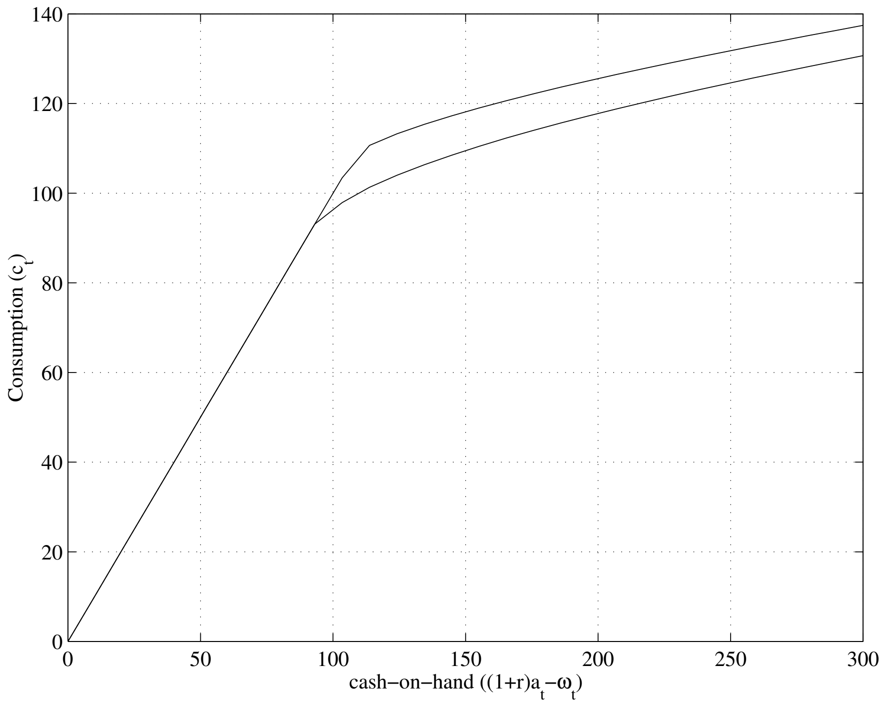
The decision rule of consumption is not differentiable in the point where assets holdings are not sufficient to guaranty a positive net position on asset holdings:

$$((1+r)a_t + \omega_t)^{-\sigma} = \beta(1+r)E_t c_{t+1}^{-\sigma}$$

Just to give you an idea of this phenomenon, we reported in figure 2.4 the consumption decision rule for two different values of ω_t as a function of cash–on–hand — which is given by $(1+r)a_t+\omega_t$.[6] This non–differentiability implies obviously that linear approximation cannot be useful in this case, as they are not defined in the neighborhood of the point that makes the household switch from the unconstrained to the constrained regime. Nevertheless, if we are

[6] We will see later on how these decision rules where computed.

Figure 2.4: Consumption decision rule



to consider an economy with tiny shocks and where the steady state lies in the unconstrained regime, the linear approximation may be sufficient as the decision rule is particularly smooth in this region because of consumption smoothing).

We therefore need to investigate alternative methods, which however require some preliminaries

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