Macroeconomics of Bank Capital and Liquidity Regulations

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Abstract

We study the transmission mechanisms of liquidity and capital regulations as well as their effects on the economy and welfare. We propose a macro–economic model in which a regulator faces the following trade–off. On the one hand, banking regulations may reduce the aggregate supply of credit. On the other hand, they promote the allocation of credit to its best uses. Accordingly, in a regulated economy there is less, but more productive lending. Based on a version of the model calibrated on US data, we find that both liquidity and capital requirements are needed, and must be set relatively high. They also mutually reinforce each other, except when liquid assets are scarce. Our analysis thus provides broad support for Basel III's "multiple metrics" framework.

Keywords: Financial Frictions, Externalities, Banking Regulation

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1 Introduction

Following the 2007–8 financial crisis, the Basel Committee on Banking Supervision (BCBS) agreed on a new regulatory framework (so–called "Basel III") that features both capital and liquidity requirements for banks. Since then, the overall economic effect of those reforms has been debated, with long run estimates of their impact on GDP varying widely (IIF, 2011, BCBS, 2010 and Fender and Lewrick, 2016). This variation reflects —*inter alia*— the difficulty to assess the net effect of the reforms given their multifaceted nature, and the complexity of the transmission channels through which they operate. Indeed, while it is generally agreed that capital and liquidity requirements can force an individual bank to internalize the adverse consequences of its excess leverage or maturity mismatch, the overall benefits for the economy could nonetheless be reduced if the requirements interacted with each other in unexpected ways; or if stacking multiple requirements had more stringent effects than intended. So far, the literature has remained silent about these issues, for it mostly focussed on the effects of capital regulation *alone*.

The aim of this paper is to provide a better understanding of the mechanics of the transmission channels of multiple banking regulations, and to offer some guidance for the design and coordination of such regulations. To do so, we develop a quantitative general equilibrium model, where banks face both capital and liquidity regulatory constraints. We study the interactions between those constraints, their synergies and potential tensions. We then use the model to assess the net welfare gain of banking regulations, identify the regulator's trade–off and devise the optimal regulatory mix.

Consistent with empirical evidence, we emphasize the role of banks in allocating credit and their impact on firm productivity. In our model, households lend to firms either directly through the corporate bond market, or indirectly through the banking sector. The characteristic that distinguishes banks from "arm's length" investors, like bond holders, is the relationship that exists between them and the firms. Relationship banking goes beyond the mere provision of cash. In addition to cash, banks provide firms with complementary services (*e.g.* advice, market intelligence, mentoring, strategic planning support, etc) that affect firms' productivity. In the model, banks are heterogeneous in terms of the productivity gains they elicit. This assumption captures the idea that some financial intermediaries are good at collecting savings, when some others are good at making corporate loans, dispensing advice, and bringing value–added to firms. By facilitating the migration of funds from the former to the latter, an interbank market enhances financial intermediation and raises aggregate productivity. There exists the usual agency problem in this market, though, as borrowing banks can always divert some of the funds towards low return assets that cannot be recovered by the lending banks. This agency problem hampers the good functioning of the interbank market and the reallocation of savings toward the most productive banks. Absent interbank transactions, some banks may engage in low–end lending activities that will later weigh on aggregate productivity.[1]

Credit Quality versus Credit Quantity. Banks' incentives for diversion are stronger when banks are more leveraged, or when they hold fewer liquid/safe (*i.e.* seizable) assets, such as government bonds. As banks and their stakeholders do not fully internalize the effects of their funding and investment decisions on the functioning of the interbank market, both bank capital and liquidity regulations are needed to address those externalities. By requiring banks to reduce their leverage and hold more liquid assets than what is privately optimal, the regulator can support interbank activity and raise the overall quality of banks' credit supply. On the other hand, liquidity requirements induce banks to substitute corporate loans with government bonds, which entails a reduction in the supply of credit. And capital requirements indirectly distort investors' asset portfolio composition, through restrictions on banks' debt issuance. In our model, the regulator must therefore balance the social benefits from a better allocation of credit against the social cost of a potentially lower credit supply and distorted portfolio choices.

Regulations: Synergies and Tensions. We consider a social planner/regulator who sets time invariant capital and liquidity requirements so as to maximize welfare. From an individual bank —or partial equilibrium— viewpoint, the two requirements reinforce each other. For example, when the bank increases its holdings of liquid/safe assets, it *de facto* reduces the volume of risky assets per unit of equity, which enhances the disciplinary effect of equity. In effect, liquidity requirements free up equity capital for interbank transactions. Hence the synergies. However, capital and liquidity regulations might also oppose each other. Tensions can arise due to investors' responses to regulatory reforms, and general equilibrium effects. Following a tightening in liquidity requirements, for example, banks demand more government bonds. In equilibrium, this exerts downward pressures on the government bond yield. Bank deposits being close substitutes to government bonds, investors re–balance their portfolio of assets toward bank deposits. As banks issue more deposits, their capital ratio deteriorates. Hence potential tensions with capital regulation.

For a version of the model calibrated on US data, we find that such tensions are rare and in general do not overcompensate the synergies. They only materialize when, following the tightening in liquidity requirements, banks demand so much liquid assets that liquid assets become scarce, as this forces investors to adjust their asset portfolio more forcefully. Hence, in our model, the optimal regulatory mix consists of capital *and* liquidity requirements. Both regulatory constraints bind. In the optimally regulated economy, banks hold slightly less liquid assets but issue significantly more

[1] Cuñat and Garicano (2009) report that, in the case of Spain, the role of bankers' experience was very significant in the run–up to the 2007-8 crisis. They find that savings banks led by a chairman without banking experience or no post–graduate education had a 2% increase in non–performing loans as of July 2009, and that this partly reflects a mis–allocation of funds toward real estate and individuals. Residential mortgages, which are relatively standardized loan contracts, do not require much screening or mentoring from the bank. But they do not support aggregate productivity either. Banks' "lazy attitude" (Manove, Padilla, and Pagano, 2001) or lack of expertise may have led them to favor this type of loans over other, more productivity enhancing, loans (see. *e.g.* ESRB, 2014).

equity than they would in the unregulated economy. Moreover, the cost of equity is higher but the cost of deposits and overall cost of funding are lower, as banks' creditors price in the higher quality of banks' credit supply (as shown in Gambacorta and Shin, 2016).

Relation to the Literature. The notion that financial institutions have an impact on allocative efficiency and aggregate productivity is not new. This is well–established in the finance–and–growth literature (Greenwood and Jovanovic, 1990, Greenwood, Sanchez, and Wang, 2013). For example, in Buera, Kaboski, and Shin (2011), financial frictions distort the allocation of capital across heterogeneous production units, lowering aggregate and sector–level productivity. For emerging market economies, those effects can be large. Hsieh and Klenow (2009) find that moving to "U.S. efficiency" would increase total factor productivity by 30%–50% in China and 40%–60% in India. In the case of developed economies, empirical evidence on the effect of the financial sector on firm productivity can be found in, for example, Hellmann and Puri (2000), who show that start–up firms financed through venture capital are the fastest to bring a product to market.[2] Using data for manufacturing firms in Spain, Gopinath, Kalemli-Ozcan, Karabarbounis, and Villegas-Sanchez (2015) document that factor mis–allocation at the micro–level induced significant aggregate productivity losses over the period 1999–2012, and empirically link this pattern to firm–level financial decisions and financial frictions.

Despite the empirical evidence, the effect of a well–functioning financial system on credit quality and allocative efficiency has been neglected in most previous cost–benefit analyses of banking regulation. In contrast, our cost–benefit analysis explicitly takes this element into account. In this respect, our approach is close to Martinez-Miera and Suarez (2014) and Christiano and Ikeda (2013). One difference is that we allow firms to issue corporate bonds, and to substitute bank loans with corporate bonds. We think this is an important feature because such substitution effects have been observed in the aftermath of the 2007–8 financial crisis (De Fiore and Uhlig, 2015). Another difference with respect to Martinez-Miera and Suarez (2014) and Christiano and Ikeda (2013) is that they focus on capital regulation, whereas we study the effects of both capital and liquidity regulations. Our work is also connected with several other recent attempts to incorporate banking regulation into otherwise standard macro–economic models. Begeneau (2015), Clerc, Derviz, Mendicino, Moyen, Nikolov, Stracca, Suarez, and Vardoulakis (2015), and Goel (2016), for instance, derive the level of bank capital requirements that maximizes welfare. Clerc, Derviz, Mendicino, Moyen, Nikolov, Stracca, Suarez, and Vardoulakis (2015) find that, depending on the risk–weights, the optimal capital requirement should be around 10.5% for business loans and 5.25% for mortgages, while Begeneau (2015) and Goel (2016) report optimal leverage ratios of 14% and 28%, respectively. As in the latter papers, our optimal leverage ratio is relatively high —around 17%. One difference

[2]Kortum and Lerner (2000) find that, in the US, increases in venture capital activity are associated with significantly higher patenting rates and innovation. There also exists empirical evidence of relationship lending banks supporting firms' productive investment (see, *e.g.* Bolton, Freixas, Gambarcorta, and Mistrulli, 2016).

is that our result takes into account the allocative effects of regulation, as well as the interactions between capital and liquidity requirements. To date, very few other macro–economic models allow for multiple regulatory requirements; those include Covas and Driscoll (2014), Van den Heuvel (2016), and Kashyap, Tsomocos, and Vardoulakis (2014). Covas and Driscoll (2014) and Van den Heuvel (2016) study the welfare costs of liquidity and capital requirements, but they neither evaluate the benefits nor analyze how those requirements interact. Kashyap, Tsomocos, and Vardoulakis (2014) study the interactions between multiple regulations and, as we do, devise the optimal regulatory mix. However, they do this within a three period Diamond–Dybvig framework, whereas we use a dynamic macro–economic framework. In our model, forward looking agents anticipate —some of— the effects of regulations and take defensive actions. For example, investors may re–balance their asset portfolio and firms may issue corporate bonds. Those reactions generate meaningful general equilibrium feedback effects onto the macro–economy. Indeed, the endogenous response of investors outside the banking sector to regulatory changes inside the banking sector is a distinctive and key element of our analysis.

Another significant strand of the literature analyzes the interactions between capital requirements and monetary policy within dynamic stochastic New–Keynesian frameworks (see, e.g, Gerali, Neri, Sessa, and Signoretti, 2010, Angeloni and Faia, 2013, De Paoli and Paustian, 2013, Collard, Dellas, Diba, and Loisel, 2016). In this literature, inefficiencies arise from both financial frictions and nominal rigidities. Our focus is different. It is on the interactions between capital and liquidity regulations. Accordingly, we deliberately confine the inefficiencies to the financial sector, and assume fully flexible prices to abstract from monetary policy considerations.

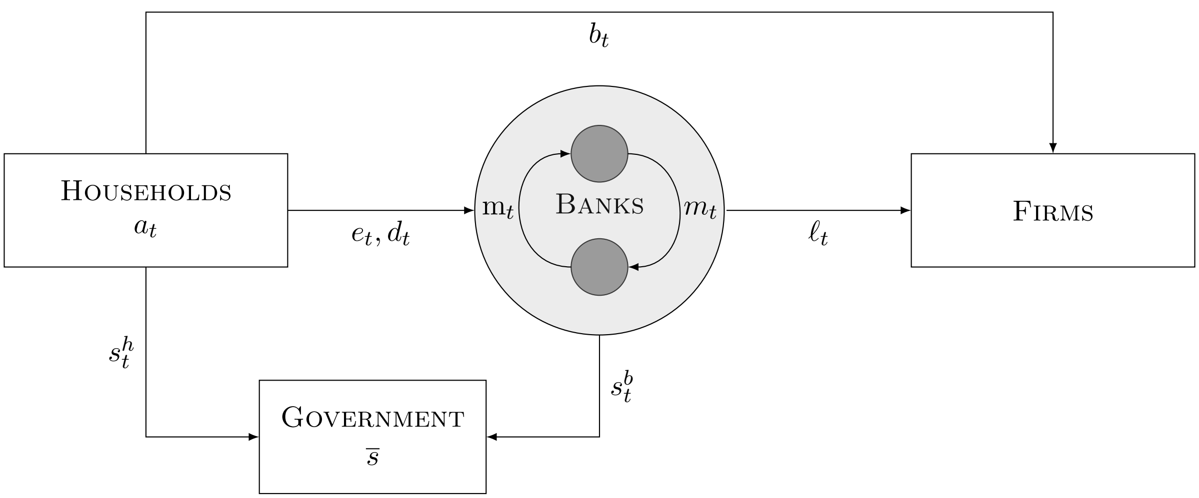
Finally, many of the above approaches share the feature that financial inefficiencies arise from the banks' risk–shifting behavior that comes along with deposit insurance. So, in a sense, banking regulation is meant to address the unintended consequences of another policy intervention. In our case, instead, regulation is meant to mitigate the agency problem that exists on interbank markets, whose dysfunctions were at the core of both the 2007–8 financial crisis and the resulting post-crisis regulatory reforms.

The paper proceeds as follows. Section 2 describes our theoretical framework, the micro-foundations of financial externalities, as well as the credit quality channel of banking regulation. In Section 3, we introduce liquidity and capital regulations, and discuss the potential synergies and tensions between the two. Section 3.2, in particular, presents our cost–benefit analysis of those regulations. Section 4 discusses some implications of our results for the conduct of regulatory reforms, in particular the use of leverage requirements as a backstop. A last section concludes.

2 Unregulated Economy

We consider an economy populated with mass one continuums of atomistic risk averse households, final good producers, material good producers, and banks, as well as with a government agency. The is no aggregate uncertainty. Households are infinitely–lived; the other agents live for one period only. In each and every period, the household supplies her labor, h_t, consumes, c_t, accumulates physical capital, k_{t+1}, and saves $\tilde{a}_{t+1}$. The household can transfer wealth across periods by investing into government bonds, s^h_{t+1}; corporate bonds, b_{t+1}, bank deposits, d_{t+1}, or bank equity, e_{t+1}. The banks that operate in period t are born at the end of period $t-1$ and die at the end of period t. (Hence, financial contracts will last for one period only.) They invest deposits and equity into corporate loans, ℓ_t, government bonds, s^b_t, or interbank loans, m_t. The final good producers (for short, "firms") produce the final goods by means of capital, k_t, labour, h_t, and material goods, x_t. Those material goods must be paid upfront, before production takes place. Therefore, final good producers must borrow at the beginning of period t in order to purchase those goods. The material good producers produce the material goods instantaneously at the beginning of period t, using final goods produced in period $t-1$. Figure 1 summarizes the flows of funds between agents in the economy, and Appendix 6.1 recaps the timing of the main decisions.

Figure 1: Financial Flows



2.1 Government

The government agency[3] issues an exogenous amount of debt s_t at the end of period $t-1$. For most of the analysis, we will assume that s_t is constant and equal to $\bar{s}$. (We relax this assumption in Section 4.5.) The distinctive feature of government debt is that it is a liquid and safe asset (see Assumption 6), which banks and households can purchase. At the end of period t, the government

[3]This government agency may comprise a central government and a central bank; this distinction will not be material for our analysis.

repays its debt by issuing new debt and levying lump–sum taxes, T_t. The gross interest rate on government debt is denoted by r_t^s. The government's budget constraint at the end of period t is

$$T_t = (r_t^s - 1)\overline{s}.$$

2.2 Households

The representative household consists of a mass one continuum of atomistic members whose decisions are made in two stages. In the first stage, the household head chooses the levels of consumption, leisure, physical capital, and savings. The decisions regarding the composition of those savings are taken in the second stage, at the level of household members.

2.2.1 Household Members' Financial Skills

A household member can invest in up to four different financial assets, namely, government bonds, corporate bonds, bank equity, and bank deposits. Financial decisions require specific financial skills.

Assumption 1 ***(Financial Investment Skills)*** *The investment in a specific asset requires specific financial skills from a household member. Skills are idiosyncratic.*

We refer to household member $q \equiv (q^{s^h}, q^b, q^d, q^e) \in [0,1]^4$ as the member with skills q^{s^h} for government bonds, q^b for corporate bonds, q^d for bank deposits, and q^e for bank equity. A household member with skill q^j must spend $1/q^j$ units of goods in order to invest one unit of the good into asset j, with $q^j \leq 1$ and $j \in \mathcal{J} \equiv \{s^h, b, d, e\}$. The remainder, $1/q^j - 1$, is a deadweight transaction cost. This setup is meant to capture the fact that some household members may, for example, face different transaction costs when they invest into corporate bonds than when they make bank deposits. We associate transaction costs to the distance between the household members and the borrowers, the effort of screening, the cost of gathering financial intelligence, etc.[4] Formally, financial skills q^{s^h}, q^b, q^d, and q^e are drawn randomly and independently from cumulative distributions $\mu_{s^h}(q^{s^h})$, $\mu_b(q^b)$, $\mu_d(q^d)$, and $\mu_e(q^e)$, respectively. To economize on notations, we denote by $\mu(q) \equiv \mu_{s^h}(q^{s^h})\mu_b(q^b)\mu_d(q^d)\mu_e(q^e)$ the joint cumulative distribution of financial investment skills.[5] Household members do not have the ability to reallocate their resources within the household after they have drawn their skills.

2.2.2 Choices

The representative household first consumes c_t, supplies labour h_t, saves $\tilde{a}_{t+1}$, and invests i_t units in physical capital, depreciates at rate δ. The household rents out her physical capital to the producers

[4] In Section 2.6.2, we calibrate those transaction costs on the basis of the observed interest rate spreads between securities and households' portfolio composition.

[5] To be clear, household member $q = (0, 0, 0, 0)$ is the least skillful, whereas household member $q = (1, 1, 1, 1)$ is the most skillful.

of consumption goods. Those decisions are taken before household members draw their financial skills. Hence, each member receives the same share of total financial wealth, $\tilde{a}_{t+1}$.

Then, the members draw their financial skills, q. Given her skills, a member chooses which financial asset to invest $\tilde{a}_{t+1}$ in. Since the returns on assets are linear, she specializes in one of the financial assets. The decision to invest into asset j is denoted by $\mathbb{1}\{j\}_{t+1}$, with $\mathbb{1}\{j\}_{t+1} = 1$ if there is an investment and $\mathbb{1}\{j\}_{t+1} = 0$ otherwise, for all $j \in \mathcal{J}$, with $\sum_{j\in\mathcal{J}} \mathbb{1}\{j\}_{t+1} = 1$.

All returns are pooled within the household at the end of period $t+1$. The household takes her decisions so as to maximize expected utility:

$$\max_{\{\tilde{a}_{t+1}, c_t, h_t, i_t\}_{t=0,\ldots,\infty}} \sum_{s=0}^{\infty} \beta^s \mathbb{E}_q \left[\max_{\{\mathbb{1}\{j\}_{t+1}\}_{t=0,\ldots,\infty}^{j\in\mathcal{J}}} u(c_{t+s}) - v(h_{t+s}) \right] \tag{1}$$

subject to the constraints:

$$c_t + \tilde{a}_{t+1} + i_t = r_t \tilde{a}_t + \rho_t k_t + w_t h_t + \pi_t^f + \pi_t^b + \pi_t^x - T_t, \tag{2}$$

$$k_{t+1} = i_t + (1-\delta) k_t, \tag{3}$$

$$r_t \tilde{a}_t \equiv r_t^s s_t^h + r_t^b b_t + r_t^d d_t + r_t^e e_t, \tag{4}$$

$$j_{t+1} \equiv \tilde{a}_{t+1} \int_{[0,1]^4} q^j \mathbb{1}\{j\}_{t+1} \mathrm{d}\mu(q) \quad \forall j \in \mathcal{J}. \tag{5}$$

where $u(\cdot)$ and $v(\cdot)$ satisfy the usual regularity conditions;[6] $\mathbb{E}_q$ is the expectation operator taken over q; r_t^j denotes the gross return on asset $j \in \mathcal{J}$; r_t denotes the overall gross return on savings; ρ_t is the gross rental rate of capital; w_t is the unit wage and h_t are the hours worked. The profits from the final good producers, the banks, and the material good producers, π_t^f, π_t^b, and π_t^x, are rebated lump–sum to the household. The maximization problem is solved backward, starting with the second stage.

Second Stage: Asset Portfolio Choice. Given her financial skills, household member q invests $\tilde{a}_{t+1}$ into asset j if and only if her expected net return on j is higher than her expected net return on any other asset $j^- \neq j$, *i.e.*:

$$\mathbb{1}\{j\}_{t+1} = 1 \Leftrightarrow \left\{ q^j > q^{j^-} \frac{r_{t+1}^{j^-}}{r_{t+1}^j} \ \forall j^- \in \mathcal{J} \setminus \{j\} \right\}, \tag{6}$$

for all assets $j \in \mathcal{J}$. Based on (6), the optimal quantity of goods $\tilde{j}_{t+1}$ the household spends into asset j (for all $j \in \mathcal{J}$) is given by

$$\tilde{j}_{t+1} = \tilde{a}_{t+1} \int_0^1 \prod_{j^- \in \mathcal{J}\setminus\{j\}} \mu_{j^-} \left(\min \left[1, q^j \frac{r_{t+1}^j}{r_{t+1}^{j^-}} \right] \right) \mathrm{d}\mu_j(q^j),$$

[6] We assume separability between the consumption and labour supply decisions, without loss of generality.

and the total amount of wealth invested into asset j (for all $j \in \mathcal{J}$) is given by

$$j_{t+1} = \tilde{a}_{t+1} \int_0^1 q^j \prod_{j^- \in \mathcal{J}\setminus\{j\}} \mu_{j^-} \left(\min \left[1, q^j \frac{r^j_{t+1}}{r^{j^-}_{t+1}} \right] \right) \mathrm{d}\mu_j(q^j). \tag{7}$$

The gap $\tilde{j}_{t+1} - j_{t+1}$ corresponds to the transaction costs the household incurs when her members invest j_{t+1} into asset j. It will be useful to define the household's total financial wealth —net of those costs, a_{t+1} as

$$a_{t+1} \equiv \sum_{j \in \mathcal{J}} j_{t+1}$$

and to express the total deadweight financial transaction costs, denoted χ_t, as

$$\chi_t \equiv \tilde{a}_{t+1} - a_{t+1}. \tag{8}$$

First Stage: Labor Supply, Consumption, Investment, Savings. The household chooses the levels of labour supply, consumption, physical investment, and savings, anticipating the second stage portfolio choices. This yields

$$v'(h_t) = u'(c_t) w_t \tag{9}$$

$$\Psi_{t,t+1} r_{t+1} = 1 \tag{10}$$

$$r_{t+1} = \rho_{t+1} + 1 - \delta, \tag{11}$$

as optimality conditions, where $\Psi_{t,t+1} \equiv \beta \frac{u'(c_{t+1})}{u'(c_t)}$ denotes the household's discount factor.

2.3 Banks

The way we model the banking sector builds upon Boissay, Collard, and Smets (2016). The representative bank is composed of a mass one continuum of atomistic members, which we dub "bankers". The bank takes its decisions in two stages. In the first stage, it raises equity, e_t, and deposits, d_t, and invests into government bonds, s^b_t. As will become clear in Assumptions 4 and 5, there exists an agency problem between the bank and its creditors. We define "deposits" as the funding that is subject to the agency problem, and "equity" as the funding that is *not* subject to the agency problem.

Figure 2: Balance Sheet

Asset			Liabilities
(cash)	n_t	d_t	*(deposits)*
(gvt bonds)	s^b_t	e_t	*(equity)*

At the time the equity, deposits, and government bond markets close, all bankers within the representative bank are endowed with the same amounts of equity, e_t, deposits, d_t, government bonds, s_t^b, and cash, n_t, where

$$n_t \equiv \mathrm{d}_t + \mathrm{e}_t - s_t^b. \tag{12}$$

Hence, they have the same balance sheet (represented in Figure 2). The second stage begins at that point. Bankers then draw their idiosyncratic expertise.

Assumption 2 ***(Banker Expertise and Firm Productivity)*** *Bankers have idiosyncratic expertise. The expertise of a banker determines her borrowers' probability of success.*

Assumption 2 rests on an interpretation of bankers as *relational* lenders. The bankers we have in mind may in practice correspond to various types of financial intermediaries, ranging from commercial banks to venture capitalists. The characteristic that distinguishes bankers from "arm's length" investors like bond holders, is the relationship that exists between them and their borrowers pre– and post–financing. Relationship banking goes beyond the mere provision of cash. In addition to cash, bankers also provide firms with complementary services in the form of advice, planning, market intelligence, mentoring, etc.[7] There is ample empirical evidence that these services bring value–added, reduce borrower delinquency, and raise firms' productivity.[8] Hellmann and Puri (2000), for example, show that start–up firms financed through venture capital are the fastest to bring a product to market. Kortum and Lerner (2000) find that, in the US, increases in venture capital activity in an industry are associated with significantly higher patenting rates and innovation, and that venture capital accounts for a disproportionate share of industrial innovations. Commercial banks too have an influence on borrower quality. Bolton, Freixas, Gambarcorta, and Mistrulli (2016), for example, provide empirical evidence that banks play a continuing role of managing firms' financial needs as they arise in response to new investment opportunities. Assumption 2 is meant to capture those effects of banks on firm productivity. Hereafter, we will think of a banker's expertise as the value adding services the banker provides to her borrowers.[9]

We refer to banker q^ℓ as the banker with expertise $q^\ell \in [0, 1]$. This expertise is essential to the firm the banker lends to, because it determines the probability that its project is successful

[7] See, *e.g.*, Casamatta (2003) and the references therein.

[8] See Lerner (1995), Hellmann and Puri (2000), Hellmann and Puri (2002), Hellmann, Lindsey, and Puri (2004), Hellmann, Nandy, and Puri (2014). Another telling example of banks' involvement into businesses is the importance of corporate leasing. In many instances, corporate loan contracts take the form of a leasing contract, whereby the bank purchases equipment on behalf of the firm, and rents it out to the firm. In Europe, notably, those loan contracts are predominantly made by banks or bank subsidiaries.

[9] Keys, Mukherjee, Seru, and Vig (2010) also find that for a borrower in a given class of risk (*e.g.* with a given FICO score), the default rate is 20% higher when the bank originating the loan does not exert its due diligence than when it does. Anecdotal evidence on how commercial banks' service quality and practices can adversely affect borrowers' ability to repay their loans can also be found in the US Consumer Financial Protection Bureau's complaint database.

(see Section 2.4).[10] More precisely, the probability that banker q^ℓ's borrowers repay their debts is q^ℓ. Bankers draw their expertise randomly and independently from cumulative distribution $\mu_\ell(q^\ell)$. We denote the contractual gross interest rate on corporate loans by $\tilde{r}_t^\ell$; since banker q^ℓ is perfectly diversified across the firms she lends to, her effective return on corporate loans is $q^\ell \tilde{r}_t^\ell$.

Assumption 3 ***(Interbank Markets)*** *Bankers have access to an interbank loan market.*

Unlike household members, bankers have the possibility to lend to and borrow from each other directly through wholesale financial markets. These markets facilitate the reallocation of funds within the banking sector: bankers with the most expertise borrow from other bankers. These exchanges may include various market–financed transfers of assets between bankers and take a variety of forms, ranging from repo transactions to the trading of treasuries on a secondary market. For short, we —loosely— refer to those markets as "interbank loan markets".[11] We denote the aggregate supply (respectively, the demand) of interbank loans by m_t (respectively m_t) and the gross return on interbank loans by r_t^m. Clearly, in a frictionless world, only banker $q^\ell = 1$ would lend to firms. This banker would borrow from the rest of the banking sector, and —given her small size— would be infinitely leveraged. However, there is a limit, determined in equilibrium, on how much a banker can borrow on the interbank market. This limit takes into account the presence of two informational frictions: asymmetric information and moral hazard.

Assumption 4 ***(Asymmetric Information)*** *Her realized value of q^ℓ is known only to banker q^ℓ.*

Asymmetric information arises from the assumption that q^ℓ is known to banker q^ℓ only, and is not verifiable *ex post*. Thus, in equilibrium the bankers that lend will be able to compute the expected value of q^ℓ for the bankers that borrow, but contracts cannot be conditioned on a banker's specific expertise.[12] Similarly, firms do not observe the expertise of the bankers they borrow from.

Assumption 5 ***(Moral Hazard)*** *A banker may divert cash and extract private benefits γ per unit of cash diverted.*

[10] Note that the expertise of the banker determines the quality of her borrower's project, irrespective of how much her loan contributes to the financing of the borrower's project in addition to bonds. In this sense, the banker's expertise is *essential.* We assume that a given firm borrows from only one banker.

[11] Our notion of "interbank market" encompasses all types of interbank transactions that help banks facilitate the reallocation of funds among themselves. Those may also include the trading of government bonds on a secondary market whereby less productive banks would purchase government bonds from more productive banks. Such secondary market transactions are in effect equivalent to treasury repo market transactions without the "close" leg (see Boissay and Cooper, 2016).

[12] While Assumption 4 is not necessary for our results, it simplifies the financial contracts and keep the model tractable. It is also in line with Cuñat and Garicano (2009), who show evidence that, in the case of Spain before the 2007–8 crisis, the quality of financial intermediation varied widely across savings banks (*Cajas*) and was related to well–hidden (*i.e.* unobservable to investors) bank manager characteristics, such as political connections, experience, and graduate education.

Once a banker obtains an interbank loan, she has the possibility to lend the funds to the firm. We assume that the cash flows generated from those loans can be seized at no cost by the banker's creditors. In this case, the banker cannot extract private benefits, and always pays back her debts. Alternatively, the banker may divert the funds, extract private benefits, abscond, and default on her debts, at the expense of the creditors. This possibility to divert funds generates a friction between the banker and her creditors. In the discussion that follows, we proceed under the assumption that the extraction of private benefits is inefficient, *i.e.* that $\gamma \leq \tilde{r}_t^\ell$ at all times. This option will nonetheless matter for the equilibrium outcome through its effects on borrowers' incentives. To deal with those incentive issues, lenders can restrict borrowing and threaten the borrower to seize her government bond holdings in case of default. To be clear, we what we call "government bonds" is all types of securities that can be seized by the creditors in a default event (see Assumption 6 below).

Assumption 6 ***(Liquid Assets)*** *Government bonds are liquid in the sense that, in case of default, they can be fully seized by creditors at no cost.*

A banker can invest her own cash n_t either into interbank loans or into corporate loans. We denote the decision to lend to other bankers by $\mathbb{1}_t\{m\}$ ($\mathbb{1}_t\{m\} = 0$ if the banker lends), and express the size of the interbank loan as a multiple ϕ_t of the banker's own cash, n_t (with no loss of generality). The optimisation problem of the representative bank and its bankers consists in maximizing the bank's end–of–period t profit with respect to s_t^b, d_t, e_t, ϕ_t, and $\mathbb{1}_t\{m\}$:

$$\begin{cases} \max\limits_{s_t^b, \mathrm{d}_t, \mathrm{e}_t} \Psi_{t-1,t} \displaystyle\int_0^1 \max\limits_{\phi_t, \mathbb{1}_t\{m\}} \pi_t^b(q^\ell) \mathrm{d}\mu_\ell(q^\ell) \\ \\ \pi_t^b(q^\ell) \equiv r_t^s s_t^b - r_t^d \mathrm{d}_t - r_t^e \mathrm{e}_t + \mathbb{1}_t\{m\} r_t^m n_t + (1 - \mathbb{1}_t\{m\}) \left(q^\ell \tilde{r}_t^\ell (1 + \phi_t) - r_t^m \phi_t \right) n_t \end{cases} \tag{13}$$

subject to (12) and the incentive compatibility constraint:

$$\gamma(1 + \phi_t) n_t - r_t^e \mathrm{e}_t \leq r_t^s s_t^b - r_t^d \mathrm{d}_t - r_t^e \mathrm{e}_t + r_t^m n_t. \tag{14}$$

The first component of the profit is the return on government bonds. The second and third components are the costs of deposits and equity. The fourth is the proceeds from the interbank loan if the banker decides to lend on the interbank market. The last term is the proceeds from the corporate loan if instead she decides to lend to the firm. In the latter case, only a fraction q^ℓ of the loans are paid back, so the return is $q^\ell \tilde{r}_t^\ell (1 + \phi_t) n_t$, net of the payment of the interbank loan, $r_t^m \phi_t n_t$. At the end of period t, the profits of the bankers within the bank are pooled together, and paid out lump–sum to the household.[13]

To prevent default, lenders must take care never allow ϕ_t to exceed a certain threshold, so that even a low q^ℓ banker has no incentive to borrow and then default. This is reflected in the incentive

[13] In equilibrium, the profit of the bank after dividend pay–outs, π_t^b, will be equal to zero.

compatibility constraint (14). The left hand side of (14) is the net return if the banker defaults. In such a case, the banker gets private benefits, $\gamma(1+\phi_t)n_t$; she defaults on interbank debt and deposits (from Assumption 5); and she pays dividends, $r_t^e \mathrm{e}_t$.[14] The right hand side of (14) is the return of the banker if she lends to other bankers.[15] It is given by the sum of the return on interbank loans (last term) and the return on government bonds, net of the cost of deposits and the dividend pay–outs.

The representative bank chooses s_t^b, d_t, and e_t in the first stage, before its bankers draw their expertise. In the second stage, the bankers draw their q^ℓs and choose ϕ_t and $\mathbb{1}_t\{m\}$ so as to maximize the overall profit of the whole bank (see the time–line in Appendix 6.1). Accordingly, we solve the above maximization problem backward, starting with the choice of ϕ_t and $\mathbb{1}_t\{m\}$.

Second Stage: Choice of ϕ_t and $\mathbb{1}_t\{m\}$. We derive the optimal decision to lend or borrow on the interbank market, given s_t^b, d_t, and e_t. From the linearity of (13), it is clear that banker q^ℓ borrows funds on the interbank market whenever the unit net return on the leveraged funds is strictly positive, *i.e.* whenever $q^\ell \tilde{r}_t^\ell > r_t^m$. Hence,

$$\mathbb{1}_t\{m\} = 0 \Longleftrightarrow q^\ell > \underline{q}_t^\ell \equiv \frac{r_t^m}{\tilde{r}_t^\ell}. \tag{15}$$

When $\mathbb{1}_t\{m\} = 0$ the first derivative of (13) with respect to ϕ_t is strictly positive, and the banker demands as much funding as possible. As a result, (14) binds and determines the banker's borrowing limit, ϕ_t:

$$\phi_t = \frac{r_t^m - r_t^d + r_t^d \frac{\mathrm{e}_t}{\mathrm{e}_t+\mathrm{d}_t} + (r_t^s - r_t^m)\frac{s_t^b}{\mathrm{e}_t+\mathrm{d}_t}}{\gamma\left(1 - \frac{s_t^b}{\mathrm{e}_t+\mathrm{d}_t}\right)} - 1. \tag{16}$$

First Stage: Choice of s_t^b, d_t, and e_t. The representative bank chooses s_t^b, d_t, and e_t so as to maximize its expected profit in (13). Using (12) and the second stage solutions (15) and (16),[16] program (13) rewrites as:

$$\max_{s_t^b, \mathrm{d}_t, \mathrm{e}_t} \Psi_{t-1,t} \left\{ (1+\Delta_t)\left(r_t^m \mathrm{e}_t + (r_t^s - r_t^m)\, s_t^b - \left(r_t^d - r_t^m\right) \mathrm{d}_t \right) - r_t^e \mathrm{e}_t \right\}, \tag{17}$$

where

$$\Delta_t \equiv \frac{\left(1 - \mu_\ell\left(\underline{q}_t^\ell\right)\right)\left(\Omega_t \tilde{r}_t^\ell - r_t^m\right)}{\gamma} \tag{18}$$

with

$$\Omega_t \equiv \int_{\underline{q}_t^\ell}^1 q^\ell \frac{\mathrm{d}\mu_\ell(q^\ell)}{1 - \mu_\ell\left(\underline{q}_t^\ell\right)}. \tag{19}$$

[14] Remember that, by our definition of equity, bankers do not divert funds away from their shareholders.

[15] For a low q^ℓ banker, the only viable alternative to diversion is to lend to other bankers.

[16] The maximisation is subject to (16), because the bank understands that its choice of s_t^b, d_t, and e_t may later affect its interbank borrowing limit.

The quantity Ω_t is the proportion of corporate loans that are paid back; it measures the quality of financial intermediation. The optimal choices of s_t^b, d_t, and e_t, are characterized by the following no–arbitrage conditions

$$r_t^s = r_t^m, \tag{20}$$

$$r_t^d = r_t^m, \tag{21}$$

and

$$r_t^e = (1 + \Delta_t) r_t^d. \tag{22}$$

The no–arbitrage condition (20) indicates that the bank is indifferent between government bonds and interbank loans. For example, if a banker draws a low q^ℓ *ex post* and lends to other bankers, then her opportunity cost of holding government bonds is the return on interbank loans r_t^m. If on the contrary the banker draws a high q^ℓ, then her opportunity cost of government bonds is the cost of borrowing on the interbank market, which is also r_t^m. Similarly, the bank is indifferent between retail and wholesale deposits (relation 21).

The spread $r_t^e - r_t^d$ in (22) is strictly positive. Despite this extra cost, the representative bank is willing to issue equity because this helps alleviate the borrowing constraint its bankers face on the interbank market. Equity is skin–in–the–game, which the bank's creditors use as a disciplinary device (see 16). With one more unit of equity, the bank can increase the interbank borrowing limit by r_t^d/γ.[17] And each additional unit of interbank funding yields $\left(1 - \mu_\ell\left(\underline{q}_t^\ell\right)\right)\left(\Omega_t \tilde{r}_t^\ell - r_t^m\right)$ as net expected gain. The no–arbitrage condition (22) thus states that the net unit gain from equity, Δ_t, must exactly compensate the bank for the extra cost, $(r_t^e - r_t^d)/r_t^d$.

Aggregate Demand and Supply of Interbank Loans. Using relation (15), the aggregate demand for interbank loans is given by

$$\mathrm{m}_t = \phi_t n_t \left(1 - \mu_\ell\left(\underline{q}_t^\ell\right)\right), \tag{23}$$

and the supply for interbank loans by,

$$m_t = \begin{cases} n_t \mu_\ell\left(\underline{q}_t^\ell\right) & \text{if } r_t^m > \gamma \\ \in \left[0, n_t \mu_\ell\left(\frac{\gamma}{\tilde{r}_t^\ell}\right)\right] & \text{if } r_t^m = \gamma \end{cases} \tag{24}$$

The aggregate interbank loan supply curve is vertical when $r_t^m = \gamma$ as, in this case, bankers are indifferent between lending to other bankers and extracting private benefits.

Aggregate Supply of Corporate Loans. Banks' optimal choices determine the aggregate supply of retail loans, which is given by:

$$\ell_t = (1 + \phi_t) n_t \left(1 - \mu_\ell\left(\underline{q}_t^\ell\right)\right). \tag{25}$$

[17] From (16), (20), and (21), it is easy to see that $\partial(\phi_t n_t)/\partial \mathrm{e}_t = r_t^m/\gamma = r_t^d/\gamma$.

2.4 Final Good Producers ("Firms")

The representative firm has one unique project, which consists in producing a homogeneous final good by means of capital k_t, labour h_t, and material goods, x_t. Capital and labour determine the firm's production capacity, represented by a constant returns to scale production function $f(\mathrm{k}_t, \mathrm{h}_t)$ that satisfies standard Inada conditions, so that the firm produces $z \min[f(\mathrm{k}_t, \mathrm{h}_t); \varsigma \mathrm{x}_t]$ consumption goods, where ς and z are strictly positive constants.

Material goods must be paid upfront at the beginning of period t. As the firm is born without resources, it must borrow at the beginning of the period in order to operate its project. The firm may finance the material goods in two ways. It can raises funding b_t through corporate bonds, at contractual rate $\tilde{r}_t^b$. Or it can borrow l_t from the banks, at contractual rate $\tilde{r}_t^\ell$. To keep things simple, we assume that each firm is randomly matched with only one single banker. Whether or not the firm's project is successful is a random outcome, which depends on the expertise of the firm's banker. When a project fails, there is no production, the firm defaults and does not repay its debts, *i.e.* neither the loan nor the bonds. At the end of the period, a successful firm pays the contractual rent on capital, $\tilde{\rho}_t$, the contractual wage, $\tilde{w}_t$, and its debts.

The objective of the firm is to maximize its expected profit,

$$\max_{\mathrm{k}_t, \mathrm{h}_t, \mathrm{x}_t, \mathrm{b}_t, \mathrm{l}_t} \pi_t^f \equiv \Omega_t \left(z \min[f(\mathrm{k}_t, \mathrm{h}_t); \varsigma \mathrm{x}_t] - \tilde{\rho}_t \mathrm{k}_t - \tilde{w}_t \mathrm{h}_t - \tilde{r}_t^b \mathrm{b}_t - \tilde{r}_t^\ell \mathrm{l}_t \right) \tag{26}$$

subject to (19) and the budget constraint:

$$\mathrm{l}_t + \mathrm{b}_t \geq p_t^x \mathrm{x}_t. \tag{27}$$

This maximization program yields the first order conditions:

$$\mathrm{x}_t = \frac{1}{\varsigma} f(\mathrm{k}_t, \mathrm{h}_t) \tag{28}$$

$$\tilde{r}_t^\ell = \tilde{r}_t^b \tag{29}$$

$$p_t^x \mathrm{x}_t = \mathrm{l}_t + \mathrm{b}_t \tag{30}$$

$$\tilde{\rho}_t = \left(z - \frac{\tilde{r}_t^\ell p_t^x}{\varsigma} \right) f_k'(\mathrm{k}_t, \mathrm{h}_t) \tag{31}$$

$$\tilde{w}_t = \left(z - \frac{\tilde{r}_t^\ell p_t^x}{\varsigma} \right) f_h'(\mathrm{k}_t, \mathrm{h}_t). \tag{32}$$

Notice that, as firms may default, the contractual wage, corporate bond yield, rental rate of capital differ from their effective counterparts by the productivity factor Ω_t as follows:

$$\rho_t \equiv \Omega_t \tilde{\rho}_t \quad ; \quad r_t^b \equiv \Omega_t \tilde{r}_t^b \quad ; \quad w_t \equiv \Omega_t \tilde{w}_t.$$

2.5 Material Good Producers

The representative material good producer transforms instantaneously every unit of consumption goods into one unit of material good at the beginning of period t. The objective of the material good producer is to maximize its profit:

$$\max_{x_t} \pi_t^x \equiv p_t^x x_t - x_t,$$

which yields the no–arbitrage condition:

$$p_t^x = 1. \tag{33}$$

2.6 Decentralized General Equilibrium

A general equilibrium of this economy is defined as follows.[18]

Definition 1 ***(Competitive General Equilibrium)*** *A competitive general equilibrium is a sequence of prices,* $\mathcal{P}_t \equiv \{r^s_{t+i}, r^m_{t+i}, r^d_{t+i}, \tilde{r}^b_{t+i}, \tilde{r}^\ell_{t+i}, r^e_{t+i}, w_{t+i}, \rho_{t+i}, p^x_{t+i}\}_{i=0}^{\infty}$*, and a sequence of quantities,* $\mathcal{Q}_t \equiv \{y_{t+i}, c_{t+i}, i_{t+i}, x_{t+i}, k_{t+i}, h_{t+i}, \tilde{a}_{t+i}, d_{t+i}, e_{t+i}, s^h_{t+i}, b_{t+i}, s^b_{t+i}, \ \ell_{t+i}\}_{i=0}^{\infty}$*, such that (i) for a given sequence of prices,* $\mathcal{P}_t$*, the sequence of quantities,* $\mathcal{Q}_t$*, solves the optimization problems of the agents, and (ii) for a sequence of quantities,* $\mathcal{Q}_t$*, the sequence of prices,* $\mathcal{P}_t$*, clears the markets, for* $i = 0, ..., +\infty$*.*

2.6.1 The Credit Quality Transmission Channel

In equilibrium, aggregate output amounts to:

$$y_t = \underbrace{\Omega_t z}_{TFP} \cdot f(k_t, h_t). \tag{34}$$

Total factor productivity (TFP) is composed of the usual exogenous technological factor, z, and an endogenous term, Ω_t which reflects the effective use of technology. Effective TFP is endogenously determined in equilibrium.

The gap between technological and effective TFP, $1 - \Omega_t$, is intimately related to the quality of financial intermediation and how savings are allocated. It depends on the distribution of expertise across bankers, $\mu_\ell(\cdot)$, as well as on the functioning of the interbank market and the financial frictions therein; in particular, the gap increases with the spread between the corporate and the interbank loan rates, $\tilde{r}^\ell_t / r^m_t$ (see relation 19). This relationship between the interest rate spread and aggregate output is at the root of a financial accelerator mechanism. A rise in exogenous productivity z typically induces a rise in the demand for credit by the firms, a rise in the corporate loan rate $\tilde{r}^\ell_t$,

[18] In Boissay, Collard, and Smets (2016) we show that there may exist multiple equilibria on the interbank market. Since those equilibria can be Pareto–ranked, we assume out coordination failures, and assume that only the Pareto–dominant equilibrium is always selected.

and therefore an increase in banks' opportunity cost of extracting private benefits. This, in turn, mitigates the moral hazard problem on the interbank market and improves the allocation of savings within the banking sector.[19] As the quality of credit improves, Ω_t goes up, which amplifies the effects of the initial rise in z. This financial accelerator mechanism is related to changes in credit quality, and distinct from Bernanke, Gertler, and Gilchrist (1999)'s. We dub this transmission channel the "credit quality channel". As we will see later, this channel is going to be the *raison-d'être* of banking regulation in our model. By enhancing the functioning of interbank markets, regulation will improve the reallocation of savings across banks, raise the quality of credit, and exert a positive effect on total factor productivity (see Section 3).

2.6.2 Calibration

The calibration assumes that the economy is *unregulated*, and is based on annual US data over the period 1970–2009. The parameters are listed in Table 2. The calibration of the real sector of the economy is most standard. The household is endowed with preferences over consumption, c_t, and labour, l_t, represented by the following utility function,

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \log(c_{t+\tau}) - \vartheta \frac{h_{t+\tau}^{1+\sigma_h}}{1+\sigma_h} \tag{35}$$

where σ_h denotes the inverse Frish labour supply elasticity and ϑ is a labour dis–utility parameter. We set $\sigma_h = 1$ and ϑ so that the household supplies one unit of labour in the deterministic steady state of the model. Technology is represented by a Cobb–Douglas production function,

$$f(\mathrm{k}_t, \mathrm{h}_t) = \mathrm{k}_t^{\alpha} \mathrm{h}_t^{1-\alpha}.$$

The capital elasticity in the production function, α, is set to 0.3, and the annual rate of depreciation of capital is set to 6% ($\delta = 0.06$). The transformation rate of material goods, ς, is set to 2.782, so that the corporate loan–to–output ratio, $1/\varsigma$, is 35.94%. The discount factor, β is set to 0.98, implying that the steady state capital–to–output ratio, k/y, is 2.32. Exogenous TFP, z, is normalized to one.

The remaining parameters pertain to the financing of the economy, and include: the supply of government bonds by the government $\bar{s}$; the private benefits, γ; and the skill distributions of household members, $\mu_j(q^j)$ (for $j \in \mathcal{J}$). For tractability reasons we assume that

$$\mu_j(q^j) = \left(q^j\right)^{\lambda^j} \tag{36}$$

for all $q^j \in [0, 1]$, with $\lambda^j \geq 0$. Parameters λ^d, λ^e, λ^{s^h}, and λ^b govern the mass of household members who have the skills to invest into deposits, equity, government bonds, and corporate bonds, respectively.

[19]This results from the increase in the overall quality of the lenders, who conduct better due diligence, provide better advice, finance safer projects that are closer to the technological frontier.

Bankers' expertise is drawn from a similar —albeit more flexible— distribution as (36):

$$\mu_\ell(q^\ell) = \begin{cases} \left(\frac{q^\ell-\theta}{1-\theta}\right)^{\lambda^\ell} & \text{if } q^\ell \in [\theta,1] \\ \\ 0 \text{ otherwise} \end{cases} \tag{37}$$

for all $q^\ell \in [0,1]$, with $\lambda^\ell \geq 0$.

All in all, we must assign values to eight financial parameters, $\overline{s}$, γ, λ^d, λ^e, λ^{s^h}, λ^b, λ^ℓ, and θ. They are jointly calibrated so that, in steady state, the model matches the seven interest rates and balance sheet ratios listed in Table 1. The implied values of the parameters are reported in Table 2.

Table 1: Calibrating Targets

Model	Value	Data
$r^m = r^d = r^s$	1.0167	Federal Fund Rate
$\tilde{r}^b$	1.0465	Moody's 3–month Seasoned Baa Corporate Bond Yield
e/d	0.1190	Banks' equity to deposit ratio
b/a	0.0658	Share of corporate bond holdings in households' financial wealth
s^h/a	0.0910	Share of government bonds in households' financial wealth
d/ℓ	1.0310	Bank deposit to loan ratio
$\phi n/d$	1.7086	Non–core to core liabilities ratio
Ω	0.9841	Proportion of non–performing loans

Note: Details on the data sources and how to compute those ratios are provided in Appendix 6.2.

Table 2: Calibration

Parameter		Values
Inverse of Frish elasticity	σ_h	1.000
Labor disutility	ϑ	0.888
Capital elasticity	α	0.300
Capital depreciation rate	δ	0.060
Material good transformation rate	ς	2.782
Discount factor	β	0.980
Exogenous TFP	z	1.000
Supply of government bonds	$\overline{s}$	0.131
Private benefits	γ	0.045
Distribution – $\mu_d(q^d)$	λ^d	456.341
Distribution – $\mu_e(q^e)$	λ^e	0.967
Distribution – $\mu_b(q^b)$	λ^b	5.062
Distribution – $\mu_{s^h}(q^{s^h})$	λ^{s^h}	55.128
Distribution – $\mu_\ell(q^\ell)$		
Curvature	λ^ℓ	0.387
Lower bound	θ	0.959

Figure 3: Cumulative Distribution Functions

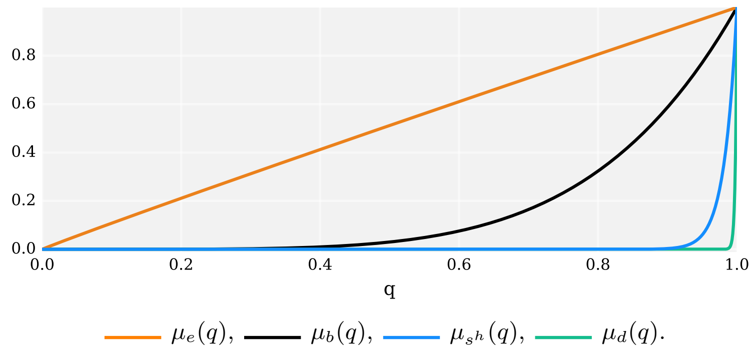


Figure 3 shows how households' skills are distributed. For most household members, the transaction costs on deposits are relatively small, whereas those on equity are relatively high (*e.g.*, the transaction cost on bank equity is above 20% for 80% of US households). Notwithstanding the several transaction costs, the sum of all those costs, χ_t, is relatively small: using our calibration, it amounts to 0.3% of aggregate output in the steady state. The curvature of the cumulative distribution functions is indicative of the elasticity of financial transaction costs to a change in the asset portfolio, as well as of the degree of substitution between the different assets. It therefore appears, for example, that government bonds and deposits (green and blue curves) are the closest substitutes. Those features will be important to keep in mind when later we discuss the link between household portfolio re–balancing and banking regulation (see Section 3.2.2).

3 Regulated Economy

We now consider a social planner/regulator whose mandate is to maximize the representative household's welfare in the steady state. Regulation is justified by the existence of financial externalities. Those hamper the functioning of the interbank market, the re–allocation of savings from the low q^ℓ–bankers to the high q^ℓ–bankers and, ultimately, aggregate productivity (see Section 2.6.1).

The expression of the interbank borrowing limit ϕ_t in (16) is useful to understand the root cause of the externalities, and to identify which form of regulatory intervention is best suited to address them. It shows that the balance sheet structure of a bank *ex ante* (*i.e.* before the q^ℓs are drawn; see Figure 2) affects its borrowing capacity on the interbank market *ex post* (*i.e.* after the q^ℓs are drawn). The latter increases monotonically with the leverage ratio, $\mathrm{e}_t/(\mathrm{d}_t+\mathrm{e}_t)$, the liquidity ratio, $s_t^b/(\mathrm{d}_t+\mathrm{e}_t)$, and the interbank loan rate, r_t^m. The externalities stem from the fact that the representative bank, who takes r_t^m as given, does not fully internalize the impact of its choice of $\mathrm{e}_t/(\mathrm{d}_t+\mathrm{e}_t)$ and $s_t^b/(\mathrm{d}_t+\mathrm{e}_t)$ on the level of ϕ_t. For example, assume it raises $s_t^b/(\mathrm{d}_t+\mathrm{e}_t)$. Then ϕ_t goes up and, holding r_t^m constant, the aggregate demand for interbank loans increases (see (23)). In equilibrium, r_t^m will adjust and rise, though. Some of the low q^ℓ borrowers will switch to lending, and the average quality of the remaining borrowers will improve. Since more expert borrowers have less incentives to divert funds, the borrowing limit ϕ_t will loosen further. The representative bank neglects those general equilibrium feedback effects. From the perspective of the regulator, it undervalues the social benefits from investing into government bonds. Similarly, it undervalues the benefits from issuing equity. These pecuniary externalities call for liquidity and capital regulations.

The timing of the model (see Appendix 6.1) implies that bankers all have the same balance sheet *ex ante*, irrespective of their expertise (see Figure 2). We restrict the regulatory toolbox to constraints on banks' balance sheets and assume that the regulator is unable to collect taxes, and cannot transfer resources across agents. The regulator sets the standards for the banks born in $t-1$ at the end of period $t-1$. While she may constrain banks' balance sheets in many different ways, on the basis of relation (16) we focus on leverage and liquidity constraints.[20]

Regulation 1 ***(Leverage and Liquidity)*** *The representative bank is required to fund itself with a minimum amount of equity (stylized "leverage ratio"),*

$$\frac{\mathrm{e}_t}{\mathrm{d}_t + \mathrm{e}_t} \geq \tau_C, \tag{38}$$

and to hold a minimum amount of government bonds (stylized "liquidity ratio"),[21]

$$\frac{s_t^b}{\mathrm{d}_t + \mathrm{e}_t} \geq \tau_L. \tag{39}$$

As she raises τ_C and τ_L, the regulator reduces bankers' incentives to default on their interbank loans, which relaxes bankers' borrowing constraint, and improves the migration of funds from low to high q^ℓ bankers. This, in turn, reduces the number of non–performing firms, and raises total factor productivity, Ω_t. The regulator may not want to tighten her standards too much, though, as there is a cost to doing so. When she requires banks to issue more equity, for example, she also *de facto* forces the households to purchase more equity. As only few household members have the suitable financial skills (see Figure 3, orange curve), the purchase of equity entails high transaction costs, χ_t. In the end, the regulator must therefore balance the social benefits from a better allocation of credit (rise in Ω_t) against higher financial transaction costs (rise in χ_t).

The optimisation problem of a regulated bank consists in maximizing the profits in (17) subject to the regulatory constraints (38) and (39). The solution of this problem is derived in Appendix 6.3. We discuss the main features of the regulated economy and its equilibrium outcome in the next sections.

[20] Indeed, one can see from (16) that, up to general equilibrium effects (captured by interest rates), the regulator can fully control bankers' borrowing limit using leverage and liquidity requirements. In Section 4.3, we discuss the effects of risk–weighted capital requirements.

[21] Note that the emphasis should be on the intention for banks to hold more liquid (*i.e.* seizable) securities, which in the context of our model take the form of government bonds.

3.1 Regulatory Synergies and Tensions

3.1.1 Synergies

From a partial equilibrium perspective (*i.e.* for given rates of return), it is easy to see from (16), (20), and (21), that the two regulations mutually reinforce each other, *i.e.*:

$$\frac{\partial^2 \phi_t}{\partial(\mathrm{e}_t/(\mathrm{d}_t+\mathrm{e}_t))\partial(s_t^b/(\mathrm{d}_t+\mathrm{e}_t))} > 0.$$

The effect of liquidity holdings on market funding is stronger when banks are better capitalized and, reciprocally, an increase in banks' capital ratio has a bigger impact on market funding when banks hold more government bonds. The intuition is the following. From depositors' and interbank lenders' perspective, cash is "risky", because bankers have the possibility to abscond with it (Assumption 5).[22] When the bank increases its holdings of liquid assets, s_t^b, it *de facto* reduces the volume of risky assets per unit of equity, which enhances the disciplinary effect of equity: the bank has relatively more skin in the game. It follows that equity is more effective in mitigating frictions when the bank holds more liquid assets. In effect, liquidity requirements free up equity capital for interbank transactions. Hence the synergies.

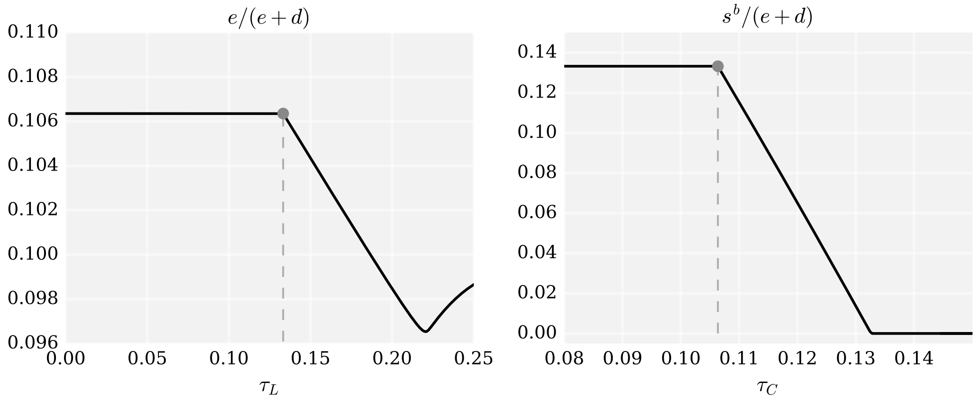
3.1.2 Tensions

Liquidity and capital regulations may also be subject to tensions. Tensions may arise due to subtle, general equilibrium forces that are related to the household re–balancing her portfolio in response to regulation. To see this, assume that banks are required to hold more government bonds. The government bond yield diminishes. As bank deposits are the closest substitutes to government bonds (see Section 2.6.2), the household demands more deposits. Following the rise in demand, the cost of deposits goes down relative to that of bank equity. So, for the banks, the opportunity cost of complying with capital regulation increases. Hence a potential tension.

Figure 4 illustrates these general equilibrium feedback effects. The right panel shows the effect of capital requirements on the bank's liquidity ratio, in the absence of any other requirement. For values of τ_C below the unregulated equilibrium capital ratio (10.6% in the steady state), capital requirements do not bind and, therefore, have no effect on banks' liquidity holdings. For this range of values, the liquidity ratio is constant. Above 10.6%, the leverage ratio requirement binds and the bank's liquidity ratio decreases as the regulator raises τ_C. The reason is that, following the rise in capital requirements, the household must reduce her deposits. As the closest substitute to deposits are government bonds, the household tends to re–balance her portfolio toward government bonds. Banks, in turn, reduce their government bond holdings.

[22] Of course, in equilibrium, there is no diversion, and there is only a distinction between safe and risky assets out of equilibrium.

Figure 4: Steady State Capital and Liquidity Ratios



Left panel: Capital ratio when the regulator imposes a liquidity requirement (τ_L) only. Right panel: Liquidity ratio when the regulator imposes a capital requirement (τ_C) only.

Similarly, the left panel shows the effect of liquidity requirements on banks' capital ratio, in the absence of any other requirements. For values of τ_L below the unregulated equilibrium liquidity ratio (13.3% in the steady state), liquidity requirements do not bind and, therefore, have no effect on banks' equity ratio. Above 13.3%, liquidity requirements bind, banks demand more government bonds, and crowd the households out of government bonds into deposits and, to a lesser extent, into corporate bonds. Banks' leverage ratio decreases, which makes it harder to comply with capital requirements, highlighting the tension that exists between the two regulations. Notice that there is an extreme case, when τ_L is above 22%, when a further tightening in the liquidity standards leads to a rise —not a drop— in banks' leverage ratio (Figure 4, left panel). In this case, the deposit rate and the government bond yield fall by so much that it becomes optimal for the household to substitute government bonds with corporate bonds, as opposed to deposits (Figure 5, left panel); hence the rebound in banks' leverage ratio. Financial dis–intermediation thus acts as a "safety valve" that helps the household cope with the falling returns (we analyze this effect in more detail in Section 4.2).

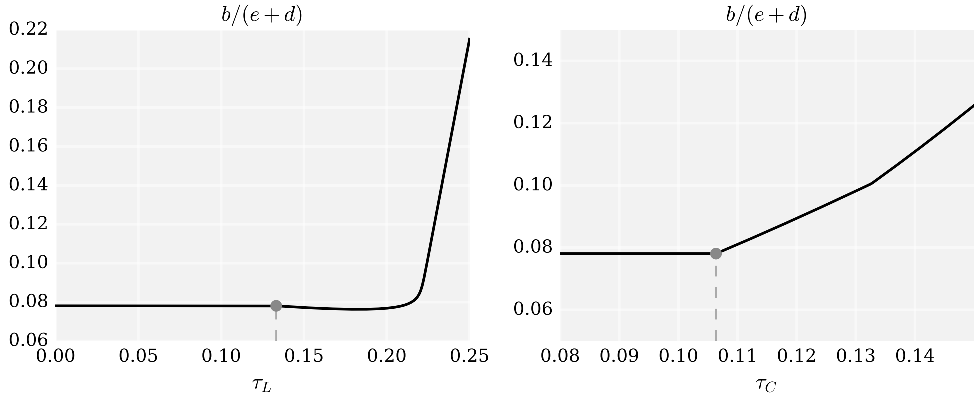
3.2 Regulatory Interactions and Welfare

This section analyzes how regulatory requirements interact with each other and affect welfare, and derives the optimal regulatory mix. The objective of the regulator is to maximize steady state welfare with respect to the regulatory requirements, given private agents' optimal behaviours.

3.2.1 Welfare with Independent Liquidity and Capital Requirements

We start by studying the effects of liquidity and capital requirements, when only one of those requirements is imposed. The right panel of Figure 6 shows how welfare varies as capital requirements tighten, in the absence of liquidity requirements. The hump shape of the relationship illustrates

Figure 5: Steady State Bond to Bank Assets ratio



Left panel: Bond to Bank Assets ratio when the regulator imposes a liquidity requirement (τ_L) only. Right panel: Bond to Bank Assets ratio when the regulator imposes a capital requirement (τ_C) only.

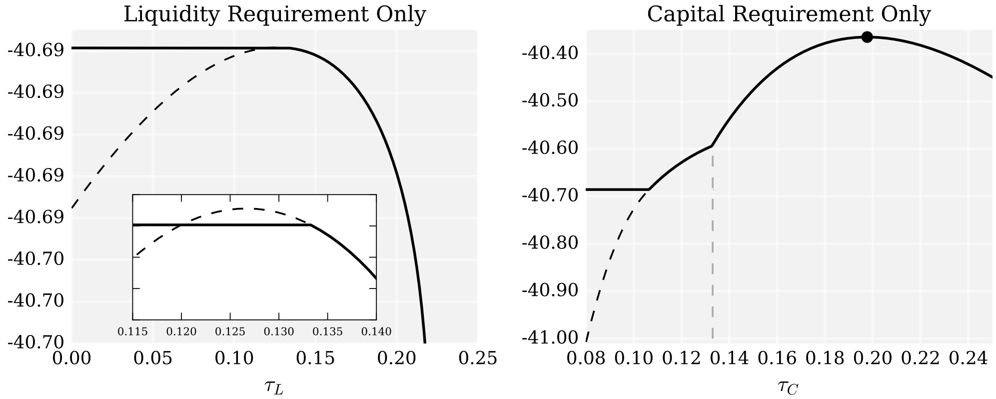
the regulator's trade–off between credit quality and credit quantity.[23] On the one hand, excess deposit funding lowers banks' interbank borrowing limit, which weighs on the demand for interbank loans. The equilibrium interbank loan rate is low, and even banks with a low q^ℓ find it profitable to lend to the firm. As a result, aggregate productivity is low, and so is welfare. In this case, it is optimal for the regulator to require banks to de–leverage. On the other hand, requiring banks to de-leverage can also weigh on welfare. Equity investment induces relatively high transaction costs for the representative household, as only few of her members possess the skills necessary to invest into equity (see Figure 3). Getting unskilled household members to invest into equity has a negative effect on the household's overall return on assets, on savings, and ultimately on the supply of credit to the economy. Figure 6 shows that, for our calibration, the optimal regulatory leverage ratio is 19.9%. This is above the 10.6% ratio that prevails in the unregulated equilibrium.[24]

Similarly, the left panel of Figure 6 shows how welfare varies as liquidity requirements tighten, in the absence of capital requirements. The relationship is also hump–shaped. However, the maximum level of welfare is reached for a liquidity ratio that is below banks' privately optimal ratio. So, if the regulator were to impose this ratio as a unique requirement, the regulation would not be binding. This means that, given our calibration, banks privately hold more liquid assets than what is socially optimal, and therefore that an optimal liquidity regulation *alone* is not effective. In the next section, we show that there is nonetheless a role for liquidity regulation, when it is used in coordination with capital regulation.

[23] The first "kink" in the welfare curve, at τ_C =10.6%, corresponds to the case where capital regulation starts binding. The second kink, at τ_C =13.2% corresponds to the case where banks do not hold government bonds anymore (*i.e.* $s^b = 0$), as shown in the right panel of Figure 4.

[24] To be clear, those ratios are not readily comparable to the requirements established under Basel III. If anything, the formulation chosen in Regulation 1 is closest in spirit to Basel III's leverage ratio, which is defined as regulatory capital over a risk-insensitive measure of total exposure. Risk-weighted capital requirements would generate comparable results only if we assumed an average risk weight of 100% on all assets.

Figure 6: Steady State Welfare Effect of Regulations



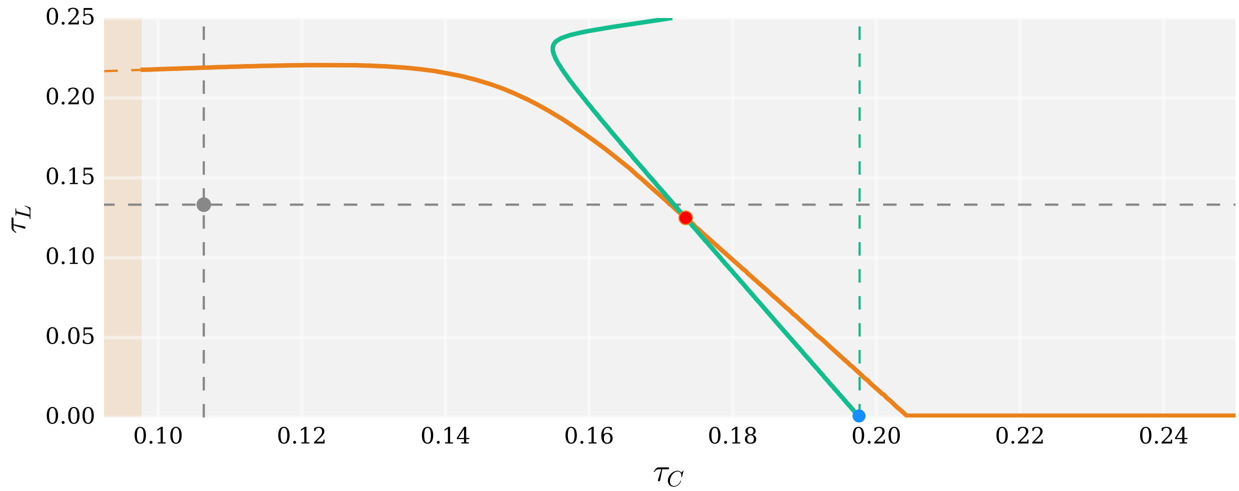
Welfare when the regulator imposes a liquidity requirement τ_L only (left panel) or a capital requirement τ_C (right panel) only. Inside box in the left panel: zoom on the top of the dashed curve.

3.2.2 Regulatory Interactions and Optimal Regulatory Mix

The previous section focused on the effects of liquidity and capital requirements, taken in isolation. We now analyze the two regulations together, and study how they interact with each other. As already discussed in Section 3.1, there can be both synergies and tensions between the two types of requirements. Whether mixing requirements altogether improves welfare is therefore *a priori* not clear. To get a sense of this, we construct two "regulatory frontiers", which we define as the liquidity (resp. capital) requirements τ_L (resp. τ_C) that deliver the highest level of welfare at the steady state for given levels of capital (resp. liquidity) requirements τ_C (resp. τ_L). Each frontier can be thought of as the best response of a regulator who would be in charge of setting τ_L (resp. τ_C) only, to another regulator who would be in charge of setting τ_C (τ_L) only. They are represented in the plain orange (liquidity frontier) and green (capital frontier) curves in Figure 7. The gray dot corresponds to the privately optimal capital and liquidity ratios, which prevail in the unregulated equilibrium. The dashed green vertical line corresponds to the outcome, when the regulator in charge of the capital requirements is "myopic", and sets the latter independently, ignoring the other regulator; as in Figure 6, the capital requirement is then equal to τ_C=19.9%. Accordingly, the blue dot corresponds to the outcome with two "myopic" regulators. The red dot corresponds to the cooperative outcome, when both requirements are set together, *e.g.* by one unique regulator.

The first result that emerges from the figure is that the cooperative outcome (red dot) coincides with the Nash non–cooperative outcome that prevails when the two regulators set the capital and liquidity standards separately, in reaction to each other (intersection of the two regulatory frontiers). Another result is that capital and liquidity requirements mutually reinforce each other *most of the time.* Tighter liquidity requirements (*i.e.* a rise in τ_L) generally allow for lower capital requirements (*i.e.* a decrease in τ_C) than would be the case on a stand–alone basis, and *vice*

Figure 7: Regulatory Frontiers



Liquidity frontier, Capital frontier, Optimal capital regulation w/o liquidity regulation, Optimal regulatory mix, Unregulated equilibrium, Outcome with two myopic regulators.
Orange area: capital requirements do not bind.

versa. This is reflected in the regulatory frontiers being downward sloping. Accordingly, setting regulatory standards independently is sub–optimal. The optimal regulation consists of a mix of capital and liquidity requirements, with τ_C =17.35% and τ_L =12.5%. For those values, both regulatory constraints bind. Moreover, in the regulated equilibrium banks hold slightly less liquid assets and issue significantly more equity than in the unregulated equilibrium (gray and red dots).

Figure 7 also illustrates that there can be certain extreme conditions under which the two regulations interact in unexpected ways. This happens when banks are required to hold most of the government bonds. In this case, represented by the upward–sloping part of the regulatory capital frontier (green curve), a further tightening in liquidity requirements that induces banks to demand even more government bonds, also provokes a massive shift of the household's savings from government bonds into deposits.[25] Hence, banks hold more liquid assets; but they are also more leveraged. The improvement in market funding conditions, which banks obtain from holding more liquid assets, does not make up for the deterioration due to them being more leveraged. At that stage, liquidity requirements have reached their limit in terms of effectiveness. They are so tight that liquid assets have become scarce and liquidity regulation has become costly to comply with. The optimal reaction of the regulator in charge of capital regulation is then to raise capital standards, in a move to force the household to substitute government bonds with securities other than bank deposits, that is, with corporate bonds. The interactions between the two requirements push the economy away from the optimal regulatory mix, which is ultimately detrimental to welfare.

[25]The high values that we find for parameters λ^{s^h} and λ^d (see Table 2) indeed indicate that *(i)* the demand for government bonds is particularly elastic to the government bond yield when the latter is low and *(ii)* that government bonds and bank deposits are the closest substitutes.

To recap, general equilibrium effects can be a source of tensions between liquidity and capital regulations. But those tensions are of second order compared to the direct, partial equilibrium effects that induce synergies.

3.2.3 Welfare with Optimal Regulatory Mix

We now compare welfare in the optimally regulated economy with welfare in the unregulated economy. The results are reported in Table 3 ("NR $\rightarrow$ ORM"). The first column is the permanent consumption gains from regulation, assuming that the economy reaches the regulated equilibrium instantaneously. The net welfare gain is positive, and corresponds to an increase in permanent annual consumption of 0.66%. This is relatively large, both economically and compared to previous studies. For example, in Van den Heuvel (2016) a 10% liquidity requirement and a 10% capital requirement induce a gross welfare loss equivalent to a 0.2% *decrease* in permanent consumption.

Table 3: Welfare Gains

	Perm. cons. gain (%)		Regulation (%)	
	St. St.	Incl. Transition	τ_C	τ_L
NR $\rightarrow$ ORM	0.6591	0.5888	17.35	12.50

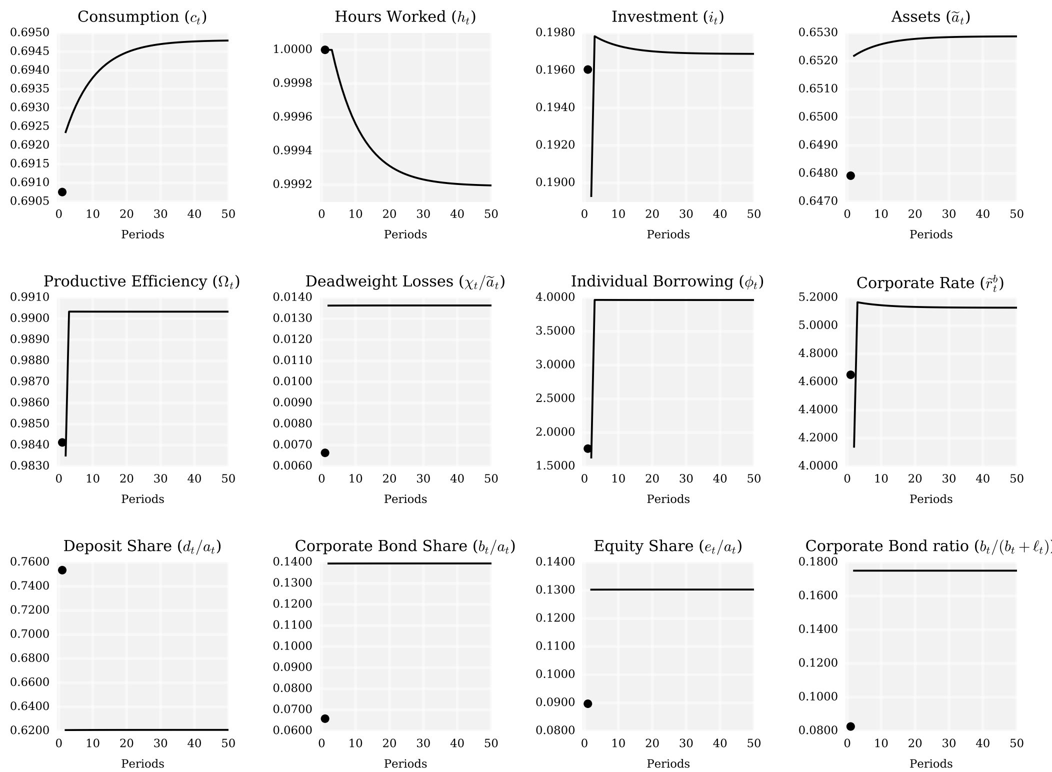
Note: NR $\rightarrow$ ORM: Permanent Consumption gain (in percent) from the non-regulated (NR) economy to the economy with the optimal regulatory mix (ORM).

One issue is whether the optimal regulatory reforms described in Table 3 are desirable, once one takes into account the social cost of the transition from the unregulated economy to the regulated economy. The transition dynamics are reported in Figure 8. Given the absence of adjustment costs, financial variables adjust fast; almost instantaneously. In contrast, real variables, such as consumption and hours worked, adjust more slowly, owing to households' desire to smooth consumption over time.

During the transition, allocative efficiency improves, the proportion of non–performing loans diminishes by 0.4pp (Ω_t rises), and aggregate productivity goes up. Firms demand more capital, labour demand and investment increase. The household invests marginally more into physical capital, and also adjusts her asset portfolio. She substitutes bank deposits with bank equity and corporate bonds. Those adjustments are costly, though: the overall cost of financial transactions, χ_t, doubles. As a result, the size of the banking sector shrinks, and the share of corporate bonds in firms' total external funding goes up.

Following the regulation, financial frictions recede and aggregate wealth rises. The household accumulates more savings, consumes more and works less. Welfare goes up, and does so monotonically. The household enjoys welfare gains even during the transition phase, which makes the regulatory reforms desirable. She does not reap the full benefit from the reforms *immediately*, though. Taking

Figure 8: Transition Toward the Regulated Economy



Note: Transition path from the unregulated to the regulated equilibrium.

delays into account lowers the total net welfare gain of regulation from 0.66% to 0.59% of permanent consumption (see Table 3). From this point of view, transition is costly, but the cost is small. This result is in line with the MAG (2010) report on the impact of the transition to stronger capital and liquidity requirements, which also points to rather small transitional effects.

4 Discussion

4.1 De–leveraging Pressures and Cost of Funding

In our model, de–leveraging pressures can be imposed either explicitly by the regulator, through regulation (as in section 3.2.2); or implicitly by banks' creditors, through market discipline (as in Section 2.3). Compared to private creditors, the regulator puts more de–leveraging pressures on banks, though, so as to get them to internalize the effects of their decisions on aggregate productivity.

Table 4: Costs of Funding

in pp	$r_t^e - r_t^m$	$r_t^d - r_t^m$	$r_t^f - r_t^m$
Non-Regulated	10.72	0.00	0.73
Optimal Regulation	14.49	-2.44	0.29
Difference	3.77	-2.44	-0.44

Note: $r_t^f \equiv \left(r_t^e e_t + r_t^d d_t + r_t^m(1-\mu(\overline{q}_t^\ell))\phi_t n_t\right) / \left(e_t + d_t + (1-\mu(\overline{q}_t^\ell))\phi_t n_t\right)$ denotes the representative bank's overall cost of funding.

Accordingly, Table 4 shows that banks' cost of equity, defined as the difference between the return the marginal investor requires for the funds and the opportunity cost of the funds, $r_t^e - r_t^m$, is 3.77pp higher in the regulated economy than in the unregulated economy. However, since regulated banks supply less deposits, their cost of deposits is lower by 2.44pp, and their total cost of funding drops by 0.44pp. This lower cost of funding reflects banks' lesser incentive to default on deposits and interbank loans, and the fact that banks' creditors price in the better quality of financial intermediation.

4.2 Dis–intermediation as a Safety Valve

The optimal regulatory mix entails a migration of activities from banks to the corporate bond market (see Figure 8). In our model, this dis–intermediation helps the household to mitigate the transaction costs associated with portfolio re–balancing, and acts as a "safety valve".

Those effects contrast with the notion that dis–intermediation reduces the scope and traction of banking regulation (see Bengui and Bianchi, 2014). To illustrate them, we consider a policy maker who can levy a tax on corporate bond revenues, and compute the optimal mix of regulations and taxes. We denote this tax by τ_B, and assume that it is paid by —and rebated lump–sum to— the household (see Appendix 6.4). If the optimal tax is positive, then we would conclude that financial

Table 5: Welfare Analysis

	Perm. cons. gain (%)	Regulation (%)		
		τ_C	τ_L	τ_B
NR $\rightarrow$ ORM+TCBR	0.6604	17.38	12.55	-0.33
ORM $\rightarrow$ ORM+TCBR	0.0013			

Note: NR $\rightarrow$ ORM+TCBR: Permanent Consumption gain (in percent) from the non-regulated (NR) economy to the economy with both the optimal regulatory mix and the tax on corporate bond revenues (OMR+TCBR). ORM $\rightarrow$ ORM+TCBR: Permanent Consumption gain (in percent) from the economy with the optimal regulatory mix (ORM) to the economy with both the optimal regulatory mix and the tax on corporate bond revenues (OMR+TCBR).

migration is detrimental to welfare. If instead it is negative —so that the policy maker subsidizes investments in corporate bonds— then we would conclude that financial migration helps to mitigate the cost of banking regulation. This would reveal the safety valve effect. The results are reported in Table 5. We find that it is optimal to subsidize —not tax— market finance; the optimal subsidy is 0.33%. By reducing the welfare cost of regulatory compliance, the subsidy frees up regulatory capacity, enables the regulator to tighten capital and liquidity standards (a comparison with Table 3 indeed shows that τ_C and τ_L are higher with the subsidy), and helps to further mitigate the agency problem on the interbank market. The welfare gain from doing so is small, though —only a 0.0013% increase in permanent consumption.

4.3 Risk–weighted Capital Requirements

This section aims at assessing the welfare gain from implementing a —stylized— risk–weighted capital requirement, as defined below, as an alternative to multiple leverage and liquidity regulations.

Regulation 2 ***(Risk–weighted Capital)*** *The representative bank is required to hold a minimum amount of equity against its risky assets (stylized "risk–weighted capital ratio"),*

$$\frac{\mathrm{e}_t}{n_t} \geq \tau_W \tag{40}$$

The question is relevant to the extent that the risk–weighted capital ratio can be viewed as a particular combination of the leverage and liquidity ratios.[26] Moreover, such a risk–dependent regulation may be effective in addressing externalities, because it requires a bank to hold equity against its risky assets only (in our case, cash), as opposed to all assets (liquid assets receiving a zero risk–weight). In this sense, it can be more targeted (provided that regulatory risk weights correspond to true risk weights) and, therefore, more frugal than leverage regulation. We are interested in how effective this is.

[26] It is clear from (38), (39), and (40) and the definition of n_t in (12) that one of those three regulations is redundant. In our model, banks' balance sheet structure is indeed too stylized (Figure 2) to warrant more than two regulations at the same time. Hence, in the optimal regulatory mix, $\tau_W \equiv \tau_C/(1-\tau_L)$.

The results are reported in Table 6. The optimal level of stylized risk–weighted capital requirement is $\tau_W = 19.81\%$. With this, the regulator can do *almost* as well in terms of welfare as with the optimal regulatory mix. She can nevertheless raise welfare further, if she complements the risk–weighted capital requirement either with a leverage ratio requirement (with $\tau_C = 17.35\%$) or with a liquidity requirement (with $\tau_L = 12.5\%$), as in Table 3. Indeed, with the additional regulatory constraint, the regulator is able not only to constrain banks' balance sheet structure but also to influence how households re–balance their asset portfolio in response to regulation. The welfare gain from the additional constraint is small, though —equivalent to a 0.0014% increase in permanent consumption.

Table 6: Welfare Analysis

	Perm. cons. gain (%)	Regulation (%)		
		τ_W	τ_C	τ_L
NR $\rightarrow$ RW	0.6576	19.81	-	-
NR $\rightarrow$ ORM*	0.6591	19.83	17.35	12.50
RW $\rightarrow$ ORM	0.0014			

Note: NR $\rightarrow$ RW: Permanent Consumption gain (in percent) from the non-regulated (NR) economy to the economy with the risk–weighted capital requirements (RW). RW $\rightarrow$ ORM: Permanent Consumption gain (in percent) from the risk–weighted capital requirements (RW) economy to the economy with optimal regulatory mix (ORM). $^\star\tau_W \equiv \tau_C/(1-\tau_L)$.

4.4 Leverage Ratio as a Backstop

In the previous section, the regulatory risk–weights correspond to the true risk–weights. In practice, though, the riskiness of banks' various types of assets is not directly observable. Under the so–called "Internal Ratings Based approach (IRB)" of the Basel III framework, banks are allowed —subject to the approval from their supervisors— to use their own credit–risk models to determine the regulatory weights they should apply to each type of assets. Basel III provides that banks should also comply with a simple leverage ratio requirement, with the idea that this additional requirement would act as a backstop if banks understated their risk–weighed assets.[27]

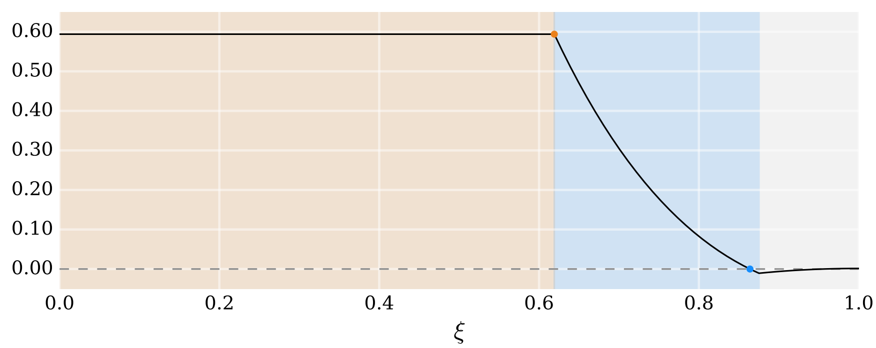
The objective of this section is to discuss the welfare gain from supplementing the risk–weighted capital regulation with a leverage regulation, when the regulatory risk-weights are inaccurate. We also determine the inaccuracy threshold for the risk–weights, above which a backstop improves welfare.

[27] See BCBS (2013) and BCBS (2016). In the latter consultative document, the Basel Committee notably argues (p. 2) that "one of the lessons from the financial crisis is that not all credit risk exposures are capable of being modeled sufficiently reliably or consistently for use in determining regulatory capital requirements. This is supported by various analyses conducted by the Basel Committee that show significant unwarranted variability in risk–weighted assets calculated under the IRB (internal rating based) approaches".

Assume that banks report ξn_t as risk–weighted assets, instead of n_t, with $\xi \in [0, 1)$. The parameter ξ can be viewed as a measure of the accuracy of the reported risk–weights. For the bank, understating the risk–weight is equivalent to lowering the effective regulatory threshold from τ_W to $\xi\tau_W$, and undermines risk–weighted capital regulation.

We compare the level of welfare in two regulatory regimes. The first regime is one where the regulator only imposes the optimal risk–weighted capital requirement, as described in Table 6, with $\tau_W = 19.81\%$. The second regime is one where it imposes the optimal regulatory mix, *i.e.* with a risk–weighted capital requirement of $\tau_W = 19.83\%$ and a leverage ratio requirement of $\tau_C = 17.35\%$ (see Table 6). Since the leverage ratio requirement is independent of ξ, it constitutes a floor for the overall requirement, a backstop. Figure 9 shows the difference in welfare between the two regulatory regimes, for various degrees of mis–reporting.

Figure 9: Leverage Ratio as a Backstop: Welfare Gains



The risk–weighted capital constraint (RWCC) binds, with or without backstop. The RWCC is slack with or without backstop. The RWCC binds without backstop, but is slack with the backstop.

The risk–weighted capital regulation is relaxed as ξ diminishes. As expected, when ξ is close to one and mis–reporting is benign, there is hardly any welfare gain from having a backstop. None of the regulatory mix and the risk weighted capital standard in Table 6 is fully effective, but both requirements bind. In this case, the leverage ratio requirement is not activated as a backstop. The backstop is activated when banks understate their risk–weighted assets by more than 12.5% (*i.e.* ξ below 0.875); on Figure 9, this corresponds to the blue and orange areas. This is the inaccuracy threshold for the risk–weights, above which having a backstop improves welfare.[28] The welfare gain increases with inaccuracy, and is maximal when banks underestimate their risk–weighted assets by more than 38% ($\xi < 0.62$). Beyond this threshold, the risk–weighted assets are so low that, without the backstop, regulation is immaterial (orange area). The maximum welfare gain of having

[28] Based on a sample of German banks, Behn, Haselmann, and Vig (2016) compare the risk–weights applied to loans under the IRB approach with the risk–weights applied to similar loans under the standardized approach (*i.e.* based on an external risk evaluation) and find that under the IRB approach banks systematically understate their risk–weights by around 15%. They also find that under the IRB approach, banks underpredict the actual default rates of the loans by 0.5 to 1 percentage points, whereas there is no such bias under the standardized approach. These results suggest that the 12.5% threshold derived from the model is a reasonable and meaningful number.

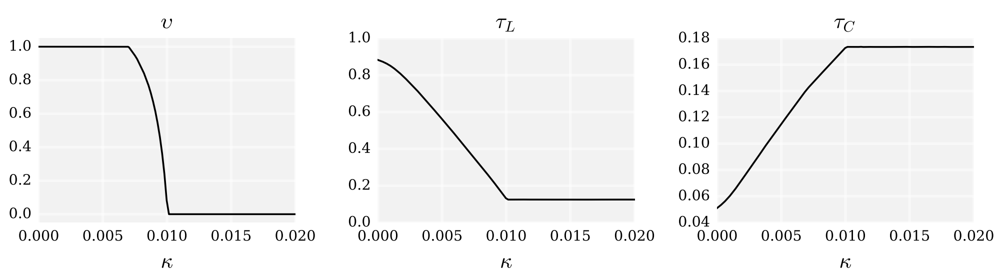
a backstop is equivalent to a 0.6% increase in permanent consumption.

4.5 Sterilization of Liquidity Regulation

One insight from our model is that liquidity regulation may have unintended feedback effects that could reduce the effectiveness of capital regulation. By raising the demand for government bonds, liquidity regulation exerts downward pressures on the yield, leads the household to substitute government bonds with deposits, which in turn gives banks an incentive to leverage up. One way to mitigate those effects is to accommodate banks' higher demand for liquid assets with an increase in the supply. We refer to such a policy as a "sterilization" policy.

The aim of this section is to derive the optimal sterilization policy. To do so, we assume that the government sterilizes a fraction υ of the liquidity requirements by issuing $\upsilon\tau_L(d_t + e_t)$ of new debt, so that the total supply of liquid assets is $\bar{s} + \upsilon\tau_L(d_t + e_t)$. To repay the new debt, the government raises additional lump sum taxes. We allow for some crowding–out effects of sterilization, by assuming that the government consumes a fraction κ of the additional cash it raises. Thus, there is a trade off. On the one hand, sterilization reduces banks' regulatory compliance cost, which is beneficial to welfare. On the other hand, government consumption crowds out private investment and consumption, which is detrimental to welfare. Figure 10 reports the optimal level of sterilization, υ, and the optimal regulatory mix, (τ_C, τ_L), as functions of κ.

Figure 10: Optimal Regulatory Mix with Sterilization



The main result is that sterilization improves welfare only when the government consumes less than 1% of its new debt. That is, when there is little crowding–out. When $\kappa \leq 0.6\%$, in particular, it is optimal to sterilize 100% of the liquidity requirements ($\upsilon = 1$), and the gain in welfare in this case is sizable, and corresponds to a 0.42% increase in permanent consumption (not shown).

The middle and right panels in Figure 10 also show how the optimal regulatory mix changes with the degree of crowding out. By mitigating the adverse portfolio re–balancing effects, sterilization frees up regulatory capacity. For the regulator, it is optimal to tighten liquidity standards, which in turn allows her to relax the capital standards. Thus, as κ goes up, τ_L decreases and τ_C increases.

5 Conclusion

We develop a general equilibrium model with a non–trivial banking sector to help understand the transmission channels of multiple —liquidity and capital— regulations. We use our model to analyze the synergies and potential tensions between those regulations, and to evaluate their effects on the macro–economy and welfare. Consistent with empirical evidence, we emphasize the role of banks in allocating credit and their impact on firm productivity. We also describe a credit quality transmission channel of banking regulations. In our model, the regulator faces the following trade–off. On the one hand, banking regulation may reduce the supply of credit to the economy. On the other hand, it improves credit quality and allocative efficiency. Accordingly, regulation tends to result in less, but more productive lending.

We find that capital and liquidity regulations generally mutually reinforce each other, except when liquid assets are scarce. Even though our results are not directly comparable to the requirements applying under Basel III, we also find that the optimal regulatory mix consists of relatively high capital and liquidity requirements: given our calibration choices, the optimal leverage ratio is around 17.3%, and the optimal liquidity ratio is around 12.5%. In the optimally regulated economy, banks hold slightly less liquid assets but issue significantly more equity than they would in the unregulated economy. Moreover, the cost of equity is higher, but the cost of deposits and average cost of funding are lower, as banks' creditors price in the higher quality of banks' credit supply. Our analysis thus provides broad support for Basel III's "multiple metrics" framework.

To parse the effects of capital and liquidity regulations, we intentionally ignored a number of frictions usually present in standard macro–economic models, such as nominal rigidities or adjustment costs, and left out of our analysis a number of other policy interventions, such as monetary or fiscal policies. Although this goes beyond the scope of this paper, it would certainly be of interest to evaluate how such policies interact with the regulatory mix of the type we consider in this paper. Finally, by construction, our steady state welfare analysis neglects the costs of rare financial crises. If we accounted for those, the net welfare gains from regulation would most likely be higher than those we found. This and other extensions are left to future research.

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6 Appendix

6.1 Timeline

1 The government issues debt $\bar{s}$. Firms produce, pay the wages, pay the rent of physical capital, pay their debts; and die. Banks pay their debts, distribute dividends; and die.

2 The household consumes c_t, invests into i_t units of physical capital goods, and saves $\tilde{a}_{t+1}$.

3 The goods market clears and closes.

4 Household members draw their financial skills (q^{s^h}, q^b, q^d, q^e) and invest $\tilde{a}_{t+1}$ into government bonds s^h_{t+1}, corporate bonds b_{t+1}, bank deposits d_{t+1}, and bank equity e_{t+1}.

5 New banks are born and demand government bonds, s^b_{t+1}, deposits, d_{t+1}, and equity e_t.

6 The government bond, deposit, and equity markets clear and close.

7 Period $t+1$ starts. New firms are born and issue corporate bonds b_{t+1}. Household members purchase corporate bonds. Bankers draw expertise q^ℓ, and invest $\mathrm{d}_{t+1} + \mathrm{e}_{t+1} - s^b_{t+1}$ into corporate loans, ℓ_{t+1}, and interbank loans, m_{t+1}.

8 Firms hire labour h_{t+1}, rent physical capital k_{t+1}, demand loans l_{t+1}, and purchase material goods, x_{t+1}.

9 The markets for labour, capital goods, material goods, corporate bonds, corporate loans, and interbank loans clear and close.

6.2 Calibration

The targeted steady state values of the key financial variables of the model are calculated as follows.

1. $r^m = 1.01674$. The real return on interbank loans matches the Federal Funds Effective Rate and is equal to 1.674%;

   → Sources: Federal Reserve Board, series RIFSPFF.N.A

   → http://www.federalreserve.gov/releases/h15/data.htm

2. $\tilde{r}^b = 1.0465$. The real corporate bond yield matches Moody's 3–month adjusted Seasoned Baa Corporate Bond Yield© and is equal to 4.65%;

   → Source: Federal Reserve Bank of Saint–Louis, series BAA

   → https://research.stlouisfed.org/fred2/series/BAA

   → http://www.federalreserve.gov/releases/h15/data.htm

3. $e/d = 0.1190$. Banks' equity to deposit ratio is equal to 11.90%;

   - e is the dollar amount of FDIC–insured commercial banks' total equity capital;
   - d is the dollar amount of FDIC–insured commercial banks' total domestic deposits;

→ Source: Federal Deposit Insurance Corporation, Tables CB14 (series "Total equity capital") and CB15 (series "Deposits")

→ https://www5.fdic.gov/hsob/HSOBRpt.asp

4. $b/a = 0.0658$. The share of corporate bond holding in households' financial wealth is equal to 6.58%;

    - b is the dollar amount of corporate and foreign bonds in household balance sheets as reported in the US Flows of Funds ($var1$) adjusted for the size of total currency and deposits including money market fund shares in households' balance sheets as reported in the US Flows of Funds ($var2$) relative to the size of deposits in banks' balance sheets as reported by the FDIC (d), *i.e.*, $b = var1 \times d/var2$;
    - a is the dollar amount of households' financial wealth and is calculated as follows: $a = b + d + e + s^h$;
    - s^h is the dollar amount of treasury securities in household balance sheets as reported in the US Flows of Funds ($var3$) adjusted for the size of deposits in households' balance sheets as reported in the US Flows of Funds ($var2$) relative to the size of deposits in banks' balance sheets as reported by the FDIC (d), *i.e.*, $s^h = var3 \times d/var2$;

    → Source: $var1$, $var2$, and $var3$ are from the US Flows of Funds, Table L.101 ("Households and Nonprofit Organization"), series FL154000025.A, FL153061505.A, and FL153063005.A, respectively

    → http://www.federalreserve.gov/datadownload/Choose.aspx?rel=Z.1

5. $s^h/a = 0.0910$. The share of government bonds in households' financial wealth is equal to 9.10%;

6. $d/\ell = 1.0310$. The bank deposit to loan ratio is equal to 103.10%;

    - ℓ is the dollar amount of corporate loans in banks' balance sheets and is calculated as follows: $\ell = b + d + e$.

7. $\phi n/d = 1.7086$. The ratio of no–core liabilities to core liabilities is equal to 170.86%;

    - $\phi n/d$ is ratio of narrow no–core liabilities to core liabilities for the US, over the period 2001–2009;

    → Source: Amidzic, Harutyunyan, Massara, Ugazio, and Walton (2015), Tables 7 and 8

    → https://www.imf.org/external/pubs/ft/wp/2015/wp1501.pdf

8. $\Omega = 0.9841$. The proportion of non–performing loans is 1.58%;

- $1 - \Omega$ is the average delinquency rate on total loans and leases at commercial banks in the US, over the period 1983–2009.

→ Source: Federal Reserve Board.

→ https://www.federalreserve.gov/releases/chargeoff/delallsa.htm

All real rates are deflated using US CPI Inflation from the Federal Reserve Bank of Saint Louis (series CPALTT01USA659N, https://research.stlouisfed.org/fred2).

6.3 Optimal Bank Balance Sheet Structure in the Regulated Economy

The optimisation problem of a regulated bank consists in maximizing the profits in (17) subject to the regulatory capital and liquidity constraints (38) and (39). We denote the Lagrange multipliers associated with those constraints by Λ_C and Λ_L, and examine the possible cases in turn.

$\Lambda_L = \Lambda_C = 0$. None of the regulatory constraints binds. One obtains the no–arbitrage conditions (20), (21), and (22).

$\Lambda_L > 0$ and $\Lambda_C > 0$. Both regulatory constraints bind. One obtains:

$$d_t = \frac{1-\tau_C}{\tau_C} e_t,$$

$$s_t^b = \frac{\tau_L}{\tau_C} e_t,$$

$$r_t^e = (1+\Delta_t)\left(r_t^m + \frac{1-\tau_C}{\tau_C}(r_t^m - r_t^d) - \frac{\tau_L}{\tau_C}(r_t^m - r_t^s)\right).$$

$\Lambda_L = 0$ and $\Lambda_C > 0$. Only the regulatory capital constraint binds. One obtains the no–arbitrage condition (20) and:

$$d_t = \frac{1-\tau_C}{\tau_C} e_t,$$

$$r_t^e = (1+\Delta_t)\left(r_t^m + \frac{1-\tau_C}{\tau_C}(r_t^m - r_t^d)\right).$$

$\Lambda_L > 0$ and $\Lambda_C = 0$. Only the regulatory liquidity constraint binds. One obtains:

$$s_t^b = \tau_L(e_t + d_t),$$

$$r_t^d = r_t^m - \tau_L(r_t^m - r_t^s),$$

$$r_t^e = (1+\Delta_t)\, r_t^d.$$

6.4 Summary of the Model

This section summarizes the equations of the model for the regulated and the unregulated economies.[29]

Representative Consumption Good Producer

1. $y_t = z\Omega_t k_t^{\alpha} h_t^{1-\alpha}$
2. $\rho_t = \left(1 - \frac{\tilde{r}_t^{\ell} p_t^x}{\varsigma z}\right) \alpha \frac{y_t}{k_t}$
3. $w_t = \left(1 - \frac{\tilde{r}_t^{\ell} p_t^x}{\varsigma z}\right) (1-\alpha) \frac{y_t}{h_t}$
4. $\tilde{r}_t^{\ell} = \tilde{r}_t^{b}$
5. $\varsigma x_t = k_t^{\alpha} h_t^{1-\alpha}$
6. $x_t = \ell_t + b_t$

Representative Intermediate Good Producer

1. $p_t^x = 1$

Government

1. $T_t = (r_t^s - 1)\overline{s}$

Representative Household

1. $\vartheta h_t^{\sigma_h} = \frac{w_t}{c_t}$
2. $k_{t+1} = i_t + (1-\delta)k_t$
3. $r_{t+1} = \rho_{t+1} + 1 - \delta$
4. $\Psi_{t,t+1} r_{t+1} = 1$
5. $s_t^h = \tilde{a}_t \int_0^1 q^{s^h} \mu_d \left(q^{s^h} \frac{r_t^s}{r_t^d}\right) \mu_b \left(q^{s^h} \frac{r_t^s}{(1-\tau_B) r_t^b}\right) \mu_e \left(q^{s^h} \frac{r_t^s}{r_t^e}\right) \mathrm{d}\mu_{s^h}(q^{s^h})$
6. $$d_t = \tilde{a}_t \int_0^{\frac{r_t^s}{r_t^d}} q^d \mu_b \left(q^d \frac{r_t^d}{(1-\tau_B) r_t^b}\right) \mu_e \left(q^d \frac{r_t^d}{r_t^e}\right) \mu_{s^h} \left(q^d \frac{r_t^d}{r_t^s}\right) \mathrm{d}\mu_d(q^d)$$
$$+ \tilde{a}_t \int_{\frac{r_t^s}{r_t^d}}^1 q^d \mu_b \left(q^d \frac{r_t^d}{(1-\tau_B) r_t^b}\right) \mu_e \left(q^d \frac{r_t^d}{r_t^e}\right) \mathrm{d}\mu_d(q^d)$$
7. $$b_t = \tilde{a}_t \int_0^{\frac{r_t^s}{(1-\tau_B) r_t^b}} q^b \mu_d \left(q^b \frac{(1-\tau_B) r_t^b}{r_t^d}\right) \mu_e \left(q^b \frac{(1-\tau_B) r_t^b}{r_t^e}\right) \mu_{s^h} \left(q^b \frac{(1-\tau_B) r_t^b}{r_t^s}\right) \mathrm{d}\mu_b(q^b)$$
$$+ \tilde{a}_t \int_{\frac{r_t^s}{(1-\tau_B) r_t^b}}^{\frac{r_t^d}{(1-\tau_B) r_t^b}} q^b \mu_d \left(q^b \frac{(1-\tau_B) r_t^b}{r_t^d}\right) \mu_e \left(q^b \frac{(1-\tau_B) r_t^b}{r_t^e}\right) \mathrm{d}\mu_b(q^b)$$
$$+ \tilde{a}_t \int_{\frac{r_t^d}{(1-\tau_B) r_t^b}}^1 q^b \mu_e \left(q^b \frac{(1-\tau_B) r_t^b}{r_t^e}\right) \mathrm{d}\mu_b(q^b)$$

[29] Note that relations 5–8 for the representative household are written under the assumption that $r_t^s < r_t^d$. Of course, those interest rates being endogenous, it may occur that $r_t^s > r_t^d$, depending on the parameters of the model and the regulatory regime. In this case those relations must be modified accordingly. Our numerical algorithm takes care of such situations.

8. $e_t = \tilde{a}_t \int_0^{\frac{r_t^s}{r_t^e}} q^e \mu_d \left(q^e \frac{r_t^e}{r_t^d} \right) \mu_b \left(q^e \frac{r_t^e}{(1-\tau_B) r_t^b} \right) \mu_{s^h} \left(q^e \frac{r_t^e}{r_t^s} \right) \mathrm{d}\mu_e(q^e)$

$+ \tilde{a}_t \int_{\frac{r_t^s}{r_t^e}}^{\frac{r_t^d}{r_t^e}} q^e \mu_d \left(q^e \frac{r_t^e}{r_t^d} \right) \mu_b \left(q^e \frac{r_t^e}{(1-\tau_B) r_t^b} \right) \mathrm{d}\mu_e(q^e)$

$+ \tilde{a}_t \int_{\frac{r_t^d}{r_t^e}}^{\frac{(1-\tau_B) r_t^b}{r_t^e}} q^e \mu_b \left(q^e \frac{r_t^e}{(1-\tau_B) r_t^b} \right) \mathrm{d}\mu_e(q^e)$

$+ \tilde{a}_t \int_{\frac{(1-\tau_B) r_t^b}{r_t^e}}^{1} q^e \mathrm{d}\mu_e(q^e)$

Representative Bank

1. $\phi_t = \dfrac{r_t^m e_t - (r_t^d - r_t^m) d_t + (r_t^s - r_t^m) s_t^b}{\gamma (e_t + d_t - s_t^b)} - 1$
2. $\ell_t = n_t$
3. $n_t = d_t + e_t - s_t^b$
4. $m_t = n_t \mu_\ell \left(\underline{q}_t^\ell \right)$
5. $\mathrm{m}_t = \phi_t n_t \left(1 - \mu_\ell \left(\underline{q}_t^\ell \right) \right)$

If none of the regulatory constraints (38) *and* (39) *binds:*

6. $r_t^e = (1 + \Delta_t) \, r_t^d$
7. $r_t^m = r_t^d$
8. $r_t^m = r_t^s$

If both the regulatory constraints (38) *and* (39) *bind:*

6. $d_t = \dfrac{1 - \tau_C}{\tau_C} e_t$
7. $s_t^b = \tau_L (e_t + d_t)$
8. $r_t^e = (1 + \Delta_t) \left(r_t^m + \dfrac{1 - \tau_C}{\tau_C} (r_t^m - r_t^d) - \dfrac{\tau_L}{\tau_C} (r_t^m - r_t^s) \right)$

If only regulatory constraint (38) *binds:*

6. $d_t = \dfrac{1 - \tau_C}{\tau_C} e_t$
7. $r_t^m = r_t^s$
8. $r_t^e = (1 + \Delta_t) \left(r_t^m + \dfrac{1 - \tau_C}{\tau_C} (r_t^m - r_t^d) \right)$

If only regulatory constraint (39) *binds:*

6. $r_t^e = (1 + \Delta_t) \, r_t^d$
7. $s_t^b = \tau_L (e_t + d_t)$
8. $r_t^m = r_t^d + \tau_L (r_t^m - r_t^s)$

Market Clearing

1. $y_t = c_t + i_t + x_{t+1} + \chi_t$
2. $s_t^h + s_t^b = \overline{s}$
3. $m_t = \mathrm{m}_t$
4. $h_t = \mathrm{h}_t$
5. $\ell_t = \mathrm{l}_t$
6. $b_t = \mathrm{b}_t$
7. $k_t = \mathrm{k}_t$
8. $\mathrm{x}_t = x_t$
9. $\mathrm{e}_t = e_t$

Definitions

1. $r_t \equiv (1 - \tau_B) r_t^b \dfrac{b_t}{\tilde{a}_t} + r_t^s \dfrac{s_t^h}{\tilde{a}_t} + r_t^d \dfrac{d_t}{\tilde{a}_t} + r_t^e \dfrac{e_t}{\tilde{a}_t}$
2. $\Psi_{t,t+1} \equiv \beta \dfrac{u'(c_{t+1})}{u'(c_t)}$
3. $\chi_t \equiv \tilde{a}_{t+1} - \overline{s} - x_{t+1}$
4. $\underline{q}_t^\ell \equiv \dfrac{r_t^m}{\tilde{r}_t^\ell}$
5. $\Omega_t \equiv \displaystyle\int_{\underline{q}_t^\ell}^1 q^\ell \dfrac{\mathrm{d}\mu_\ell(q^\ell)}{1 - \mu_\ell\left(\underline{q}_t^\ell\right)}$
6. $r_t^b \equiv \Omega_t \cdot \tilde{r}_t^b$
7. $\Delta_t \equiv \dfrac{\left(1 - \mu_\ell\left(\underline{q}_t^\ell\right)\right)\left(\Omega_t \tilde{r}_t^\ell - r_t^m\right)}{\gamma}$