Imperfect Information and Inflation Dynamics*

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Abstract

The original version of the new Keynesian (NK) model has important empirical limitations, in particular with regard to inflation, output and interest rate dynamics. Some of its recent extensions fare better empirically but only with the help of controversial pricing schemes. We offer an alternative approach that relies on imperfect information. We demonstrate that under plausible informational imperfections, the presence of a signal extraction problem leads to inflation persistence, realistic inflation and output dynamics and a liquidity effect.

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Introduction

The New Keynesian model has gained wide acceptance. Nevertheless, the original version of the model has a number of important implications that seem to be at variance with the empirical evidence. Most prominent among them are the predicted monotone dynamics of inflation, output and nominal interest rates in response to a monetary shock. This is inconsistent with evidence presented by Christiano, Eichenbaum and Evans (2005) suggesting that following a monetary shock, there is a delayed, hump shaped response of inflation, a hump shaped response of output and a liquidity effect.

A great deal of recent work has been devoted to addressing these difficulties. Broadly speaking, two distinct approaches have been pursued. The first, advocated by Mankiw and Reis, 2002, adheres to full rationality and assumes information rather than price stickiness. While this seems attractive and is reminiscent of Lucas' imperfect information model, practically, it does not provide a solution to the problem. As shown by Collard and Dellas, 2004, the sticky information version requires an implausibly large amount of stickiness (what Collard and Dellas term a time warp) in order to generate satisfactory macro-dynamics.

The second approach involves the partial abandonment of rational expectations. For instance, Gali and Gertler (1999) assume the existence of myopic agents who set prices in a mechanical fashion. In a similar vein, Christiano, Eichenbaum and Evans (2005) assume that a fraction of the agents index their prices to past inflation[1]. With the inclusion also of several *real* rigidities, the modified version of the NK model has proved very successful in generating inertial movements in inflation, output and the nominal interest rate (Christiano, Eichenbaum and Evans, 2005). Nonetheless, the problem with this approach is that, in addition to abandoning full rationality, it is also at variance with observed pricing patterns. As a recent ECB report documents (Dhyne et al. 2005), nominal prices remain flat in between infrequent price jumps. A specification that relies on price indexation in order to generate realistic inflation dynamics is problematic (see Collard and Dellas, 2005).

In this paper we propose a third approach, which matches the empirical success of the Christiano et al (2005) model regarding macro-dynamics *without* having a need for a backward price indexation scheme. We emphasize the same signal extraction problem that was at the heart of Lucas' 1972, imperfect information, rational expectations model but in the context of a model of the NK variety. That is, in a model where firms (and or workers) set prices optimally for more than one period[2]. Unlike the original Lucas model which required that nominal aggregates

[1] Minford and Peel (2004) argue that allowing such agents to adjust prices based on expected rather than on past inflation eliminates the new Phillips curve.

[2] Woodford, 2002, examines the dynamic effects of nominal shocks in a model with signal extraction but with flexible prices and wages. We have found that such a model does not perform as well as its fixed, staggered prices

(such as the nominal interest rate, inflation, the money stock etc.) could only be observed with long lags in order to generate persistence (a feature that undermined its plausibility), our formulation fares well even when relatively fast and accurate observation of nominal aggregates is permitted[3].

We show that a specification with a plausibly small amount of imperfect information is considerably superior to the standard, full information version. It also performs as well as the leading NK model(Christiano et al. 2005) regarding macroeconomic dynamics and has similar properties regarding unconditional properties. Signal extraction allows the model to produce a weak instantaneous response to current shocks. A delayed, hump shaped response of inflation and output following a monetary shock. And a liquidity effect.

The remaining of the paper is organized as follows. Section 1 presents the model. Section 2 discusses the calibration. Section 3 presents the main results. The last section contains the conclusions.

1 The model

The set up is the new Keynesian model with price rigidities, augmented to include various real rigidities. Below we describe the behavior of the firms and the households. The production side of the economy consists of two sectors: one producing intermediate goods and the other a final good. The intermediate good is produced with capital and labor and the final good with intermediate goods. The final good is homogeneous and can be used for consumption (private and public) and investment purposes.

1.1 Final sector

The final good, Y is produced by combining intermediate goods, X_i, by perfectly competitive firms. The production function is given by

$$y_t = \left(\int_0^1 y_{it}^{\theta} \mathrm{d}i \right)^{\frac{1}{\theta}} \tag{1}$$

where $\theta \in (-\infty, 1)$. Profit maximization and free entry lead to the general price index

$$P_t = \left(\int_0^1 P_{it}^{\frac{\theta}{\theta-1}} \mathrm{d}i \right)^{\frac{\theta-1}{\theta}} \tag{2}$$

The final good may be used for consumption — private or public — and investment purposes.

counterpart.

[3]Naturally, *some* imperfect information with regard to these variables must remain, otherwise the signal extraction problem collapses. We offer a more detailed discussion of this issue later in section 2.

1.2 Intermediate goods producers

Each firm i, $i \in (0,1)$, produces an intermediate good by means of capital and labor according to a constant returns–to–scale technology, represented by the Cobb–Douglas production function

$$y_(it) = a_t(u_{it}k_{it})^{\alpha} n_{it}^{1-\alpha} \text{ with } \alpha \in (0,1) \tag{3}$$

where k_{it} and n_{it} respectively denote the physical capital and the labor input used by firm i in the production process. a_t is an exogenous, stationary, stochastic, technology shock, whose properties will be defined later. Assuming that each firm i operates under perfect competition in the input markets, the firm determines its production plan by minimizing its total cost

$$\min_{\{u_{it}k_{it}, n_{it}\}} P_t W_t n_{it} + P_t z_t u_{it} k_{it}$$

subject to (3). This leads to the following expression for total costs:

$$P_t s_t y_{it}$$

where the real marginal cost, S, is given by $\frac{W_t^{1-\alpha} z_t^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha} a_t}$.

Intermediate goods producers are monopolistically competitive, and therefore set prices for the good they produce. We follow Calvo, 1983, in assuming that firms set their prices for a stochastic number of periods. In each and every period, a firm either gets the chance to adjust its price (an event occurring with probability γ) or it does not. If it does not get the chance, then we will assume that it sets its price according to one of two scenaria. In the first one, which will be used in the version of the model with the signal extraction formulation, the price is assumed to remain fixed until the firm gets a call that allows it to reset its price optimally. In our view, this is the more realistic scenario as the evidence on price setting suggests that firms set their prices infrequently and discretely, and in between price jumps, prices remain constant. The second scenario is the one that has been suggested by Christiano et al., 2005, where the firms set prices according to

$$P_{it} = \xi_t P_{it-1} \tag{4}$$

where $\xi_t = \pi_{t-1}$ with $\pi_t = P_t/P_{t-1}$. That is, the firms index their prices to the lagged, economy wide rate of inflation. This scheme is quite popular in the literature in spite of the fact that it is not rational[4], and it also introduces a completely free parameter.

On the other hand, a firm i that sets its price optimally in period t chooses a price, $P_t^{\star}$, in order to maximize:

$$\max_{P_t^{\star}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \Phi_{t+\tau}(1-\gamma)^{\tau} \left(P_t^{\star}\Xi_{t,\tau} - P_{t+\tau}s_{t+\tau}\right) y_{it+\tau}$$

[4]The firms could easily index their price to the expected aggregate rate of inflation instead. Such information is as readily available as that on lagged inflation from surveys, central bank forecasts or targets and so on.

subject to the total demand it faces

$$y_{it+\tau} = \left(\frac{P_t^\star \Xi_{t,\tau}}{P_{t+\tau}}\right)^{\frac{1}{\theta-1}} y_{t+\tau}$$

and where

$$\Xi_{t+\tau} = \begin{cases} \prod_{\ell=0}^{\tau-1} \xi_{t+\ell} & \text{for } \tau \geqslant 1 \\ 1 & \tau = 0 \end{cases}$$

$\Phi_{t+\tau}$ is an appropriate discount factor derived from the household's evaluation of future relative to current consumption. This leads to the price setting equation

$$P_t^\star = \frac{1}{\theta} \frac{\mathbb{E}_t \sum_{\tau=0}^{\infty} (1-\gamma)^\tau \Phi_{t+\tau} P_{t+\tau}^{\frac{2-\theta}{1-\theta}} \Xi_{t,\tau}^{\frac{1}{\theta-1}} s_{t+\tau} y_{t+\tau}}{\mathbb{E}_t \sum_{\tau=0}^{\infty} (1-\gamma)^\tau \Phi_{t+\tau} \Xi_{t,\tau}^{\frac{\theta}{\theta-1}} P_{t+\tau}^{\frac{1}{\theta-1}} y_{t+\tau}} \tag{5}$$

Since the price setting scheme is independent of any firm specific characteristic, all firms that reset their prices will choose the same price.

In each period, a fraction γ of contracts ends and $(1-\gamma)$ survives. Hence, from (2) and the price mechanism, the aggregate intermediate price index writes

$$P_t = \left(\gamma P_t^{\star \frac{\theta}{\theta-1}} + (1-\gamma)(\xi_t P_{t-1})^{\frac{\theta}{\theta-1}}\right)^{\frac{\theta-1}{\theta}} \tag{6}$$

1.3 The Household

There exists an infinite number of households distributed over the unit interval and indexed by $j \in [0,1]$. Households have market power over the labor services they provide. The preferences of household j are given by

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau \left[\log(c_{t+\tau} - \vartheta c_{t+\tau-1}) + \frac{\nu^m}{1-\sigma_m}\left(\frac{M_{t+\tau}}{P_{t+\tau}}\right)^{1-\sigma_m} - \frac{\nu^h}{1+\sigma_h} h_{jt+\tau}^{1+\sigma_h}\right] \tag{7}$$

where $0 < \beta < 1$ is a constant discount factor, c_t denotes consumption in period t, M_t/P_t is real balances and h_{jt} is the quantity of labor supplied by the representative household of type j. ϑ is the parameter of habit persistence.

In each period, household j faces the budget constraint

$$E_t B_{t+1} Q_t + M_t + P_t(c_t + i_t + a(u_t)k_t) = B_t + M_{t-1} + P_t z_t u_t k_t + W_{jt} h_{jt} + \Omega_t + \Pi_t \tag{8}$$

where B_t is state contingent deliveries of the final good and Q_t is the corresponding price of the asset that delivers these goods. M_t is end of period t money holdings. P_t, the nominal price of goods. c_t and i_t are consumption and investment expenditure respectively; k_t is the amount of physical capital owned by the household and leased to the firms at the real rental rate z_t. Only a fraction u_t of the capital stock is utilized in any period, which involves an increasing and convex cost $a(u)$. W_{jt} is the nominal wage (specific to individual j). Ω_t is a nominal lump-sum transfer received from the monetary authority and Π_t denotes the profits distributed to the household by the firms.

Capital accumulates according to the law of motion

$$k_{t+1} = (1-\delta)k_t + \Phi(i_t, i_{t-1}, k_t) \tag{9}$$

where $\delta \in [0,1]$ denotes the rate of depreciation. $\Phi(\cdot)$ is a general specification that allows the modelling of either capital or investment adjustment costs (its properties will be discussed later).

The workers have monopoly power over their labor services. They sell these services to competitive firms that produce aggregate labor services using the following technology:

$$n_t = \left(\int_0^1 h_{jt}^{\theta_w} \mathrm{d}j\right)^{\frac{1}{\theta_w}} \tag{10}$$

where $\theta_w \in (-\infty, 1)$. Profit maximization and free entry condition on the market implies that aggregate wage takes the form

$$W_t = \left(\int_0^1 W_{jt}^{\frac{\theta_w}{\theta_w - 1}} \mathrm{d}i\right)^{\frac{\theta_w - 1}{\theta_w}} \tag{11}$$

In the current version we report results corresponding to the case of flexible wages. The results do not differ when we allow for nominal wage sluggishness in place or along side price stickiness.

1.4 The monetary authorities

Our specification of monetary policy involves an exogenous money supply rule, where money evolves according to

$$M_t = \exp(\mu_t) M_{t-1} \tag{12}$$

The gross growth rate of the money supply, μ_t, is assumed to follow an exogenous stochastic process whose properties will be defined later.

1.5 The government

The government finances government expenditure on the domestic final good using lump sum taxes. The stationary component of government expenditures is assumed to follow an exogenous stochastic process, whose properties will be defined later.

2 Parametrization

For comparison purposes, the parametrization of the model relies heavily on Christiano, Eichenbaum and Evans, 2005. The model is parameterized on US quarterly data for the post WWII period. When necessary, the data are taken from the Federal Reserve Database.[5] The parameters are reported in table 1.

The accumulation function $\Phi(i_t, i_{t-1}, k_t)$ is assumed to take the following form

$$\Phi(i_t, i_{t-1}, k_t) = \left(1 - \omega S\left(\frac{i_t}{i_{t-1}}\right) - (1-\omega)\frac{\varphi}{2}\left(\frac{i_t}{k_t} - \delta\right)^2 \frac{k_t}{i_t}\right) i_t$$

The function $S(\cdot)$ satisfies $S(1) = S'(1) = 0$ and $S''(1) = \varphi > 0$. $\Phi(i_t, i_{t-1}, k_t)$ nests two theories of accumulation frictions. Setting $\omega = 1$ we recover the specification used in Christiano et al., 2005, where investment is subject to adjustment costs. Setting $\omega = 0$, we obtain the standard capital adjustment costs specification. The capital utilization function $a(u_t)$ satisfies $a(1) = 0$, $a''(1)/a'(1) = 1/\sigma_a$. $\sigma_a = 100$. The investment adjustment cost parameter φ is chosen so that the model can match the first order autocorrelation of output (0.84). This gives rise to $\varphi = 0.33$. The results do not differ when we borrow the value of φ from Christiano et al. 2005, ($\varphi = 2.5$) instead of calibrating it.

The stochastic technology shock, $a_t = \log(A_t/\overline{A})$, is assumed to follow a stationary AR(1) process of the form

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$$

with $|\rho_a| < 1$ and $\varepsilon_{a,t} \rightsquigarrow \mathcal{N}(0, \sigma_a^2)$. $\rho_a = 0.95$. The value of σ_a was selected so that the model matches the volatility of output (1.49). This gave $\sigma_a = 0.0042$, a value close to that used by King and Rebelo (2000).

The government spending shock[6] is assumed to follow an AR(1) process

$$\log(g_t) = \rho_g \log(g_{t-1}) + (1-\rho_g)\log(\overline{g}) + \varepsilon_{g,t}$$

with $|\rho_g| < 1$ and $\varepsilon_{g,t} \sim \mathcal{N}(0, \sigma_g^2)$. The persistence parameter is set to, ρ_g, of 0.968 and the standard deviation of innovations is $\sigma_g = 0.0104$. The government spending to output ratio is set to 0.20.

The gross growth rate of money evolves according to

$$\mu_t = (1-\rho_\mu)\overline{\mu} + \rho_\mu \mu_{t-1} + \epsilon_{\mu t}$$

[5]URL:`http://research.stlouisfed.org/fred/`

[6]The –logarithm of the– government expenditure series is first detrended using a linear trend.

where $|\rho_m| < 1$, $\overline{\mu} = E(\mu_t)$ and $\epsilon_{\mu t}$ is a Gaussian white noise process with mean 0 and standard deviation σ_μ. We set $\rho_\mu = 0.5$ and choose the value of σ_μ in order to match the volatility of inflation (0.16). This resulted in $\sigma_\mu = 0.0017$. The nominal growth of the economy, $\overline{\mu}$, is set such that the average quarterly rate of inflation over the period is $\overline{\pi} = 1.2\%$ per quarter.

Table 1: Calibration: Benchmark case

Preferences		
Discount factor	β	0.988
Habit persistence	ϑ	0.650
Inverse labor supply elasticity	σ_h	1.000
Money demand elasticity	σ_m	10.500
Technology		
Capital elasticity of intermediate output	α	0.281
Parameter of markup	θ	0.850
Depreciation rate	δ	0.025
Probability of price resetting	γ	0.250
Shocks and policy parameters		
Steady state money supply growth (gross)	μ	1.012
Share of government spending	g/y	0.200

Table 2: Shocks

	ρ	σ
Technology	0.9500	0.0042
Fiscal	0.9684	0.0104
Money supply	0.5000	0.0017

Information

We now specify the structure of information. We assume that the agents observe their own, individual specific variables (consumption, technology shock, capital stock and so on) but cannot always aggregate this information. As a result they do not fully know the true current aggregate state of the economy and can only gradually learn about it using the Kalman filter, based on a set of perfectly and imperfectly observable aggregate variables (signals). We will assume that some of the aggregate variables are perfectly observed, some are not observed at all and some are observed with error by the agents. In particular, for mis–measured variable x

$$x_t^\star = x_t^{\mathrm{T}} + \xi_t$$

where x_t^{T} denotes the value of the variable under perfect information and ξ_t is a noisy process that satisfies $E(\xi_t) = 0$ for all t; $E(\xi_t \varepsilon_{a,t}) = E(\xi_t \varepsilon_{g,t}) = 0$; and

$$E(\xi_t \xi_k) = \begin{cases} \sigma_\xi^2 & \text{if } t = k \\ 0 & \text{Otherwise} \end{cases}$$

Knowledge of the *aggregate* state of the economy matters for the agents because individual price setting depends on expectations of future nominal marginal cost and marginal revenue, which in turn depend on future aggregate prices, wages and so on. Obviously, for the informational considerations emphasized in this paper to be operative, it is essential that the specification of information satisfy two principles. First, it does not allow agents to immediately infer the true state of the economy based on the available signals. Otherwise, the signal extraction problem disappears, the agents always know the state of the economy and the model collapses to that under perfect information. There are various ways to ensure a meaningful signal extraction problem. One is to be have many variables that are subject to noise. An equivalent but perhaps less transparent way is to reduce the model through substitution to a small number of equations and variables and then impose noise on a smaller number of variables.

The second principle is that the informational constraints are sensible. That is, the location, timing and amount of noise is plausible. This requires fairly clear signals on easily observable aggregate variables (such as the nominal interest rate). And mis-perceptions about monetary aggregates and other nominal variables that are neither implausibly large nor too persistent. Without loss of generality we assume that the agents receive noisy signals on the key aggregate nominal variables, $\{R, \pi, \mu_\}$, and in particular on the vector $\{R_t, \pi_t, \pi_{t-1}, \pi_{t-2}, \mu_t, \mu_{t-1}, \mu_{t-2}\}$. We also assume that new information on the lagged *pi*'s and *mu*'s becomes available as time progresses (due perhaps to data revisions). We calibrated the variance of the noise on these variables by matching the first eight periods in the IRF of inflation to a money shock in the Christiano et al. 2005. The model is thus, by construction, able to generate inertial behavior in inflation comparable to that generated by the Christiano et al. model. Consequently, its plausibility can be assessed by checking whether the amount of required noise is realistic and its location plausible and also whether the implications of the model for the other variables is satisfactory. The calibrated values for the variance of noise appear in table 3.

As can be seen, the model under imperfect information and learning (signal extraction) requires negligible noise on nominal interest rate observations. The amount of noise on inflation and money supply is somewhat larger but also quite small. Note, that it is smaller than that typically used in models of learning in the literature (see, for instance, Woodford, 2002). The BEA reports "preliminary" and revised values for the GDP deflator. The standard deviation of the difference between announced and revised values for the GDP deflator from 1999-2003 is 0.48%. Coenen, Levin and Wieland, 2005, give some information on data revisions in the Euro area. Over the period January 1999-February 2001 they report that the mean absolute revision for the GDP price deflator was 0.11% and 0.14% one and two quarters respectively after the initial publication. For M3 they report monthly mean absolute revisions of (in percent) {0.16, 0.08, 0.06, 0.03, 0.05, 0.04, 0.03} for the months following the initial release.

Table 3: Volatility of noise

R_t	π_t	π_{t-1}	π_{t-2}	μ_t	μ_{t-1}	μ_{t-2}
2.23045e-4	3.1301e-3	1.5707e-3	7.8817e-4	8.2173e-3	4.1161e-3	2.0618e-3

3 The results

The model is log–linearized around its deterministic steady state and then solved. The solution method for the case in which the agents solve a signal extraction problem is detailed in appendix A.

Figure 1 presents the response of inflation, output, the nominal and the real interest rate to a 1% shock to the growth rate of the money supply under three model specifications: The original version of the NK model with full rationality, that is, without any backward price indexation (standard version). The version with indexation (Christiano et al. 2005, CEE). And the version with full rationality and signal extraction. In all three cases, the model includes three real rigidities, namely, habit persistence, variable capital utilization and investment adjustment costs[7].

As can be seen, the three versions perform comparably with one exception. Namely, the response of inflation. The standard version cannot generate inflation inertia. This finding confirms the well known fact (see Collard and Dellas, 2005) that price staggering does not suffice to produce plausible dynamics.

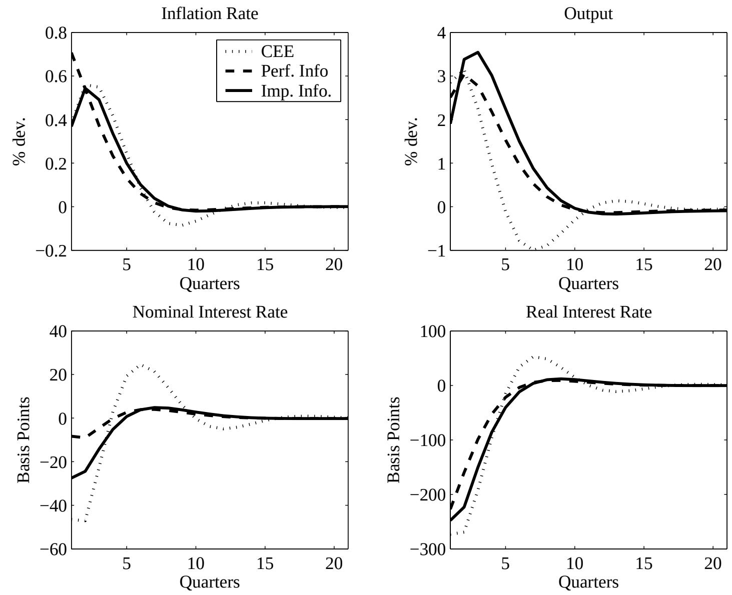
The existence of real rigidities is crucial. In their absence, neither the CEE nor the signal extraction model possess a strong, internal propagation mechanism, so the dynamics of inflation would lack sufficient inertia. As a matter of fact, there is a trade off between the degree of imperfect information and the strength of real rigidities required to produce inertial behavior.

It should also be emphasized, that unlike the CEE model where investment adjustment costs play a crucial role for producing a liquidity effect, the model with signal extraction can produce a liquidity effect without this feature.

Table 4 reports unconditional moments both in the data and in the three model specifications.

[7]Using capital in place of investment adjustment costs makes no difference for the behavior of the model with signal extraction.

Figure 1: IRF to a money supply shock



Note: Three model specifications: a) Standard, fully rational (Perf. Info)
b) Backward indexation (CEE), c) Fully rational with signal extraction (Imp. Info)

The performance of the models is comparable. Their main weaknesses are to be found in the under-prediction of volatilities (in particular of consumption and the nominal interest rate) as well as their implication of counter-cyclicality in the interest rates. Note, that the model with signal extraction does somewhat better along the last dimension. Canzoneri et al. 2004 argue that there exists no model that can adequately capture interest rate behavior, so this weakness is not specific to these models.

4 Conclusions

The new Keynesian model has provided a popular framework for the analysis of monetary policy. Nevertheless, in spite of its overall success, the model has had difficulties accounting for the empirical behavior of inflation (and also of nominal interest rates, unless investment adjustment costs are introduced). Its recent extensions, in particular the Christiano et al. 2005 and Smets and Wouters 2004 versions fare much better empirically but this success is due to the adoption of hard to defend pricing schemes. In particular, backward pricing indexation schemes not only abandon the discipline imposed by rational expectations but also seem to be at variance with the behavior of prices as documented for instance in the ECB project on inflation persistence.

In this paper we have argued that there may exist another, more satisfactory solution. This solution embeds Lucas' signal extraction problem into the Christiano et al. 2005 model but

Table 4: HP moments

Var.	Std	Rel. Std	$\rho(\cdot, y)$	$\rho(1)$	$\rho(2)$
		Data			
y	1.49	1.00	1.00	0.88	0.70
c	0.80	0.54	0.86	0.87	0.69
i	6.03	4.04	0.92	0.83	0.61
h	1.88	1.26	0.83	0.92	0.73
π	0.16	0.11	0.32	0.33	0.24
R_{nom}	0.40	0.27	0.21	0.81	0.57
R_{real}	0.33	0.22	0.10	0.73	0.50
		(b) Standard NK (no indexation)			
y	1.35	1.00	1.00	0.87	0.64
c	0.21	0.16	0.86	0.88	0.65
i	4.12	3.06	0.94	0.90	0.70
h	0.83	0.61	0.88	0.76	0.47
π	0.16	0.12	0.61	0.56	0.22
R_{nom}	0.01	0.01	-0.53	0.80	0.45
R_{real}	0.12	0.09	-0.63	0.56	0.21
		(a) CEE (indexation)			
y	1.49	1.00	1.00	0.84	0.55
c	0.23	0.16	0.90	0.86	0.60
i	4.49	3.01	0.95	0.88	0.63
h	0.87	0.58	0.90	0.74	0.38
π	0.16	0.11	0.54	0.81	0.43
R_{nom}	0.04	0.03	-0.75	0.74	0.29
R_{real}	0.18	0.12	-0.71	0.70	0.28
		(b) Signal extraction			
y	1.51	1.00	1.00	0.83	0.56
c	0.21	0.14	0.91	0.87	0.62
i	4.19	2.77	0.94	0.89	0.68
h	1.08	0.71	0.91	0.74	0.42
π	0.16	0.11	0.74	0.62	0.27
R_{nom}	0.02	0.02	-0.23	0.67	0.30
R_{real}	0.15	0.10	-0.57	0.66	0.28

Note: All series are HP–filtered. Data cover the period 1960:1–2002:4, except for aggregate weekly hours that run from 1964:1 to 2002:4. Output is defined as $C+I+G$. C is nondurables and services, I includes investment and durables. π is the CPI based inflation rate, R_{nom} is the federal fund rate, and $R_{real} = R_{nom} - \pi$. Std. is standard deviation, Rel. Std is standard deviation of the variable relative to that of output, $\rho(\cdot, y)$ is its correlation with output and $\rho(1)$ and $\rho(2)$ the first and second order autocorrelation.

leaves out backward price indexation. This combination allows short lived mis-perceptions of the state of the economy to constrain initial responses but also propagate shocks over time through the real rigidities. Under a small and empirically plausible amount of imperfect information (noise) the model can generate dynamics for the key macroeconomic variables that are virtually indistinguishable from those arising in its most successful rivals.

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A Solution Method

The log–linear version of the system of dynamic equation characterizing the equilibrium may be written as

$$M_{cc}Y_t = M_{cs}\begin{pmatrix} X_t^b \\ X_t^f \end{pmatrix} + M_{ce}\begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} \tag{13}$$

$$M_{ss0}\begin{pmatrix} X_{t+1}^b \\ X_{t+1|t}^f \end{pmatrix} + M_{ss1}\begin{pmatrix} X_t^b \\ X_t^f \end{pmatrix} + M_{se1}\begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} = M_{sc0}Y_{t+1|t} + M_{sc1}Y_t + \begin{pmatrix} M_e u_{t+1} \\ 0 \end{pmatrix} \tag{14}$$

$$S_t = C^0\begin{pmatrix} X_t^b \\ X_t^f \end{pmatrix} + C^1\begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} + v_t \tag{15}$$

Y is a vector of n_y control variables, S is a vector of n_s signals used by the agents to form expectations, X^b is a vector of n_b predetermined (backward looking) state variables (including shocks to fundamentals), X^f is a vector of n_f forward looking state variables, finally u and v are two Gaussian white noise processes with variance–covariance matrices Σ_{uu} and Σ_{vv} respectively and $E(uv') = 0$.

$X_{t+i|t} = E(X_{t+i}|\mathcal{I}_t)$ for $i \geqslant 0$ and where $\mathcal{I}_t$ denotes the information set available to the agents at the beginning of period t. The information set available to the agents consists of *i)* the structure of the model and *ii)* the history of the observable signals they are given in each period:

$$\mathcal{I}_t = \{S_{t-j}, j \geqslant 0, M_{cc}, M_{cs}, M_{ce}, M_{ss0}, M_{ss1}, M_{sc0}, M_{sc1}, M_{se1}, M_e, C^0, C^1, \Sigma_{uu}, \Sigma_{vv}\}$$

Therefore, it is when we specify the signals that we may impose the information structure of the agents.

Before solving the system, note that, from (13), we have

$$Y_t = B^0\begin{pmatrix} X_t^b \\ X_t^f \end{pmatrix} + B^1\begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} \tag{16}$$

where $B^0 = M_{cc}^{-1}M_{cs}$ and $B^1 = M_{cc}^{-1}M_{ce}$, such that

$$Y_{t|t} = B\begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} \tag{17}$$

with $B = B^0 + B^1$.

A.1 Solving the system

Step 1: We first solve for the expected system:

$$M_{ss0}\begin{pmatrix} X^b_{t+1|t} \\ X^f_{t+1|t} \end{pmatrix} + (M_{ss1}+M_{se1})\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} = M_{sc0}Y_{t+1|t} + M_{sc1}Y_{t|t} \tag{18}$$

Plugging (17) in (18), we get

$$\begin{pmatrix} X^b_{t+1|t} \\ X^f_{t+1|t} \end{pmatrix} = W \begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} \tag{19}$$

where

$$W = -\left(M_{ss0} - M_{sc0}B\right)^{-1}\left(M_{ss1} + M_{se1} - M_{sc1}B\right)$$

After getting the Jordan form associated to (19) and applying standard methods for eliminating bubbles, we get

$$X^f_{t|t} = G X^b_{t|t}$$

¿From which we get

$$\begin{aligned} X^b_{t+1|t} &= (W_{bb} + W_{bf}G)X^b_{t|t} = W^b X^b_{t|t} \qquad (20)\\ X^f_{t+1|t} &= (W_{fb} + W_{ff}G)X^b_{t|t} = W^f X^b_{t|t} \qquad (21) \end{aligned}$$

Step 2: We go back to the initial system to get and write

Then, (14) rewrites

$$\begin{aligned} M_{ss0}\begin{pmatrix} X^b_{t+1} \\ X^f_{t+1|t} \end{pmatrix} + M_{ss1}\begin{pmatrix} X^b_{t} \\ X^f_{t} \end{pmatrix} + M_{se1}\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} &= M_{sc0}B\begin{pmatrix} X^b_{t+1|t} \\ X^f_{t+1|t} \end{pmatrix} + M_{sc1}B^0\begin{pmatrix} X^b_{t} \\ X^f_{t} \end{pmatrix} \\ &\quad + M_{sc1}B^1\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} + \begin{pmatrix} M_e u_{t+1} \\ 0 \end{pmatrix} \end{aligned}$$

Taking expectations, we have

$$\begin{aligned} M_{ss0}\begin{pmatrix} X^b_{t+1|t} \\ X^f_{t+1|t} \end{pmatrix} + M_{ss1}\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} + M_{se1}\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} &= M_{sc0}B\begin{pmatrix} X^b_{t+1|t} \\ X^f_{t+1|t} \end{pmatrix} + M_{sc1}B^0\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} \\ &\quad + M_{sc1}B^1\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} \end{aligned}$$

Substracting, we get

$$M_{ss0}\begin{pmatrix} X^b_{t+1} - X^b_{t+1|t} \\ 0 \end{pmatrix} + M_{ss1}\begin{pmatrix} X^b_{t} - X^b_{t|t} \\ X^f_{t} - X^f_{t|t} \end{pmatrix} = M_{sc1}B^0\begin{pmatrix} X^b_{t} - X^b_{t|t} \\ X^f_{t} - X^f_{t|t} \end{pmatrix} + \begin{pmatrix} M_e u_{t+1} \\ 0 \end{pmatrix} \tag{22}$$

which rewrites

$$\begin{pmatrix} X^b_{t+1} - X^b_{t+1|t} \\ 0 \end{pmatrix} = W^c\begin{pmatrix} X^b_{t} - X^b_{t|t} \\ X^f_{t} - X^f_{t|t} \end{pmatrix} + M_{ss0}^{-1}\begin{pmatrix} M_e u_{t+1} \\ 0 \end{pmatrix} \tag{23}$$

where, $W^c = -M_{ss0}^{-1}(M_{ss1} - M_{sc1}B^0)$. Hence, considering the second block of the above matrix equation, we get

$$W_{fb}^c(X_t^b - X_{t|t}^b) + W_{ff}^c(X_t^f - X_{t|t}^f) = 0$$

which gives

$$X_t^f = F^0 X_t^b + F^1 X_{t|t}^b$$

with $F^0 = -W_{ff}^{c\,-1} W_{fb}^c$ and $F^1 = G - F^0$.

Now considering the first block we have

$$X_{t+1}^b = X_{t+1|t}^b + W_{bb}^c(X_t^b - X_{t|t}^b) + W_{bf}^c(X_t^f - X_{t|t}^f) + M^2 u_{t+1}$$

from which we get, using (20)

$$X_{t+1}^b = M^0 X_t^b + M^1 X_{t|t}^b + M^2 u_{t+1}$$

with $M^0 = W_{bb}^c + W_{bf}^c F^0$, $M^1 = W^b - M^0$ and $M^2 = M_{ss0}^{-1} M_e$..

We also have

$$S_t = C_b^0 X_t^b + C_t^0 X_t^f + C_b^1 X_{t|t}^b + C_f^1 X_{t|t}^f + v_t$$

from which we get

$$S_t = S^0 X_t^b + S^1 X_{t|t}^b + v_t$$

where $S^0 = C_b^0 + C_f^0 F^0$ and $S^1 = C_b^1 + C_f^0 F^1 + C_f^1 G$

Finally, we get

$$Y_t = B_b^0 X_t^b + B_t^0 X_t^f + B_b^1 X_{t|t}^b + B_f^1 X_{t|t}^f$$

from which we get

$$Y_t = \Pi^0 X_t^b + \Pi^1 X_{t|t}^b$$

where $\Pi^0 = B_b^0 + B_f^0 F^0$ and $\Pi^1 = B_b^1 + B_f^0 F^1 + B_f^1 G$

A.2 Filtering

Since our solution involves terms in $X_{t|t}^b$, we would like to compute this quantity. However, the only information we can exploit is a signal S_t that we described previously. We therefore use a Kalman filter approach to compute the optimal prediction of $X_{t|t}^b$.

In order to recover the Kalman filter, it is a good idea to think in terms of expectation errors. Therefore, let us define

$$\widehat{X}_t^b = X_t^b - X_{t|t-1}^b$$

and

$$\widehat{S}_t = S_t - S_{t|t-1}$$

Note that since S_t depends on $X^b_{t|t}$, only the signal relying on $\widetilde{S}_t = S_t - S^1 X^b_{t|t}$ can be used to infer anything on $X^b_{t|t}$. Therefore, the policy maker revises its expectations using a linear rule depending on $\widetilde{S}^e_t = S_t - S^1 X^b_{t|t}$. The filtering equation then writes

$$X^b_{t|t} = X^b_{t|t-1} + K(\widetilde{S}^e_t - \widetilde{S}^e_{t|t-1}) = X^b_{t|t-1} + K(S^0 \widehat{X}^b_t + v_t)$$

where K is the filter gain matrix, that we would like to compute.

The first thing we have to do is to rewrite the system in terms of state–space representation. Since $S_{t|t-1} = (S^0 + S^1)X^b_{t|t-1}$, we have

$$\begin{aligned}
\widehat{S}_t &= S^0(X^b_t - X^b_{t|t}) + S^1(X^b_{t|t} - X^b_{t|t-1}) + v_t \\
&= S^0 \widehat{X}^b_t + S^1 K(S^0 \widehat{X}^b_t + v_t) + v_t \\
&= S^\star \widehat{X}^b_t + \nu_t
\end{aligned}$$

where $S^\star = (I + S^1 K)S^0$ and $\nu_t = (I + S^1 K)v_t$.

Now, consider the law of motion of backward state variables, we get

$$\begin{aligned}
\widehat{X}^b_{t+1} &= M^0(X^b_t - X^b_{t|t}) + M^2 u_{t+1} \\
&= M^0(X^b_t - X^b_{t|t-1} - X^b_{t|t} + X^b_{t|t-1}) + M^2 u_{t+1} \\
&= M^0 \widehat{X}^b_t - M^0(X^b_{t|t} + X^b_{t|t-1}) + M^2 u_{t+1} \\
&= M^0 \widehat{X}^b_t - M^0 K(S^0 \widehat{X}^b_t + v_t) + M^2 u_{t+1} \\
&= M^\star \widehat{X}^b_t + \omega_{t+1}
\end{aligned}$$

where $M^\star = M^0(I - KS^0)$ and $\omega_{t+1} = M^2 u_{t+1} - M^0 K v_t$.

We therefore end–up with the following state–space representation

$$\widehat{X}^b_{t+1} = M^\star \widehat{X}^b_t + \omega_{t+1} \tag{24}$$

$$\widehat{S}_t = S^\star \widehat{X}^b_t + \nu_t \tag{25}$$

For which the Kalman filter is given by

$$\widehat{X}^b_{t|t} = \widehat{X}^b_{t|t-1} + PS^{\star\prime}(S^\star P S^{\star\prime} + \Sigma_{\nu\nu})^{-1}(S^\star \widehat{X}^b_t + \nu_t)$$

But since $\widehat{X}^b_{t|t}$ is an expectation error, it is not correlated with the information set in $t-1$, such that $\widehat{X}^b_{t|t-1} = 0$. The prediction formula for $\widehat{X}^b_{t|t}$ therefore reduces to

$$\widehat{X}^b_{t|t} = PS^{\star\prime}(S^\star P S^{\star\prime} + \Sigma_{\nu\nu})^{-1}(S^\star \widehat{X}^b_t + \nu_t) \tag{26}$$

where P solves

$$P = M^{\star} P M^{\star\prime} + \Sigma_{\omega\omega}$$

and $\Sigma_{\nu\nu} = (I + S^1 K)\Sigma_{vv}(I + S^1 K)'$ and $\Sigma_{\omega\omega} = M^0 K \Sigma_{vv} K' M^{0'} + M^2 \Sigma_{uu} M^{2'}$

Note however that the above solution is obtained for a given K matrix that remains to be computed. We can do that by using the basic equation of the Kalman filter:

$$\begin{aligned} X^b_{t|t} &= X^b_{t|t-1} + K(\widetilde{S}^e_t - \widetilde{S}^e_{t|t-1}) \\ &= X^b_{t|t-1} + K(S_t - S^1 X^b_{t|t} - (S_{t|t-1} - S^1 X^b_{t|t-1})) \\ &= X^b_{t|t-1} + K(S_t - S^1 X^b_{t|t} - S^0 X^b_{t|t-1}) \end{aligned}$$

Solving for $X^b_{t|t}$, we get

$$\begin{aligned} X^b_{t|t} &= (I + KS^1)^{-1}(X^b_{t|t-1} + K(S_t - S^0 X^b_{t|t-1})) \\ &= (I + KS^1)^{-1}(X^b_{t|t-1} + KS^1 X^b_{t|t-1} - KS^1 X^b_{t|t-1} + K(S_t - S^0 X^b_{t|t-1})) \\ &= (I + KS^1)^{-1}(I + KS^1) X^b_{t|t-1} + (I + KS^1)^{-1} K(S_t - (S^0 + S^1) X^b_{t|t-1})) \\ &= X^b_{t|t-1} + (I + KS^1)^{-1} K \widehat{S}_t \\ &= X^b_{t|t-1} + K(I + S^1 K)^{-1} \widehat{S}_t \\ &= X^b_{t|t-1} + K(I + S^1 K)^{-1}(S^{\star} \widehat{X}^b_t + \nu_t) \end{aligned}$$

where we made use of the identity $(I + KS^1)^{-1} K \equiv K(I + S^1 K)^{-1}$. Hence, identifying to (26), we have

$$K(I + S^1 K)^{-1} = P S^{\star\prime}(S^{\star} P S^{\star\prime} + \Sigma_{\nu\nu})^{-1}$$

remembering that $S^{\star} = (I + S^1 K) S^0$ and $\Sigma_{\nu\nu} = (I + S^1 K)\Sigma_{vv}(I + S^1 K)'$, we have

$$K(I+S^1K)^{-1} = PS^{0'}(I+S^1K)'((I+S^1K)S^0PS^{0'}(I+S^1K)'+(I+S^1K)\Sigma_{vv}(I+S^1K)')^{-1}(I+S^1K)S^0$$

which rewrites as

$$\begin{aligned} K(I + S^1 K)^{-1} &= PS^{0'}(I + S^1 K)' \left[(I + S^1 K)(S^0 P S^{0'} + \Sigma_{vv})(I + S^1 K)'\right]^{-1} \\ K(I + S^1 K)^{-1} &= PS^{0'}(I + S^1 K)'(I + S^1 K)'^{-1}(S^0 P S^{0'} + \Sigma_{vv})^{-1}(I + S^1 K)^{-1} \end{aligned}$$

Hence, we obtain

$$K = PS^{0'}(S^0 P S^{0'} + \Sigma_{vv})^{-1} \tag{27}$$

Now, recall that

$$P = M^{\star} P M^{\star\prime} + \Sigma_{\omega\omega}$$

Remembering that $M^{\star} = M^0(I+KS^0)$ and $\Sigma_{\omega\omega} = M^0K\Sigma_{vv}K'M^{0\prime} + M^2\Sigma_{uu}M^{2\prime}$, we have

$$\begin{aligned} P &= M^0(I-KS^0)P\left[M^0(I-KS^0)\right]' + M^0K\Sigma_{vv}K'M^{0\prime} + M^2\Sigma_{uu}M^{2\prime} \\ &= M^0\left[(I-KS^0)P(I-S^{0\prime}K') + K\Sigma_{vv}K'\right]M^{0\prime} + M^2\Sigma_{uu}M^{2\prime} \end{aligned}$$

Plugging the definition of K in the latter equation, we obtain

$$P = M^0\left[P - PS^{0\prime}(S^0PS^{0\prime}+\Sigma_{vv})^{-1}S^0P\right]M^{0\prime} + M^2\Sigma_{uu}M^{2\prime} \tag{28}$$

A.3 Summary

We finally end–up with the system of equations:

$$\begin{aligned} X^b_{t+1} &= M^0X^b_t + M^1X^b_{t|t} + M^2u_{t+1} & (29) \\ S_t &= S^0_bX^b_t + S^1_bX^b_{t|t} + v_t & (30) \\ Y_t &= \Pi^0_bX^b_t + \Pi^1_bX^b_{t|t} & (31) \\ X^f_t &= F^0X^b_t + F^1X^b_{t|t} & (32) \\ X^b_{t|t} &= X^b_{t|t-1} + K(S^0(X^b_t - X^b_{t|t-1}) + v_t) & (33) \\ X^b_{t+1|t} &= (M^0+M^1)X^b_{t|t} & (34) \end{aligned}$$

to describe the dynamics of our economy.

This may be recasted as a standard state–space problem as

$$\begin{aligned} X^b_{t+1|t+1} &= X^b_{t+1|t} + K(S^0(X^b_{t+1} - X^b_{t+1|t}) + v_{t+1}) \\ &= (M^0+M^1)X^b_{t|t} + K(S^0(M^0X^b_t + M^1X^b_{t|t} + M^2u_{t+1} - (M^0+M^1)X^b_{t|t}) + v_{t+1}) \\ &= KS^0M^0X^b_t + ((I-KS^0)M^0 + M^1)X^b_{t|t} + KS^0M^2u_{t+1} + Kv_{t+1} \end{aligned}$$

Then

$$\begin{pmatrix} X^b_{t+1} \\ X^b_{t+1|t+1} \end{pmatrix} = M_{\mathrm{X}}\begin{pmatrix} X^b_t \\ X^b_{t|t} \end{pmatrix} + M_{\mathrm{E}}\begin{pmatrix} u_{t+1} \\ v_{t+1} \end{pmatrix}$$

where

$$M_{\mathrm{X}} = \begin{pmatrix} M^0 & M^1 \\ KS^0M^0 & ((I-KS^0)M^0 + M^1) \end{pmatrix} \text{ and } M_{\mathrm{E}} = \begin{pmatrix} M^2 & 0 \\ KS^0M^2 & K \end{pmatrix}$$

and

$$Y_t = M_{\mathrm{Y}}\begin{pmatrix} X^b_t \\ X^b_{t|t} \end{pmatrix}$$

where

$$M_{\mathrm{Y}} = \begin{pmatrix} \Pi^0_b & \Pi^1_b \end{pmatrix}$$