Home Bias in Goods and Assets *

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Abstract

We show that international trade in goods offers a compelling resolution of the portfolio home bias puzzle. A simple model with traded and non–traded goods implies that investors can achieve full international risk diversification if their foreign equity position (as a % of GDP) matches their country's degree of openness (the imports to GDP share). Empirical evidence on the international equity holdings of financially mature economies strongly supports this implication.

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Introduction

In a typical country, consumers tend to consume mostly goods that are produced domestically. This phenomenon has been termed home bias in consumption. Similarly, investors tend to invest most of their wealth in domestic assets, and most of the capital in any country is owned by the domestic residents. This phenomenon has been termed portfolio home bias. Are these two biases linked?

Stockman and Dellas, 1988, (henceforth, SD) argue that this is indeed the case[1]: the presence of consumption home bias –due to the existence of non–traded goods– can induce a similar bias in portfolio holdings. In SD's endowment economy with a *separable* utility function between traded and non–traded goods, the optimal portfolio of a domestic investor involves full ownership of the firms that produce the domestic non–traded goods and a fully diversified ownership of the firms that produce the tradable goods. If the share of non–traded goods in consumption is large, the model can generate substantial portfolio home bias.

The existing literature has not viewed the SD model as offering a full resolution to the puzzle[2]. The conventional wisdom at the time put the share of non–traded goods in the consumption basket in the US at around 50%. With this share, the SD model implies that 75% of domestic wealth will be invested in domestic assets. But this figure falls significantly short of the actual degree of portfolio home bias observed in the US[3]. Recent work by Berstein et al., 2005, however, suggests that the true CPI share of non traded goods is much higher (close to 80%). With this figure, the SD model implies that the share of wealth invested in domestic equity is around 90%, a number very close to its real world counterpart.

[1]Obstfeld and Rogoff, 2000, suggest that these two biases may not only have a common source but they may also be connected to a number of other biases –puzzles– in international finance. The quest for a unifying theory is ongoing. Naturally, there is also a large literature that attempts to explain portfolio home bias without making use of consumption home bias (see, for instance, the survey paper by Lewis, 1996). However, it has been met with limited success so far.

[2]There exists a large literature that examines whether the portfolio home bias that emanates from non–traded goods can be augmented with bias arising from traded goods if the SD assumption of separability between traded and non–traded goods is lifted (Tesar, 1993, Baxter, Jermann, and King, 1998, Serrat, 2001, Pesenti and Van Wincoop, 2002). In the models of Tesar and Pesenti–Van Wincoop there may be home bias in the shares of the tradeable sector depending on the value of the preferences and covariance parameters. Unfortunately, these papers rely on partial equilibrium analysis and thus their results may not hold in general equilibrium. Baxter, Jermann, and King, 1998, on the other hand, use a general equilibrium, two-period exchange economy. They argue that the model *cannot* generates a home bias in the traded goods portfolio. And that equity holdings in the non–traded goods may well be home biased yet not sufficiently so in order to make the total portfolio exhibit home bias. Nevertheless, the generality of these results is severely limited by the fact that their analysis ignores the equilibrium effects of the demand for dynamic hedging. Serrat, 2001, uses a dynamic, general equilibrium model and finds that the domestic investors fully own the equity of the firms that produce the domestic non–traded good and that there is also home bias in the equity positions in traded goods. Kollmann, 2006a, has disputed the latter claim, arguing that the correct solution to the model of Serrat does not involve any portfolio home bias in *traded* goods equity.

[3]See Tesar and Werner, 1995, French and Poterba, 1991.

The evidence thus offers support to the SD model. But this test relies heavily on our ability to separate goods into the traded and non–traded categories, a rather ambiguous enterprize. Fortunately, there exists an appealing alternative. The main implication of the SD model regarding portfolio allocation can be cast in terms of the degree of openness rather than in terms of traded vs non–traded goods[4]. In particular, the model implies that investors can achieve full international risk diversification if the share of wealth invested in foreign equity (as a % of GDP) is equal to the share of imports in GDP.

Table 1 reports these two shares for the US, Japan, UK, Germany and France. The first column of the table gives the share of foreign asset holdings in total domestic —equity— portfolio over the period 1995–2004. The second column reports the imports to GDP share over the same period. The match between the figures in these two columns is remarkably high (the correlation coefficient is 0.92). The fact that a similarly good match is obtained when considering other sub–periods or even individual years or when looking at a larger set of countries (see section 2.2) suggests that this stylized fact is very robust[5].

Table 1: Foreign Equity and Import Shares, 1995-2004

	Equity Share	Import Share
US	0.1203	0.1334
JAP	0.0829	0.0923
UK	0.2623	0.2888
GER	0.3676	0.2906
FR	0.2033	0.2437
	Corr = 0.922	

Note: See section 2.2 for details on the computatin of the shares. Data sources: Lane–Milesi-Feretti, 2006 for foreign equity assets and liabilities. World Development indicators for stock market capitalization. And IFS for import shares.

How robust is the *prediction* of the model regarding the match of imports and equity shares? Typically, the theoretical implications reported in the portfolio literature tend to be extremely sensitive to even small variation in the key parameters of the model (see, for instance, Kollmann, 2006b). Our model does not suffer from this weakness. We establish that departures from separability as well as plausible variation in the main parameters of the model do not affect

[4]Of course, there exists a correspondence between these two categories in the model. In SD, a share of 10.5% for imports (the US figure over the last thirty years) translates into a 79% share for non traded goods in the CPI. This coincides with the figure reported by Berstein et al., 2005.

[5]Heathcote and Perri, 2004, perform a related exercise involving foreign asset holdings and the degree of openness. While they use a different data set and time period and their measures of openness and asset holdings differ from ours, their results are broadly consistent with the notion that trade in goods is an important determinant of asset trade. See also, Aviat and Coeurdacier, 2007, for a confirming test of this relationship in the context of the gravity model.

materially the properties of optimal portfolios. An additional finding of interest is that the model can also deliver home bias in the equities of the traded goods industries if the consumption of traded goods has a foreign bias[6]. There exists no empirical evidence on the presence or absence of home bias in traded goods equity[7] and hence it is not known whether portfolio bias in traded goods represents a desirable feature or not. If it were, our model indicates how it could be obtained.

The rest of the paper is structured as follows. Section 1 contains a description of the model as well as the solution for country portfolios. The solution method is quite simple and is closely related to that of Kollmann, 2006b. Section 2 describes and discusses the main results. Section 3 concludes. A technical appendix provides a formal description of the properties of portfolios.

1 The Model

The world consists of two countries, indexed by $i = 1, 2$. In each period, each country receives an exogenous endowment of a traded, $Y_{it} > 0$ and a non–traded $Z_{it} > 0$, good. The goods are perishable. We use $\mathscr{Y}_t = \{Y_{it}, Z_{it}; i = 1, 2\}$ to denote the vector of endowments.

1.1 The household

Country i is inhabited by a representative agent whose preferences are described by

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \frac{C_{it}^{1-\sigma} - 1}{1-\sigma} \text{ with } \sigma > 0 \tag{1}$$

C_{it} denotes total consumption in country i. It consists of traded and non traded goods according to the specification

$$C_{it} = \left(\omega_i^{\frac{1}{\rho}} C_{it}^{y\,\frac{\rho-1}{\rho}} + (1-\omega_i)^{\frac{1}{\rho}} C_{it}^{z\,\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \omega_i \in (0, 1) \text{ and } \rho > 0 \tag{2}$$

where C_{it}^y (resp. C_{it}^z) denotes the consumption of traded (resp. non–traded) goods in country i at period t.

[6] A foreign bias in the consumption of traded goods is a standard implication of trade theory. It is also a likely feature of the CPI; see the discussion in section 2

[7] Kollmann, 2006b, draws on Kang and Stulz, 1997, who report the existence of home bias in the shares of Japanese manufacturing, to claim the existence of home bias in the equity of traded goods. This interpretation is not justified as domestic consumption of foreign manufactures contains a significant domestic non–traded component (see Berstein et al., 2005). Interestingly, Kang and Stulz report that foreign investors hold larger equity positions in the manufacturing sector and lower ones in the wholesale and retail distribution as well as in services. This finding highlights the importance of the distinction between traded and non–traded goods for portfolio choice and offers support to the key thesis of our paper.

The traded good aggregate combines domestic and foreign goods according to

$$C_{it}^{y} = \left(\alpha_i^{\frac{1}{\varphi}} {C_{iit}^{y}}^{\frac{\varphi-1}{\varphi}} + (1-\alpha_i)^{\frac{1}{\varphi}} {C_{ijt}^{y}}^{\frac{\varphi-1}{\varphi}} \right)^{\frac{\varphi}{\varphi-1}} \quad \alpha_i \in (0,1) \text{ and } \varphi > 0 \tag{3}$$

where C_{ijt}^{y} denotes the consumption of the traded good j in country i at period t.

The individuals have access to an equity market where the shares of the firms that own the endowments of the four goods (the four "trees") can be traded. The budget constraint of the representative household in country i takes the form

$$\sum_{j=1}^{2} \left[Q_{jt}^{y} S_{ijt+1}^{y} + Q_{jt}^{z} S_{ijt+1}^{z} + P_{jt}^{y} C_{ijt}^{y} \right] + P_{it}^{z} C_{it}^{z} = \sum_{j=1}^{2} \left[(Q_{jt}^{y} + P_{jt}^{y} Y_{jt}) S_{ijt}^{y} + (Q_{jt}^{z} + P_{jt}^{z} Z_{jt}) S_{ijt}^{z} \right] \tag{4}$$

where P_{jt}^{y} and P_{jt}^{z} are the prices of the traded and non–traded good j respectively. S_{ijt}^{y} denotes the number of shares of traded good j owned by the households in country i at the beginning of period t while S_{ijt}^{z} is the number of shares of the non–traded good. The price of traded goods shares is Q_{jt}^{y} and that of non–traded is Q_{jt}^{z}. The traded goods shares yield a dividend of $P_{jt}^{y} Y_{jt}$ and the non–traded ones $P_{jt}^{z} Z_{jt}$. Note that there are four assets (equities) in the model and four independent sources of uncertainty. This implies that the equity markets in this model can support the complete asset markets allocation of resources up to a linear approximation. As in Kollmann, 2006b, we will use this equivalence to determine asset holdings.

The household's consumption/portfolio choices are determined by maximizing (1) subject to (2)–(4). The domestic traded good will be used as the numéraire good. Then the evolution of asset prices is given by the standard Euler equations

$$Q_{jt}^{y} \lambda_t^i = \beta \mathbb{E}_t \lambda_{t+1}^i (Q_{jt+1}^{y} + P_{jt+1}^{y} Y_{jt+1}) \tag{5}$$

$$Q_{jt}^{z} \lambda_t^i = \beta \mathbb{E}_t \lambda_{t+1}^i (Q_{jt+1}^{z} + P_{jt+1}^{z} Z_{jt+1}) \tag{6}$$

where $i, j = 1, 2$. Since asset markets are complete and the two economies are perfectly symmetric,[8] we have $\lambda_t^1 = \lambda_t^2$.

Market clearing requires that

$$Z_{1t} = C_{1t}^{z} \tag{7}$$

$$Z_{2t} = C_{2t}^{z} \tag{8}$$

$$Y_{1t} = C_{11t}^{y} + C_{21t}^{y} \tag{9}$$

$$Y_{2t} = C_{12t}^{y} + C_{22t}^{y} \tag{10}$$

[8] Relaxing the perfect symmetry assumption would simply have implied that $\lambda_t^1 \propto \lambda_t^2$ where the proportionality factor be given by the relative initial wealth ratio.

The equilibrium satisfies the FOCs of the optimization problems of the representative agents in the two countries and the market clearing conditions. Since asset markets are effectively complete, the solution of the model can be determined without any need to know equity shares. The model is solved using a perturbation method and takes the form

$$X_t^e = G_x(\mathscr{Y}_t)$$

where $X_t^e \in \{C_{ijt}^{ye}, C_{ijt}^{ze}, P_{jt}^{ye}, P_{jt}^{ze}, Q_{jt}^{ye}, Q_{jt}^{ze}; i,j=1,2\}$. We will explain below how to use $X_t^e = G_x(\mathscr{Y}_t)$ to determine $S_{ijt+1}^{ye}, S_{ijt+1}^{ze}$.

1.2 Specification of the endowments

The endowment process for the traded goods takes the form[9]

$$\begin{aligned} y_{1t} - \overline{y}_1 &= \rho_{11}^y (y_{1t-1} - \overline{y}_1) + \rho_{12}^y (y_{2t-1} - \overline{y}_2) + \varepsilon_{1t}^y \\ y_{2t} - \overline{y}_2 &= \rho_{21}^y (y_{1t-1} - \overline{y}_1) + \rho_{22}^y (y_{2t-1} - \overline{y}_2) + \varepsilon_{2t}^y \end{aligned}$$

where the eigenvalues of the matrix $A_y = \begin{pmatrix} \rho_{11}^y & \rho_{12}^y \\ \rho_{21}^y & \rho_{22}^y \end{pmatrix}$ all lie inside the unit circle, and $(\varepsilon_{1t}^y, \varepsilon_{2t}^y) \rightsquigarrow \mathscr{N}(0, \Sigma_y)$.

Similarly for the non–traded goods

$$\begin{aligned} z_{1t} - \overline{z}_1 &= \rho_{11}^z (z_{1t-1} - \overline{z}_1) + \rho_{12}^z (z_{2t-1} - \overline{z}_2) + \varepsilon_{1t}^z \\ z_{2t} - \overline{z}_2 &= \rho_{21}^z (z_{1t-1} - \overline{z}_1) + \rho_{22}^z (z_{2t-1} - \overline{z}_2) + \varepsilon_{2t}^z \end{aligned}$$

where the eigenvalues of the matrix $A_z = \begin{pmatrix} \rho_{11}^z & \rho_{12}^z \\ \rho_{21}^z & \rho_{22}^z \end{pmatrix}$ all lie inside the unit circle, and $(\varepsilon_{1t}^z, \varepsilon_{2t}^z) \rightsquigarrow \mathscr{N}(0, \Sigma_z)$.

1.3 Solving for asset holdings

Asset holdings are indeterminate in the deterministic steady state around which the system is log-linearized and solved. This creates a difficulty. Our solution method is a variation of that of Kollmann, 2006b (which in turn is related to Baxter et al., 1998).

Let us focus on the domestic economy ($i = 1$). We define wealth, in utility terms, as

$$\Omega_t^1 \equiv \lambda_t^1 \left(Q_{1t}^y S_{11t+1}^y + Q_{2t}^y S_{12t+1}^y + Q_{1t}^z S_{11t+1}^z + Q_{2t}^z S_{12t+1}^z \right) \tag{11}$$

where λ_t^1 are the Lagrange multipliers associated with the budget constraints (4).

[9] We denote $y_{it} = \log(Y_{it})$, i=1,2. Likewise for z.

Using the definition of wealth, the budget constraint of the household can be rewritten as

$$\begin{aligned}\Omega_t^1 + \lambda_t^1(C_{11t}^y + P_{2t}^y C_{12t}^y + P_{1t}^z C_{1t}^z) =& \lambda_t^1 \frac{Q_{1t}^y + Y_{1t}}{Q_{1t-1}^y} Q_{1t-1}^y S_{11t}^y + \lambda_t^1 \frac{Q_{2t}^y + P_{2t}^y Y_{2t}}{Q_{2t-1}^y} Q_{2t-1}^y S_{12t}^y \\ &+ \lambda_t^1 \frac{Q_{1t}^z + P_{1t}^z Z_{1t}}{Q_{1t-1}^z} Q_{1t-1}^z S_{11t}^z + \lambda_t^1 \frac{Q_{2t}^z + P_{2t}^z Z_{2t}}{Q_{2t-1}^z} Q_{2t-1}^z S_{12t}^z\end{aligned} \tag{12}$$

Updating equation (12) by one period, and making use of the household's Euler equations for asset decisions and the fact that shares are predetermined leads to a stochastic difference equation in wealth

$$\Omega_t^1 = \beta \mathbb{E}_t \left[\Omega_{t+1}^1 + \lambda_{t+1}^1 (C_{11t+1}^y + P_{2t+1}^y C_{12t+1}^y + P_{1t+1}^z C_{1t+1}^z)\right]$$

This equation determines wealth, Ω_t^1.

Let us define the asset returns as

$$\begin{aligned} R_{1t}^y &= \tfrac{Q_{1t}^y + Y_{1t}}{Q_{1t-1}^y}, & R_{2t}^y &= \tfrac{Q_{2t}^y + P_{2t}^y Y_{2t}}{Q_{2t-1}^y}, \\ R_{1t}^z &= \tfrac{Q_{1t}^z + P_{1t}^z Z_{1t}}{Q_{1t-1}^z}, & R_{2t}^z &= \tfrac{Q_{2t}^z + P_{2t}^z Z_{2t}}{Q_{2t-1}^z}. \end{aligned}$$

Let also define the shares of economy i total wealth that are accounted by each asset as

$$\begin{aligned} \alpha_{i1t+1}^y &= \tfrac{\lambda_t^1 Q_{1t}^y S_{11t+1}^y}{\Omega_t^1} & \alpha_{i2t+1}^y &= \tfrac{\lambda_t^1 Q_{2t}^y S_{12t+1}^y}{\Omega_t^1} \\ \alpha_{i1t+1}^z &= \tfrac{\lambda_t^1 Q_{1t}^z S_{11t+1}^z}{\Omega_t^1} & \alpha_{i2t+1}^z &= 1 - \alpha_{i1t+1}^y - \alpha_{i2t+1}^y - \alpha_{i1t+1}^z \end{aligned}$$

Using these definitions in the domestic budget constraint gives

$$\frac{\Omega_t^1}{\lambda_t^1} + C_{11t}^y + P_{2t}^y C_{12t}^y + P_{1t}^z C_{1t}^z = \frac{\Omega_{t-1}^1}{\lambda_{t-1}^1}\left((R_{1t}^y - R_{2t}^z)\alpha_{11t}^y + (R_{2t}^y - R_{2t}^z)\alpha_{12t}^y + (R_{1t}^z - R_{2t}^z)\alpha_{11t}^z\right) + R_{2t}^z \frac{\Omega_{t-1}^1}{\lambda_{t-1}^1}$$

or, equivalently

$$(R_{1t}^y - R_{2t}^z)\alpha_{11t}^y + (R_{2t}^y - R_{2t}^z)\alpha_{12t}^y + (R_{1t}^z - R_{2t}^z)\alpha_{11t}^z = \frac{\lambda_{t-1}^1}{\Omega_{t-1}^1}\left(\frac{\Omega_t^1}{\lambda_t^1} + C_{11t}^y + P_{2t}^y C_{12t}^y + P_{1t}^z C_{1t}^z\right) - R_{2t}^z$$

The last equation can be written in more compact form as

$$M(\mathscr{Y}_t) \bullet \alpha_t = L(\mathscr{Y}_t) \text{ with } \alpha_t = \{\alpha_{11t}^y, \alpha_{12t}^y, \alpha_{11t}^z, \alpha_{12t}^z\} \tag{13}$$

Since shares are predetermined, equation 13 implies

$$(M(\mathscr{Y}_t) - \mathbb{E}_{t-1} M(\mathscr{Y}_t)) \bullet \alpha_t = L(\mathscr{Y}_t) - \mathbb{E}_{t-1} L(\mathscr{Y}_t) \tag{14}$$

Market completeness implies that the last relation should hold for all realizations of shocks. Projecting equation (14) on each shock gives a system of four equations in four unknowns,

$\{\alpha_{1t}^y, \alpha_{2t}^y, \alpha_{1t}^z, \alpha_{1t}^z\}$ and allows us to determine asset shares. To a first order approximation, this defines a linear system that delivers constant shares.[10] Since the four wealth shares comprising α must sum to unity, we actually have five equations in these four unknowns. Markets are effectively complete if there is a solution for the four wealth shares satisfying all five equations–thus making equation 14 hold for all realizations of the four shocks in the model. In performing our calculations, we have checked to make sure that this is indeed the case (i.e., that four stocks do effectively complete the markets in our model).

2 The results

2.1 Parametrization

Our baseline parametrization corresponds to the model of SD and is reported in Table 2. Setting $\sigma\rho = 1$ implies that the marginal utility of traded good is not affected by variation in the consumption of the non–traded good (and vice versa). The value of ρ used by Stockman and Tesar, 1995, is $\rho = 0.44$. Hence, a natural choice in this case is to set $\sigma = 2$ and $\rho = 0.5$. As in SD, we set $\alpha_1 = \alpha_2 = 0.5$ so there is no bias in the consumption of traded goods. The average degree of openness (import share) in the US over 1970–2005 is 10.5%. This implies a value for the share of traded goods in the CPI,[11] $\omega = 0.21$ ($\omega(1 - \alpha) = 0.21 \times 0.50 = 0.105$). This share is virtually identical to that reported by Burstein et al. 2005. It is considerably smaller than that used in the earlier literature because it takes into account the non–traded services and goods associated with the distribution of traded goods. It is computed as follows (see Burstein et al, 2005): In the US, the share of traded in the CPI, computed the traditional way, is 0.429. Burstein et al, calculate that approximately 50% of that involves non–traded distribution costs. Hence, the true share of traded goods in CPI is only about 0.21, of which half approximately comes from imported goods and the other half comes from local goods[12].

The elasticity of substitution between domestic and foreign traded goods is set to 1.5 as in Backus et al., 1995. The discount factor is set to 0.99. The endowment processes are assumed to be identical. In fact, the form of the processes does not actually matter for our results.[13] More

[10]This first order approximation with time invariant shares will prove useful for an intuitive exposition of our results. We have however also computed a second order approximation to equation (14). In this case, there is time variation in share holdings, but the average values are virtually identical to the ones from the first order approximation. The interested reader is referred to a companion technical appendix to this paper in which we solve a simplified version of the model using a higher order (up to 8–th order) projection approach. The results strongly indicate that low order approximations have very good accuracy properties when computing asset holdings.

[11]Note that this result assumes that the relative price of the non–traded good is equal to one in steady state. This is ensured by selecting the ratio of the averages of the endowments ($\overline{z}/\overline{y} = (1 - \omega)/\omega$).

[12]Burstein et al, 2005, report a total import content in US consumption of 9.1%. Local goods are mostly exportable goods that are consumed locally.

[13]This result is not related to the log–linear approximation and also obtains when we use higher order projection methods. (See companion technical appendix)

precisely, neither persistence nor volatilities are of any consequences for the determination of wealth and equity shares (but the average level of the process does matter). In the benchmark, traded and non–traded endowments are such that the relative price of non–traded good is equal to unity.

We also report results with an alternative parametrization that represents a case of particular interest. Namely, it involves a bias in the consumption of traded goods. As mentioned above, Burstein et al., 2005, assign half of the traded goods to imports (10.5%) and the other half to local (exportable) goods. But it seems reasonable to assume that some of these local goods may actually be non–tradeables[14]. Let us —arbitrarily— assume that about one–fourth of this 10.5% of local goods is indeed non–traded (say, 2.5%) rather than exportables. This implies that the true share of domestic exportables in the domestic consumption basket is only 8% rather that the 10.5% we used earlier ($10.5\% - 2.5\%$). With this assumption, the true share of traded goods decreases from $\omega = 0.21$ to $\omega = 0.21 - 0.025 = 0.185$ and $\alpha_1 = 0.08/0.185 = 0.43$.[15] We call this the case of bias in traded consumption.

Table 2: Parametrization

Parameter		Separable	Consumption Bias
	Preferences		
Discount Factor	β	0.99	0.99
Risk aversion	σ	2.00	2.00
	Total Consumption Bundle		
Share of traded	ω	0.21	0.185
Substitution traded/nontraded	ρ	0.50	0.50
	Traded Goods Bundle		
Share of Domestic Traded	$\alpha_1 = \alpha_2$	0.50	0.43
Substitution Domestic/Foreign	φ	1.50	1.50
	Endowments		
Persistence	$\rho^y_{11} = \rho^y_{22} = \rho^z_{11} = \rho^z_{22}$	0.85	0.85
Spillover	$\rho^y_{12} = \rho^y_{21} = \rho^z_{12} = \rho^z_{21}$	0.05	0.05
Volatility	$\sigma^y_1 = \sigma^y_2 = \sigma^z_1 = \sigma^z_2$	0.01	0.01
Correlation	$\mathrm{Corr}(\varepsilon^y_1, \varepsilon^y_2)$, $\mathrm{Corr}(\varepsilon^z_1, \varepsilon^z_2)$	0.00	0.00

[14] The behavior of the prices of these goods lies in between that of non–traded and traded. See Burstein et al., 2005.

[15] In both cases, the steady state levels of the endowments are adjusted so that the relative price of the foreign traded good is one.

2.2 Discussion

Table 3 reports wealth shares, *i.e.* the share of total wealth of a domestic agent that is held in the form of one of the four assets available, and equity shares *i.e.* the share of the value of equity in a particular industry that is owned by domestic agents. The first row corresponds to the separable case of SD; and the second to the case with consumption bias in traded goods. As detailed below in section (2.2.1), if utility is separable and there is no consumption bias in traded goods, then the model implies that the average share of foreign equity held in domestic equity portfolio is equal to the average share of imports in GDP. That is, the model predicts that

$$s_{ij} = \frac{A_{ij}}{A_{ii} + A_{ij}} = t_i$$

where s_{ij} is the share of the domestic investors equity wealth that is invested in foreign equity and t_i is the imports shares. s_{ij} can be re-written as

$$s_{ij} = \frac{A_{ij}}{A_{ii} + A_{ij}} = \frac{A_{ij}/y_i}{A_{ii}/y_i + A_{ij}/y_i} = \frac{A_{ij}/y_i}{(A_{ii}/A_i)(A_i/y_i) + A_{ij}/y_i}$$

or

$$s_{ij} = \frac{\text{FOREIGN ASSETS}}{\text{STOCK MARKET CAPITALIZATION - FOREIGN LIABILITIES} + \text{FOREIGN ASSETS}}$$

where all of these quantities are measures as a % of GDP.

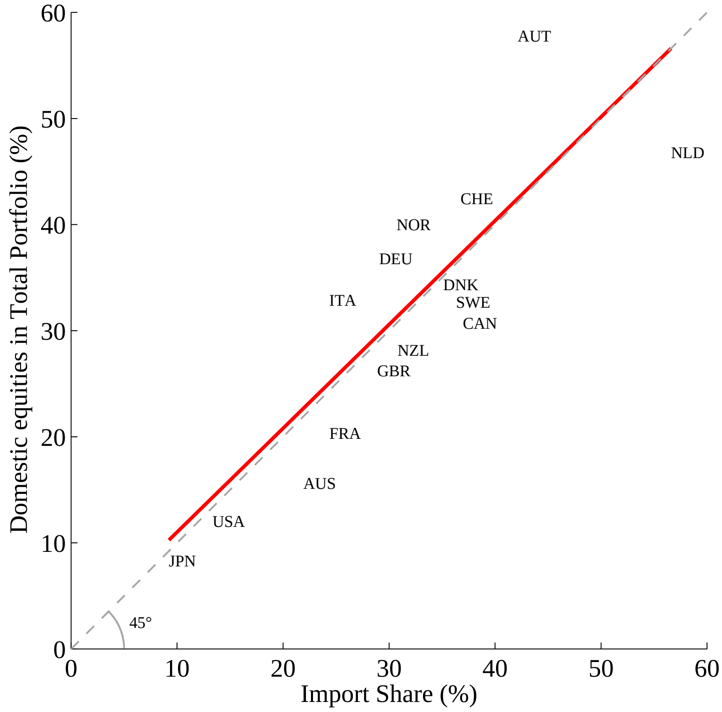
We have used data on foreign equity assets and liabilities from Lane–Milesi–Feretti, 2006, and data on market capitalization from the World Development Indicators to compute s_{ij} for financially developed countries. Graph 2.2 reports this share as well as the imports–GDP share. As in Table 1 the match is nearly perfect. A similarly good match obtains when we look at individual years, other period averages and so on. The result is very robust. The fit is not as good when less developed countries are included but this is not surprising given the widespread use of capital controls and the presence of severe official and unofficial financial impediments in those countries.

Table 3: Shares: The separable case

	Wealth Shares				Equity Shares			
	α_{11}^y	α_{11}^z	α_{12}^y	α_{12}^z	S_{11}^y	S_{11}^z	S_{12}^y	S_{12}^z
Separable	0.1050	0.7900	0.1050	0.0000	0.5000	1.0000	0.5000	0.0000
Cons. Bias	0.1060	0.8150	0.0790	0.0000	0.5729	1.0000	0.4271	0.0000

Consequently, the SD model provides a solution to the portfolio home bias puzzle. This was not realized at the time because of two reasons. First, SD did not draw the implications of the

Figure 1: Portfolio equity assets and imports (share of GDP, average 1995–2004, developed countries)



Note: Foreign equity assets and liabilities from Lane–Milesi-Feretti, 2006. Market capitalization from the World Development Indicators, 2006. The regression line is $PEA_i = \underset{(5.5745)}{1.2256} + \underset{(0.1720)}{0.9785}\, IM_i$ and has $R^2 = 0.71$, where PEA_i is the foreign equity asset measure (s_{ij}) and IM_i is the import share (standard errors in parenthesis).

model for the equality of foreign equity and import shares (and even if they had done so, the lack of suitable data at that time would not have allowed them to test the model). And second, only recently has it been established that the share of non–traded in consumption is much higher than previously thought.

Having showed that trade in goods is sufficient for understanding international portfolios we now provide the underlying intuition. This also helps shed some light on the driving forces behind portfolio allocations in general. We then proceed to investigate the conditions under which the model also generates home bias in traded goods equity.

2.2.1 Separable utility ($\sigma\rho = 1$), no consumption bias ($\alpha = 0.5$)

Observation 1: Investors choose to hold 100% and 0% of the domestic and foreign non–traded good equity respectively.

Variation in the endowment of the domestic non–traded good affects domestic residents who own equity in the non–traded goods sector in two ways. First, it affects the value of the stream of dividends in proportion to the number of shares held. And second, it affects the expenditure needed to finance the consumption of the non–traded good (because of the price change). If a domestic investor holds 100% of the domestic non–traded good equity and also consumes 100% of that good then the gain (loss) as an investor exactly offsets the loss (gain) as a consumer.

If utility is separable ($\sigma\rho = 1$) then traded and non–traded goods are neither substitutes nor complements. That is, variation in consumption of the non–traded good does not change the agents' utility from consumption of the traded good (and vice versa). In this case it is optimal to hold 100% of the domestic non–traded good equity (and 0% of the foreign) in order to prevent any international wealth redistribution following shocks to the non–traded good endowment. There is no need to have any redistribution —which would lead to changes in the relative shares of the traded good consumption bundle across the two countries— as the marginal utility of the consumption of the traded goods is independent of the amount of the non–traded good consumed.

Observation 2: Investors choose to hold a 50% share in each of the two traded goods –domestic and foreign– industries.

With separability between traded and non traded gods and similar preferences over traded goods across countries, SD show that the efficient level of consumption of traded goods is $c^y_{11t} = c^y_{21t} = y_{1t}/2$ and $c^y_{12t} = c^y_{22t} = y_{2t}/2$ (see also the solutions for c_{iit} in the Appendix A.4). This consumption pattern can be supported if each country holds a 50% share in each of the two traded goods.

We can now establish that such a portfolio implies that the average share of domestic wealth invested in foreign equity is equal to the average degree of trade openness (imports to GDP share). The average degree of openness in country 1 is

$$t_1 = \frac{P_2^y C_{12}^y}{Y_1 + P_1^z Z_1} = \frac{P_2^y C_{12}^y / Y_1}{1 + P_1^z Z_1 / Y_1}$$

Recall that the steady state levels of the endowments were selected such that the relative prices are equal to unity. We show in appendix A.2 that this implies $P_2^y C_{12}^y / Y_1 = C_{12}^y / Y_1 = 1 - \alpha = 0.5$. The share of imports is then

$$t_1 = \frac{0.5}{1 + Z_1 / Y_1}$$

The average share of country 1 wealth invested in foreign equity is

$$s_{12} = \frac{Q_2^y S_{12}^y + Q_2^z S_{12}^z}{Q_1^y S_{11}^y + Q_2^y S_{12}^y + Q_1^z S_{11}^z + Q_2^z S_{12}^z}$$

Using the equity shares reported above ($S_{12}^z = 0$ and $S_{12}^y = S_{11}^y = 0.5$) and the fact that in a symmetric equilibrium $Q_1^y = Q_2^y = Q^y$ and $Q_1^z = Q_2^z = Q^z$, s_{12} reduces to

$$s_{12} = \frac{0.5}{1 + Q^z / Q^y}$$

As shown in the appendix A.2, the average ratio of stock prices Q^z/Q^y is equal to the average ratio of the dividends associated with these stocks, Z_1/Y_1. Consequently, the model implies $s_{12} = t_1$.

2.2.2 Non–separable utility ($\sigma\rho \neq 1$), no consumption bias ($\alpha = 0.5$)

We now relax the assumption of separability by allowing $\sigma\rho$ to depart from unity. We study the effects of a non–traded shock on consumption. The following proposition describes the changes in efficient consumption[16].

Proposition 1 *In equilibrium, the impact effect of a non–traded goods shock satisfies*

$$\frac{\partial c_{iit}^y}{\partial z_{it}} \gtreqless 0, \; \frac{\partial c_{iit}^y}{\partial z_{jt}} \lesseqgtr 0, \; \frac{\partial c_{ijt}^y}{\partial z_{it}} \gtreqless 0, \; \frac{\partial c_{ijt}^y}{\partial z_{jt}} \lesseqgtr 0, \; \frac{\partial c_{it}^y}{\partial z_{it}} \gtreqless 0, \; \frac{\partial c_{it}^y}{\partial z_{jt}} \lesseqgtr 0, \; \Longleftrightarrow \sigma\rho \lesseqgtr 1$$

The effects of traded goods shocks on efficient consumption allocations as well as the supporting portfolios can be studied in a similar fashion (see the appendix A.5).

Let us assume that $\sigma\rho < 1$, so that non–traded and traded are *complements*. In an efficient equilibrium, we want consumption of traded goods to be higher in the country that has experienced

[16] The interested reader is referred to Appendix A.6 for a formal proof of all the propositions.

a positive endowment shock in its non–traded sector, because the increase in the consumption of non–traded goods increases the marginal utility of consumption of the traded good. But this implies that we want this country to experience a redistribution of income in its favor. This would happen if some of the shares of firms producing domestic non–traded goods were held by foreign investors. In this case, the gain to the domestic agents as consumers of the non–traded good would exceed their loss as investors in that good, because while they consume 100% of it they hold less than 100% of its equity. The foreign agents suffer an investment loss without reaping any consumption benefit. The resulting income redistribution allows the domestic agents to claim a larger proportion of the traded goods bundle. Hence, the optimal portfolio here involves holding a large share –almost unity– in the domestic non–traded sector and a small[17] share in the foreign non–traded sector. This is illustrated in the top row of Table 4. Here we have used the same parametrization as in the separable case (with $\sigma = 2$) except for the elasticity of substitution between traded and non traded which takes the value $\rho = 0.4$.

Table 4: Shares: The non-separable case

	Wealth Shares				Equity Shares			
	α_{11}^y	α_{11}^z	α_{12}^y	α_{12}^z	S_{11}^y	S_{11}^z	S_{12}^y	S_{12}^z
$\rho = 0.4$	0.1050	0.7741	0.1050	0.0159	0.5000	0.9798	0.5000	0.0202
$\rho = 0.6$	0.1050	0.8073	0.1050	-0.0173	0.5000	1.0219	0.5000	-0.0219

Exactly the same type of reasoning establishes that the domestic agents will want to hold more than 100% of the domestic non–traded equity and will want to short foreign non traded goods equity if traded and non–traded goods are substitutes, that is, if $\sigma\rho > 1$. The second row of Table 4 reports asset shares under the assumption that $\rho = 0.6$.

2.2.3 Separable utility ($\sigma\rho = 1$), consumption bias ($\alpha \neq 0.5$)

Generating portfolio bias in traded goods industries

We now turn to the role of consumption bias in traded goods. We assume that $\rho < 1$. In this case, the domestic and foreign traded goods are substitutes ($d^2u/dc_{11}^y dc_{12}^y < 0$) if the traded goods are more substitutable among themselves than they are with the non–traded goods, that is, if $\varphi > \rho$. This is an assumption that we will maintain throughout this paper.[18]

Suppose that there is home bias in traded goods consumption ($\alpha_i > 0.5$). Suppose also that $\sigma\rho = 1$. *Does this make the domestic investors want to hold an equity share in the domestic traded that exceeds or falls short of 0.5?*

[17] At least for small deviations from $\sigma\rho = 1$.

[18] Standard calibrations typically set $\varphi > 1$ and $\rho < 1$.

Consider a positive shock to the domestic traded endowment. Let us hold consumption of the foreign traded good constant and allow the consumption of the domestic traded good to increase by the same proportion in the two countries. Because of the home bias in traded consumption, domestic consumption of traded goods increases by more than foreign consumption of traded goods. If $\varphi > \rho$, the marginal utility of both the domestic and foreign traded good is *decreasing* in total traded good consumption. This implies that the marginal utility of both traded goods has decreased more at home that abroad, violating the risk sharing principle. For the marginal utilities to be equalized the foreign consumer must increase her consumption of the traded goods by a larger extent than the domestic consumer. In other words, the ratio of domestic to foreign expenditures of traded goods must decrease. This result is described in the next proposition.

Proposition 2 *Let Θ_t denote the ratio of domestic to foreign traded expenditures*

$$\Theta_t = \frac{C^y_{11t} + P^y_{2t} C^y_{12t}}{C^y_{21t} + P^y_{2t} C^y_{22t}}$$

and let us assume $\sigma\rho = 1$, we then have

$$\frac{\partial \log(\Theta_t)}{\partial y_{1t}} \lesseqgtr 0 \Longleftrightarrow \rho \lesseqgtr 1 \text{ and } \alpha \gtreqless \frac{1}{2}$$

In order to support the efficient equilibrium, asset holdings must be such that dividend income abroad increases by more than dividend income at home following a positive shock to the domestic endowment of the traded good. *What portfolio shares will deliver this?* The next proposition gives addresses this question.

Proposition 3 *When the elasticity of substitution between traded and non traded goods is less than unity ($\rho < 1$), traded goods are suitably substitutable among themselves ($\varphi > \rho + (1-\rho)/4\alpha(1-\alpha)$) and there is home bias in traded goods ($\alpha > 0.5$), then the optimal portfolio allocation exhibits foreign portfolio bias in traded goods industries.*

In order to understand this result, recall that the holdings of non–traded goods equity are not affected by the value of α as long as $\sigma\rho = 1$, hence, these shares remain at 100% and 0% respectively. In contrast, the appropriate holdings of traded goods equity depend on the value of α. If $\alpha > 0.5$ (home bias in the consumption of traded goods) then the supporting portfolio of traded goods equity must have a foreign bias ($S^y_{11} < 0.5, S^y_{12} > 0.5$), as long as $\varphi > 1$. With $\varphi > 1$, the increase in the endowment of the domestic traded good will lower its relative price but less than one for one. Holding shares $S^y_{11} < 0.5$ still allows the domestic consumer to consume more of the domestic tradeable because of its lower relative price. But, at the same time, it makes relative dividend incomes move in favor of the foreign agents, which allows them to claim a larger share of world tradeables (we argued above that this is a property of an efficient equilibrium).

The numerical results corresponding to the case of consumption bias under the parametrization in the second column of Table 2 appear in the second row of Table 3.

Under foreign bias in the consumption of tradeables, $\alpha < 0.5$, the opposite pattern obtains. That is, there is *home bias* in the portfolio of traded goods equity. Nevertheless, bias in the consumption of traded is not necessary to generate overall home bias. If the share of non–traded goods in the CPI is not too much below 50%, there will be home bias in consumption independent of whether α is greater or smaller than 0.5. But in any case, for the sake of assessing the model's prediction for bias in the traded goods equity, it is of interest to consider whether α is likely to exceed or to fall short of 0.5 in the real world.

We have already discussed how the CPI decomposition creates a presumption[19] that $\alpha < 0.5$. We have also carried a detailed study of the Swiss CPI. The documentation of the CPI only provides —precise— information on the domestic (or imports) content of each item that enters the basket. A possible —but arbitrary— classification that does not rely on an *ad hoc* assignment of items to the various categories (non–traded, domestic trade, foreign traded) is to assign an item to the non–traded category if the domestic content is 100%, to the foreign tradeable if the import content is more than 50% and to the domestic tradeable in all other cases. Such a classification produces the values 60%, 11%, 29% for non–traded, domestic tradeable and foreign tradeable respectively. While it is clear that this classification is far from ideal as something may have 100% domestic content and still be a tradeable good, we view these figures as indicating that the likelihood of foreign bias in traded goods is not negligible.

Finally, a strong case for foreign bias in the consumption of traded goods can be made based on trade theory. Standard trade theory implies the existence of international specialization, with countries typically producing a small range of traded goods and exporting most of their tradeables production in exchange for a much broader set of foreign produced traded goods. A similar prediction arises in new trade models, to the extent that there are gains from specialization, as it would be the case in the presence of returns to scale, country specific factors and so on.

2.3 Sensitivity Analysis

We now turn to the investigation of how variation in the key parameters of the model affects the composition of optimal portfolios. Table 5 provides information on the role of deviations from separability and from symmetry in the consumption of traded goods. Tables 6 and 7 provide

[19]Unfortunately, as Bernstein et al., 2005, discuss there exists no direct information on the domestic exportable goods component of the CPI.

information on the sensitivity of the results around the two baseline parametrizations reported in Table 2. The results are very robust. There is actually only two cases, which do not seem empirically relevant, where the model fails to generate home bias in portfolio. Namely, when the elasticity of substitution between traded and non–traded goods, ρ, is very high (say, $\rho = 5$). And when the share of imports in the domestic consumption basket is very high ($\omega = 0.75$), see Table 6. Note also that higher values of the elasticity of substitution between domestic and foreign traded (as suggested by Obstfeld and Rogoff, 2000) do not undermine home bias. This is encouraging because there is great uncertainty regarding the precise value of this elasticity[20].

3 Conclusions

Investors tend to invest most of their wealth in domestic assets, and most of the capital in any country is owned by the domestic residents. This is true even in countries that appear to be well integrated within the world capital markets. A large literature has attempted to provide an explanation to this "puzzling" phenomenon, with rather limited success so far. In this paper we show that international trade in goods can fully resolve the home portfolio bias. In particular, the Stockman–Dellas, 1989, model with traded and non–traded goods implies that international equity positions should match import shares. This is precisely the pattern observed in the real world. To the extent that import shares fall short of 50%, portfolios will exhibit a home bias.

In addition to accounting for overall portfolio home bias, the model can -with a small extension-also generate home bias in the portfolios of traded goods equity[21]. For this, it needs foreign bias in the consumption of traded goods. Such bias is a key prediction of trade theory: Trade allows countries to produce few and to consume many traded goods. It seems also consistent with the —meagre— empirical evidence available.

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[20]The robustness of our results in the presence of large variation in the parameters values is another advantage of our proposed solution to the portfolio bias relative to other approaches. Those other approaches produce results that are extremely sensitive to even slight but plausible variation in key parameters.

[21]A model without non–traded goods but home bias in the consumption of traded goods can also account for portfolio home bias in traded goods equity. But for this it requires that the elasticity of substitution between domestic and foreign traded goods falls within a narrow range strictly *below* unity (see Kollmann, 2006b). This requirement may be problematic as typical estimates of the elasticity of substitution between domestic and foreign traded goods not only exceed unity but they can be quite high.

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Table 5: The role of separability and of consumption bias

	Wealth Shares				Equity Shares			
	α_{11}^{y}	α_{11}^{z}	α_{12}^{y}	α_{12}^{z}	S_{11}^{y}	S_{11}^{z}	S_{12}^{y}	S_{12}^{z}
$\sigma\rho < 1$								
$\alpha < 0.5$	0.1365	0.7502	0.0735	0.0398	0.6499	0.9496	0.3501	0.0504
$\alpha = 0.5$	0.1050	0.7525	0.1050	0.0375	0.5000	0.9525	0.5000	0.0475
$\alpha > 0.5$	0.0735	0.7502	0.1365	0.0398	0.3501	0.9496	0.6499	0.0504
$\sigma\rho = 1$								
$\alpha < 0.5$	0.1278	0.7900	0.0822	0.0000	0.6087	1.0000	0.3913	0.0000
$\alpha = 0.5$	0.1050	0.7900	0.1050	0.0000	0.5000	1.0000	0.5000	0.0000
$\alpha > 0.5$	0.0822	0.7900	0.1278	0.0000	0.3913	1.0000	0.6087	0.0000
$\sigma\rho > 1$								
$\alpha < 0.5$	0.1175	0.8374	0.0925	-0.0474	0.5596	1.0601	0.4404	-0.0601
$\alpha = 0.5$	0.1050	0.8363	0.1050	-0.0463	0.5000	1.0587	0.5000	-0.0587
$\alpha > 0.5$	0.0925	0.8374	0.1175	-0.0474	0.4404	1.0601	0.5596	-0.0601

Note: Here we assume that $\omega = 0.21$ as in our first benchmark calibration. $\rho\sigma < 1$ corresponds to the case $\rho = 0.25$ while $\rho\sigma > 1$ denotes $\rho = 0.75$. The case $\alpha < 1$ assumes $\alpha = 0.4$ and $\alpha > 0.5$ assumes $\alpha = 0.6$.

Table 6: Shares: Sensitivity analysis (Benchmark: Separable case)

	Wealth Shares				Equity Shares			
	α_{11}^{y}	α_{11}^{z}	α_{12}^{y}	α_{12}^{z}	S_{11}^{y}	S_{11}^{z}	S_{12}^{y}	S_{12}^{z}
Benchmark	0.1050	0.7900	0.1050	0.0000	0.5000	1.0000	0.5000	0.0000
Consumption share of traded goods: ω								
0.10	0.0222	0.9556	0.0222	0.0000	0.5000	1.0000	0.5000	0.0000
0.50	0.3950	0.2100	0.3950	0.0000	0.5000	1.0000	0.5000	0.0000
0.75	0.4857	0.0287	0.4857	0.0000	0.5000	1.0000	0.5000	0.0000
Elasticity of substitution traded vs nontraded: ρ								
0.25	0.1050	0.7525	0.1050	0.0375	0.5000	0.9525	0.5000	0.0475
0.75	0.1050	0.8363	0.1050	-0.0463	0.5000	1.0587	0.5000	-0.0587
5.00	0.1050	-0.0488	0.1050	0.8388	0.5000	-0.0618	0.5000	1.0618
Consumption share of domestic traded good in consumption of traded: α								
0.25	0.2100	0.7900	0.0000	0.0000	1.0000	1.0000	0.0000	0.0000
0.75	0.0000	0.7900	0.2100	0.0000	0.0000	1.0000	1.0000	0.0000
Elasticity of substitution between traded goods: φ								
0.60	0.1050	0.7900	0.1050	0.0000	0.5000	1.0000	0.5000	0.0000
2.00	0.1050	0.7900	0.1050	0.0000	0.5000	1.0000	0.5000	0.0000
5.00	0.1050	0.7900	0.1050	0.0000	0.5000	1.0000	0.5000	0.0000
Elasticity of intertemporal substitution: σ								
0.50	0.1050	0.9716	0.1050	-0.1816	0.5000	1.2299	0.5000	-0.2299
4.00	0.1050	0.8197	0.1050	-0.0297	0.5000	1.0376	0.5000	-0.0376

Table 7: Shares: Sensitivity analysis (Benchmark: Consumption Bias)

	Wealth Shares				Equity Shares			
	α_{11}^{y}	α_{11}^{z}	α_{12}^{y}	α_{12}^{z}	S_{11}^{y}	S_{11}^{z}	S_{12}^{y}	S_{12}^{z}
Benchmark	0.1060	0.8150	0.0790	0.0000	0.5729	1.0000	0.4271	0.0000
Consumption share of traded goods: ω								
0.10	0.0295	0.9484	0.0220	0.0000	0.5729	1.0000	0.4271	0.0000
0.50	0.4669	0.1850	0.3481	0.0000	0.5729	1.0000	0.4271	0.0000
0.75	0.5588	0.0246	0.4166	0.0000	0.5729	1.0000	0.4271	0.0000
Elasticity of substitution traded vs nontraded: ρ								
0.25	0.1111	0.7795	0.0739	0.0355	0.6008	0.9565	0.3992	0.0435
0.75	0.0999	0.8570	0.0851	-0.0420	0.5398	1.0515	0.4602	-0.0515
5.00	0.3017	-0.5284	-0.1167	1.3434	1.6309	-0.6483	-0.6309	1.6483
Consumption share of domestic traded good in consumption of traded: α								
0.25	0.1850	0.8150	0.0000	0.0000	1.0000	1.0000	0.0000	0.0000
0.75	0.0000	0.8150	0.1850	0.0000	0.0000	1.0000	1.0000	0.0000
Elasticity of substitution between traded goods: φ								
0.60	0.0764	0.8150	0.1086	0.0000	0.4129	1.0000	0.5871	0.0000
2.00	0.0992	0.8150	0.0858	0.0000	0.5361	1.0000	0.4639	0.0000
5.00	0.0942	0.8150	0.0908	0.0000	0.5089	1.0000	0.4911	0.0000
Elasticity of intertemporal substitution: σ								
0.50	0.1113	0.9760	0.0737	-0.1610	0.6016	1.1975	0.3984	-0.1975
4.00	0.1069	0.8424	0.0781	-0.0274	0.5777	1.0336	0.4223	-0.0336

—Technical Appendix—

A First Order Conditions and the log-linearized version

A.1 Efficient Allocation

Let us focus on the efficient allocation problem, we have the following set of conditions in equilibrium

$$C_t^1 = \left(\omega^{\frac{1}{\rho}} C_{1t}^{y}{}^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} C_{1t}^{z}{}^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{15}$$

$$C_t^2 = \left(\omega^{\frac{1}{\rho}} C_{2t}^{y}{}^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} C_{2t}^{z}{}^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{16}$$

$$C_{1t}^y = \left(\alpha^{\frac{1}{\varphi}} C_{11t}^{y}{}^{\frac{\varphi-1}{\varphi}} + (1-\alpha)^{\frac{1}{\varphi}} C_{12t}^{y}{}^{\frac{\varphi-1}{\varphi}}\right)^{\frac{\varphi}{\varphi-1}} \tag{17}$$

$$C_{2t}^y = \left((1-\alpha)^{\frac{1}{\varphi}} C_{21t}^{y}{}^{\frac{\varphi-1}{\varphi}} + \alpha^{\frac{1}{\varphi}} C_{22t}^{y}{}^{\frac{\varphi-1}{\varphi}}\right)^{\frac{\varphi}{\varphi-1}} \tag{18}$$

$$\Lambda_t P_{1t}^z = (1-\omega)^{\frac{1}{\rho}} C_{1t}^{z}{}^{-\frac{1}{\rho}} C_t^1{}^{\frac{1}{\rho}-\sigma} \tag{19}$$

$$\Lambda_t = \omega^{\frac{1}{\rho}} \alpha^{\frac{1}{\varphi}} C_{11t}^{y}{}^{-\frac{1}{\varphi}} C_{1t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}} C_t^1{}^{\frac{1}{\rho}-\sigma} \tag{20}$$

$$\Lambda_t P_{2t}^y = \omega^{\frac{1}{\rho}} (1-\alpha)^{\frac{1}{\varphi}} C_{12t}^{y}{}^{-\frac{1}{\varphi}} C_{1t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}} C_t^1{}^{\frac{1}{\rho}-\sigma} \tag{21}$$

$$\Lambda_t P_{2t}^z = (1-\omega)^{\frac{1}{\rho}} C_{2t}^{z}{}^{-\frac{1}{\rho}} C_t^2{}^{\frac{1}{\rho}-\sigma} \tag{22}$$

$$\Lambda_t P_{2t}^y = \omega^{\frac{1}{\rho}} \alpha^{\frac{1}{\varphi}} C_{22t}^{y}{}^{-\frac{1}{\varphi}} C_{2t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}} C_t^2{}^{\frac{1}{\rho}-\sigma} \tag{23}$$

$$\Lambda_t = \omega^{\frac{1}{\rho}} (1-\alpha)^{\frac{1}{\varphi}} C_{21t}^{y}{}^{-\frac{1}{\varphi}} C_{2t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}} C_t^2{}^{\frac{1}{\rho}-\sigma} \tag{24}$$

$$Y_{1t} = C_{11t}^y + C_{21t}^y \tag{25}$$

$$Y_{2t} = C_{12t}^y + C_{22t}^y \tag{26}$$

$$Z_{1t} = C_{1t}^z \tag{27}$$

$$Z_{2t} = C_{2t}^z \tag{28}$$

As can be immediately seen from (19)–(24), setting $\rho\sigma = 1$ corresponds to a separable utility function as in this case

$$\begin{aligned}
\Lambda_t P_{1t}^z &= (1-\omega)^{\frac{1}{\rho}} C_{1t}^{z}{}^{-\frac{1}{\rho}} \\
\Lambda_t &= \omega^{\frac{1}{\rho}} \alpha^{\frac{1}{\varphi}} C_{11t}^{y}{}^{-\frac{1}{\varphi}} C_{1t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}} \\
\Lambda_t P_{2t}^y &= \omega^{\frac{1}{\rho}} (1-\alpha)^{\frac{1}{\varphi}} C_{12t}^{y}{}^{-\frac{1}{\varphi}} C_{1t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}} \\
\Lambda_t P_{2t}^z &= (1-\omega)^{\frac{1}{\rho}} C_{2t}^{z}{}^{-\frac{1}{\rho}} \\
\Lambda_t P_{2t}^y &= \omega^{\frac{1}{\rho}} \alpha^{\frac{1}{\varphi}} C_{22t}^{y}{}^{-\frac{1}{\varphi}} C_{2t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}} \\
\Lambda_t &= \omega^{\frac{1}{\rho}} (1-\alpha)^{\frac{1}{\varphi}} C_{21t}^{y}{}^{-\frac{1}{\varphi}} C_{2t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}}
\end{aligned}$$

A.2 Some steady state results

Let us focus on the steady state of the problem, we have the following set of conditions in equilibrium

$$C^1 = \left(\omega^{\frac{1}{\rho}} C_1^{y\,\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} C_1^{z\,\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{29}$$

$$C^2 = \left(\omega^{\frac{1}{\rho}} C_2^{y\,\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} C_2^{z\,\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{30}$$

$$C_1^y = \left(\alpha^{\frac{1}{\varphi}} C_{11t}^{y\,\frac{\varphi-1}{\varphi}} + (1-\alpha)^{\frac{1}{\varphi}} C_{12}^{y\,\frac{\varphi-1}{\varphi}}\right)^{\frac{\varphi}{\varphi-1}} \tag{31}$$

$$C_2^y = \left((1-\alpha)^{\frac{1}{\varphi}} C_{21}^{y\,\frac{\varphi-1}{\varphi}} + \alpha^{\frac{1}{\varphi}} C_{22}^{y\,\frac{\varphi-1}{\varphi}}\right)^{\frac{\varphi}{\varphi-1}} \tag{32}$$

$$\Lambda P_1^z = (1-\omega)^{\frac{1}{\rho}} C_1^{z\,-\frac{1}{\rho}} C^{1\,\frac{1}{\rho}-\sigma} \tag{33}$$

$$\Lambda = \omega^{\frac{1}{\rho}} \alpha^{\frac{1}{\varphi}} C_{11t}^{y\,-\frac{1}{\varphi}} C_1^{y\,\frac{1}{\varphi}-\frac{1}{\rho}} C^{1\,\frac{1}{\rho}-\sigma} \tag{34}$$

$$\Lambda P_2^y = \omega^{\frac{1}{\rho}} (1-\alpha)^{\frac{1}{\varphi}} C_{12}^{y\,-\frac{1}{\varphi}} C_1^{y\,\frac{1}{\varphi}-\frac{1}{\rho}} C^{1\,\frac{1}{\rho}-\sigma} \tag{35}$$

$$\Lambda P_2^z = (1-\omega)^{\frac{1}{\rho}} C_2^{z\,-\frac{1}{\rho}} C^{2\,\frac{1}{\rho}-\sigma} \tag{36}$$

$$\Lambda P_2^y = \omega^{\frac{1}{\rho}} \alpha^{\frac{1}{\varphi}} C_{22}^{y\,-\frac{1}{\varphi}} C_2^{y\,\frac{1}{\varphi}-\frac{1}{\rho}} C^{2\,\frac{1}{\rho}-\sigma} \tag{37}$$

$$\Lambda = \omega^{\frac{1}{\rho}} (1-\alpha)^{\frac{1}{\varphi}} C_{21}^{y\,-\frac{1}{\varphi}} C_2^{y\,\frac{1}{\varphi}-\frac{1}{\rho}} C^{2\,\frac{1}{\rho}-\sigma} \tag{38}$$

$$Y_1 = C_{11t}^y + C_{21}^y \tag{39}$$

$$Y_2 = C_{12}^y + C_{22}^y \tag{40}$$

$$Z_1 = C_1^z \tag{41}$$

$$Z_2 = C_2^z \tag{42}$$

$$Q_1^y = \beta(Q_1^y + Y_1) \tag{43}$$

$$Q_2^y = \beta(Q_2^y + P_2^y Y_2) \tag{44}$$

$$Q_1^z = \beta(Q_1^z + P_1^z Z_1) \tag{45}$$

$$Q_2^z = \beta(Q_2^z + P_2^z Z_2) \tag{46}$$

Note that defining $\overline{P}_1^y = \left(\alpha + (1-\alpha) P_2^{y\,1-\varphi}\right)^{\frac{1}{1-\varphi}}$ and $\overline{P}_2^y = \left(\alpha P_2^{y\,1-\varphi} + (1-\alpha)\right)^{\frac{1}{1-\varphi}}$, equations (34)–(35) and (37)–(38) reduce to

$$\Lambda \overline{P}_1^y = (1-\omega)^{\frac{1}{\rho}} C_1^{y\,-\frac{1}{\rho}} C^{1\,\frac{1}{\rho}-\sigma} \tag{47}$$

$$\Lambda \overline{P}_2^y = (1-\omega)^{\frac{1}{\rho}} C_2^{y\,-\frac{1}{\rho}} C^{2\,\frac{1}{\rho}-\sigma} \tag{48}$$

Let focus on the case of a symmetric equilibrium: $Y_1 = Y_2 = Y$, $Z_1 = Z_2 = Z$, $C_1^z = C_2^z$, $C^1 = C^2$, $C_{11}^y = C_{22}^y$ and $C_{12}^y = C_{21}^y$. We first give conditions such that $\overline{P}_1^y = \overline{P}_2^y = P_2^y = P_1^z = P_2^z = 1$. Since we will restrict ourselves to a symmetric economy, it will be sufficient to only consider the domestic economy. In this case, using (33) and (47), we get

$$\frac{P_1^z}{\overline{P}_1^y} = \frac{(1-\omega)^{\frac{1}{\rho}} C_1^{z-\frac{1}{\rho}}}{(1-\omega)^{\frac{1}{\rho}} C_1^{y-\frac{1}{\rho}}}$$

Hence, we have

$$\frac{P_1^z}{\overline{P}_1^y} = 1 \Longleftrightarrow \frac{C_1^z}{C_1^y} = \frac{1-\omega}{\omega}$$

Note that once we have $\overline{P}_1^y = 1$, it follows from its definition that $P_2^y = 1$, and therefore $\overline{P}_2^y = 1$. Also note that, in equilibrium, we have $C_1^z = Z_1$, and $C_1^y = C_{11}^y + P_2^y C_{12}^y = C_{11}^y + C_{12}^y = C_{11}^y + C_{21}^y$ (the last step follows from symmetry). Since in equilibrium $C_{11}^y + C_{21}^y = Y_1$, it follows that $C_1^y = Y_1$. Hence, in order for relative prices to be equal to unity in the steady state it is sufficient that the ratio of endowment be given by

$$\frac{Z_1}{Y_1} = \frac{Z_2}{Y_2} = \frac{Z}{Y} = \frac{1-\omega}{\omega}$$

This is the assumption we make in the paper.

With relative prices equal to unity, in a symmetric equilibrium equations (34)–(35) and (37)–(38) imply

$$\frac{\alpha}{C_{11}^y} = \frac{1-\alpha}{C_{12}^y} = \frac{\alpha}{C_{22}^y} = \frac{1-\alpha}{C_{21}^y}$$

which produces

$$\frac{\alpha}{C_{11}^y} = \frac{1}{C_{11}^y + C_{21}^y} = \frac{1}{Y_1} \Longleftrightarrow \frac{C_{11}^y}{Y_1} = \alpha$$

and similarly,

$$\frac{C_{11}^y}{Y_1} = \frac{C_{22}^y}{Y_2} = \alpha \text{ and } \frac{C_{12}^y}{Y_1} = \frac{C_{21}^y}{Y_2} = 1-\alpha$$

Asset prices in the deterministic steady state can be easily determined. With relative prices equal to unity, we have

$$Q_i^x = \frac{\beta}{1-\beta} x_i$$

where $i = \{1, 2\}$ and $x = \{y, z\}$. A direct consequence is then that $Q^z/Q^y = Z/Y$.

A.3 Log–linear Representation

The log-linear version of this system is given by

$$c_t^1 = \varpi c_{1t}^y + (1-\varpi) z_{1t} \tag{49}$$

$$c_t^2 = \varpi c_{2t}^y + (1-\varpi) z_{2t} \tag{50}$$

$$c_{1t}^y = \alpha c_{11t}^y + (1-\alpha) c_{12t}^y \tag{51}$$

$$c_{2t}^y = (1-\alpha) c_{21t}^y + \alpha c_{22t}^y \tag{52}$$

$$\frac{1-\sigma\rho}{\rho} c_{1t} - \frac{z_{1t}}{\rho} = \lambda_t + p_{1t}^z \tag{53}$$

$$\left(\frac{1}{\varphi} - \frac{1}{\rho}\right) c_{1t}^y + \frac{1-\sigma\rho}{\rho} c_{1t} - \frac{c_{11t}^y}{\varphi} = \lambda_t \tag{54}$$

$$\left(\frac{1}{\varphi} - \frac{1}{\rho}\right) c_{1t}^y + \frac{1-\sigma\rho}{\rho} c_{1t} - \frac{c_{12t}^y}{\varphi} = \lambda_t + p_{2t}^y \tag{55}$$

$$\frac{1-\sigma\rho}{\rho} c_{2t} - \frac{z_{2t}}{\rho} = \lambda_t + p_{2t}^z \tag{56}$$

$$\left(\frac{1}{\varphi} - \frac{1}{\rho}\right) c_{2t}^y + \frac{1-\sigma\rho}{\rho} c_{2t} - \frac{c_{22t}^y}{\varphi} = \lambda_t + p_{2t}^y \tag{57}$$

$$\left(\frac{1}{\varphi} - \frac{1}{\rho}\right) c_{2t}^y + \frac{1-\sigma\rho}{\rho} c_{2t} - \frac{c_{21t}^y}{\varphi} = \lambda_t \tag{58}$$

$$y_{1t} = \alpha c_{11t}^y + (1-\alpha) c_{21t}^y \tag{59}$$

$$y_{2t} = (1-\alpha) c_{12t}^y + \alpha c_{22t}^y \tag{60}$$

where $\varpi = \omega^{\frac{1}{\rho}} \overline{y}^{\frac{\rho-1}{\rho}} / (\omega^{\frac{1}{\rho}} \overline{y}^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} \overline{z}^{\frac{\rho-1}{\rho}})$

A.4 Solving the Log–linearized Version

From (59) and (60) we get

$$c_{21t}^y = \frac{y_{1t} - \alpha c_{11t}^y}{1-\alpha} \tag{61}$$

$$c_{12t}^y = \frac{y_{2t} - \alpha c_{22t}^y}{1-\alpha} \tag{62}$$

Plugging this last result in (51) and (52), we get

$$c_{1t}^y = y_{2t} + \alpha c_{11t}^y - \alpha c_{22t}^y \tag{63}$$

$$c_{2t}^y = y_{1t} + \alpha c_{22t}^y - \alpha c_{11t}^y \tag{64}$$

therefore, using (49) and (50), we obtain

$$c_t^1 = \varpi \alpha c_{11t}^y - \varpi \alpha c_{22t}^y + \varpi y_{2t} + (1-\varpi) z_{1t} \tag{65}$$

$$c_t^2 = \varpi \alpha c_{22t}^y - \varpi \alpha c_{11t}^y + \varpi y_{1t} + (1-\varpi) z_{2t} \tag{66}$$

Using (61)–(66) in (54) and (58), we get

$$\lambda_t = \left(\frac{1}{\varphi} - \frac{1}{\rho}\right)(y_{2t} + \alpha c^y_{11t} - \alpha c^y_{22t}) + \frac{1-\sigma\rho}{\rho}(\varpi\alpha c^y_{11t} - \varpi\alpha c^y_{22t} + \varpi y_{2t} + (1-\varpi)z_{1t}) - \frac{c^y_{11t}}{\varphi}$$
$$\lambda_t = \left(\frac{1}{\varphi} - \frac{1}{\rho}\right)(y_{1t} + \alpha c^y_{22t} - \alpha c^y_{11t}) + \frac{1-\sigma\rho}{\rho}(\varpi\alpha c^y_{22t} - \varpi\alpha c^y_{11t} + \varpi y_{1t} + (1-\varpi)z_{2t}) - \frac{c^y_{21t}}{\varphi}$$

computing the difference, we get

$$\begin{aligned}&\left(\frac{2\alpha(\rho-\varphi)}{\rho\varphi} + \frac{2\alpha\varpi(1-\sigma\rho)}{\rho} - \frac{1}{\varphi(1-\alpha)}\right)c^y_{11t} - \left(\frac{2\alpha(\rho-\varphi)}{\rho\varphi} + \frac{2\alpha\varpi(1-\sigma\rho)}{\rho}\right)c^y_{22t}\\ &= \left(\frac{(\rho-\varphi)}{\rho\varphi} + \frac{\varpi(1-\sigma\rho)}{\rho} - \frac{1}{\varphi(1-\alpha)}\right)y_{1t} - \left(\frac{(\rho-\varphi)}{\rho\varphi} + \frac{\varpi(1-\sigma\rho)}{\rho}\right)y_{2t}\\ &\quad + \frac{(1-\varpi)(1-\sigma\rho)}{\rho}(z_{2t} - z_{1t})\end{aligned} \tag{67}$$

Likewise, using (61)–(66) in (55) and (57), we get

$$\lambda_t + p^y_{2t} = \left(\frac{1}{\varphi} - \frac{1}{\rho}\right)(y_{2t} + \alpha c^y_{11t} - \alpha c^y_{22t}) + \frac{1-\sigma\rho}{\rho}(\varpi\alpha c^y_{11t} - \varpi\alpha c^y_{22t} + \varpi y_{2t} + (1-\varpi)z_{1t}) - \frac{c^y_{12t}}{\varphi}$$
$$\lambda_t + p^y_{2t} = \left(\frac{1}{\varphi} - \frac{1}{\rho}\right)(y_{1t} + \alpha c^y_{22t} - \alpha c^y_{11t}) + \frac{1-\sigma\rho}{\rho}(\varpi\alpha c^y_{22t} - \varpi\alpha c^y_{11t} + \varpi y_{1t} + (1-\varpi)z_{2t}) - \frac{c^y_{22t}}{\varphi}$$

computing the difference, we get

$$\begin{aligned}&\left(\frac{2\alpha(\rho-\varphi)}{\rho\varphi} + \frac{2\alpha\varpi(1-\sigma\rho)}{\rho} - \frac{1}{\varphi(1-\alpha)}\right)c^y_{22t} - \left(\frac{2\alpha(\rho-\varphi)}{\rho\varphi} + \frac{2\alpha\varpi(1-\sigma\rho)}{\rho}\right)c^y_{11t}\\ &= \left(\frac{(\rho-\varphi)}{\rho\varphi} + \frac{\varpi(1-\sigma\rho)}{\rho} - \frac{1}{\varphi(1-\alpha)}\right)y_{2t} - \left(\frac{(\rho-\varphi)}{\rho\varphi} + \frac{\varpi(1-\sigma\rho)}{\rho}\right)y_{1t}\\ &\quad + \frac{(1-\varpi)(1-\sigma\rho)}{\rho}(z_{1t} - z_{2t})\end{aligned} \tag{68}$$

Let us denote $\gamma = \frac{(\rho-\varphi)}{\rho\varphi} + \frac{\varpi(1-\sigma\rho)}{\rho}$, $\psi = 1/\varphi(1-\alpha)$ and $\zeta = (1-\varpi)(1-\sigma\rho)/\rho$, c^y_{11t} and c^y_{22t} are solution to the system

$$\begin{pmatrix} 2\alpha\gamma - \psi & -2\alpha\gamma \\ -2\alpha\gamma & 2\alpha\gamma - \psi \end{pmatrix}\begin{pmatrix} c^y_{11t} \\ c^y_{22t} \end{pmatrix} = \begin{pmatrix} \gamma - \psi & -\gamma & -\zeta & \zeta \\ -\gamma & \gamma - \psi & \zeta & -\zeta \end{pmatrix}\begin{pmatrix} y_{1t} \\ y_{2t} \\ z_{1t} \\ z_{2t} \end{pmatrix}$$

which has solution

$$\begin{pmatrix} c^y_{11t} \\ c^y_{22t} \end{pmatrix} = \frac{1}{(\psi - 4\alpha\gamma)}\begin{pmatrix} \psi - \gamma(1+2\alpha) & (1-2\alpha)\gamma & \zeta & -\zeta \\ (1-2\alpha)\gamma & \psi - \gamma(1+2\alpha) & -\zeta & \zeta \end{pmatrix}\begin{pmatrix} y_{1t} \\ y_{2t} \\ z_{1t} \\ z_{2t} \end{pmatrix}$$

We have

$$\begin{aligned}
\psi - 4\alpha\gamma &= \frac{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{\rho\varphi(1-\alpha)} \\
\psi - \gamma(1+2\alpha) &= \frac{(\rho - (1+2\alpha)(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{\rho\varphi(1-\alpha)} \\
(1-2\alpha)\gamma &= \frac{(1-2\alpha)(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{\rho\varphi(1-\alpha)} \\
\zeta &= \frac{(1-\varpi)(1-\sigma\rho)}{\rho}
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
c^y_{11t} =& \frac{\rho - (1+\alpha(1-2\alpha))(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{1t} + \frac{(1-2\alpha)(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{2t} \\
&+ \frac{(1-\varpi)(1-\sigma\rho)\varphi(1-\alpha)}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{1t} - \frac{(1-\varpi)(1-\sigma\rho)\varphi(1-\alpha)}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{2t}
\end{aligned}$$

and

$$\begin{aligned}
c^y_{22t} =& \frac{(1-2\alpha)(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{1t} + \frac{\rho - (1+\alpha(1-2\alpha))(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{2t} \\
&- \frac{(1-\varpi)(1-\sigma\rho)\varphi(1-\alpha)}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{1t} + \frac{(1-\varpi)(1-\sigma\rho)\varphi(1-\alpha)}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{2t}
\end{aligned}$$

Using 61 and 62, we get

$$\begin{aligned}
c^y_{21t} =& \frac{\rho - (1-(1-\alpha)(1-2\alpha))(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{1t} - \frac{\alpha(1-2\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{2t} \\
&- \frac{\alpha(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{1t} + \frac{\alpha(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{2t}
\end{aligned}$$

and

$$\begin{aligned}
c^y_{12t} =& -\frac{\alpha(1-2\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{1t} + \frac{\rho - \alpha(1-(1-\alpha)(1-2\alpha))(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{2t} \\
&+ \frac{\alpha(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{1t} - \frac{\alpha(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{2t}
\end{aligned}$$

The traded good aggregates are then given by

$$\begin{aligned}
c^y_{1t} =& \frac{\alpha(\rho - 2(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{1t} + \frac{(1-\alpha)(\rho - 2\alpha(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{2t} \\
&+ \frac{2\alpha(1-\alpha)(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{1t} - \frac{2\alpha(1-\alpha)(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{2t} \\
c^y_{2t} =& \frac{(1-\alpha)(\rho - 2\alpha(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{1t} + \frac{\alpha(\rho - 2(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{2t} \\
&- \frac{2\alpha(1-\alpha)(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{1t} + \frac{2\alpha(1-\alpha)(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{2t}
\end{aligned}$$

and the consumption aggregates take the form

$$
\begin{aligned}
c_t^1 =& \frac{\varpi\alpha(\rho - 2(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{1t} + \frac{\varpi(1-\alpha)(\rho - 2\alpha(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{2t} \\
&+ \frac{(1-\varpi)(\rho(1-4\alpha(1-\alpha)) + 2\alpha(1-\alpha)\varphi(2 - \varpi(1-\sigma\rho)))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{1t} \\
&- \frac{2\varpi\alpha(1-\alpha)(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{2t} \\
c_t^2 =& \frac{\varpi(1-\alpha)(\rho - 2\alpha(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{1t} + \frac{\varpi\alpha(\rho - 2(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{2t} \\
&- \frac{2\varpi\alpha(1-\alpha)(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{1t} \\
&+ \frac{(1-\varpi)(\rho(1-4\alpha(1-\alpha)) + 2\alpha(1-\alpha)\varphi(2 - \varpi(1-\sigma\rho)))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{2t}
\end{aligned}
$$

Prices are then given by

$$
\begin{aligned}
p_{2t}^y =& \frac{1 - \varpi(1-\sigma\rho)}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}(y_{1t} - y_{2t}) + \frac{(1-2\alpha)(1-\varpi)(1-\sigma\rho)}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}(z_{1t} - z_{2t}) \\
p_{1t}^z =& \frac{\rho(\alpha\varphi + (1-\alpha)\rho) - (1-\alpha)(\rho + 2\alpha\varphi)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{\rho\varphi(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{1t} \\
&+ \frac{(1-\alpha)((\rho - 2\alpha\varphi)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)) - \rho(\rho - \varphi))}{\rho\varphi(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{2t} \\
&+ \frac{(1-\alpha)(1-\sigma\rho)((1-2\alpha)(1-\varpi)\rho + 2\alpha\varphi(1+\varpi)) - (4\alpha(1-\alpha)\varphi + (1-4\alpha(1-\alpha))\rho)}{\rho(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{1t} \\
&- \frac{((1-2\alpha)\rho + 2\alpha\varphi)(1-\alpha)(1-\varpi)(1-\sigma\rho)}{\rho(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{2t} \\
p_{2t}^z =& \frac{\rho(\alpha\rho + (1-\alpha)\varphi) - \alpha(\rho + 2(1-\alpha)\varphi)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{\rho\varphi(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{1t} \\
&- \frac{\alpha(\rho(\rho - \varphi) - (\rho - 2(1-\alpha)\varphi)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{\rho\varphi(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{2t} \\
&- \frac{\alpha(1-\varpi)(1-\sigma\rho)(2(1-\alpha)\varphi + (1-2(1-\alpha))\rho)}{\rho(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{1t} \\
&+ \frac{\alpha(1-\sigma\rho)((1-2(1-\alpha))(1-\varpi)\rho + 2(1-\alpha)\varphi(1+\varpi)) - (4\alpha(1-\alpha)\varphi + (1-4\alpha(1-\alpha))\rho)}{\rho(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{2t}
\end{aligned}
$$

We can finally get λ_t

$$
\begin{aligned}
\lambda_t =& - \frac{\varphi(1 - \varpi(1-\sigma\rho))(\rho - 2\alpha(1-\alpha))(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{\rho\varphi(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{1t} \\
&+ \frac{2\alpha(1-\alpha)\varphi(1 - \varpi(1-\sigma\rho))(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{\rho\varphi(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} y_{2t} \\
&+ \frac{\alpha(1-\varpi)(1-\sigma\rho)(\rho - 2(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{\rho(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{1t} \\
&+ \frac{(1-\alpha)(1-\varpi)(1-\sigma\rho)(\rho - 2\alpha(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{\rho(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))} z_{2t}
\end{aligned}
$$

A.5 Properties of the Solution

In the following, we review some properties of the log–linear solution of the model and report the proofs of the main propositions reported in the main text.

Lemma 1 *The dominator of each coefficient involved in the solution of the efficient allocation is positive.*

Proof: The denominator of each coefficient is always proportional to

$$\Delta \equiv \rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))$$

which rewrites

$$\Delta = \rho(1-4\alpha(1-\alpha)) + \rho\sigma\varpi\varphi 4\alpha(1-\alpha) + 4\alpha(1-\alpha)\varphi(1-\varpi)$$

By definition, $0 \leqslant \varpi \leqslant 1$ and since $0 \leqslant \alpha \leqslant 1$, we have $4\alpha(1-\alpha) \leqslant 1$. The result follows.
Q.E.D. □

Result 1 *The effects of traded goods shocks on efficient consumption: In equilibrium, the impact effect of a traded goods shock satisfies*

$$\frac{\partial c^y_{iit}}{\partial y_{it}} > 0 \Longleftrightarrow \alpha > \frac{1}{2} \text{ or } \varphi > \frac{\alpha(1-2\alpha)\rho}{(1+\alpha(1-2\alpha))(1-\varpi(1-\sigma\rho))} \text{ and } \alpha < \frac{1}{2} \tag{69}$$

$$\frac{\partial c^y_{iit}}{\partial y_{jt}} > 0,\ \frac{\partial c^y_{ijt}}{\partial y_{it}} < 0 \Longleftrightarrow \varphi \lesseqgtr \frac{\rho}{1-\varpi(1-\sigma\rho)} \text{ and } \alpha \lesseqgtr \frac{1}{2} \tag{70}$$

Proof: Let us first prove the first part of the result. Given that the denominator is positive, and looking at the solution for c^y_{iit}, the sign of the response to y_{it} is given by the sign of

$$\rho - (1+\alpha(1-2\alpha))(\rho - \varphi(1-\varpi(1-\sigma\rho)))$$

This quantity is strictly positive as long as

$$\varphi > \frac{\alpha(1-2\alpha)\rho}{(1+\alpha(1-2\alpha))(1-\varpi(1-\sigma\rho))}$$

When $\alpha > 0.5$, the right hand side of the inequality is negative. Since $\varphi \geqslant 0$ by assumption, the inequality is always satisfied in that case. When $\alpha < 0.5$, the inequality must hold.

Let us now prove the second part of the result. Given that the denominator of the solution is positive, $\alpha \in (0,1)$, $\varpi \in (0,1)$, $\varphi > 0$ and $\rho > 0$, the sign of the coefficient in front of y_{jt} in the solution of c^y_{iit} and c^y_{ijt} is entirely determined by the sign of $(1-2\alpha)(\rho-\varphi(1-\varpi(1-\sigma\rho)))$. The result then follows.
Q.E.D. □

Result 2 *The effects of endowment shocks on the relative price of traded goods: The impact effect of an endowment shock on the relative price of the foreign traded good satisfies*

1. *Shocks on traded*

$$\frac{\partial p_{2t}^y}{\partial y_{1t}} > 0,\ \frac{\partial p_{2t}^y}{\partial y_{2t}} < 0$$

2. *Shocks on non–traded*

$$\frac{\partial p_{2t}^y}{\partial z_{1t}} > 0,\ \frac{\partial p_{2t}^y}{\partial z_{2t}} < 0 \Longleftrightarrow \sigma\rho \lesseqgtr 1 \text{ and } \alpha \lesseqgtr \frac{1}{2}$$

Proof:

1. We proved in result 1 that the denominator of the coefficient on endowments is positive. Furthermore, $1 - \varpi(1 - \sigma\rho)$ is positive. The result then follows.
2. Since the denominator of the coefficient is positive, the sign of the impact effect of a shock on non–traded is given by the sign of $(1 - 2\alpha)(1 - \rho\sigma)$. The result follows.

Q.E.D. □

Lemma 2 *Assume that $\sigma\rho = 1$ and that the elasticity of substitution between traded and non traded goods is less than unity ($\rho < 1$), then the relative price of the foreign traded good increases less than one for one following an increase in the domestic endowment, provided domestic and foreign traded goods are sufficiently good substitutes ($\varphi > \rho + \frac{1-\rho}{4\alpha(1-\alpha)}$)*

Proof:Lemma 2 Since $\rho\sigma = 1$, $\partial p_{2t}^y/\partial y_{1t}$ reduces to

$$\frac{\partial p_{2t}^y}{\partial y_{1t}} = \frac{1}{\rho - 4\alpha(1-\alpha)(\rho - \varphi)}$$

In the case where $\rho < 1$ (the case we consider) we have

$$\frac{\partial p_{2t}^y}{\partial y_{1t}} < 1 \Longleftrightarrow \rho - 4\alpha(1-\alpha)(\rho - \varphi) > 1$$

which amounts to

$$\varphi > \rho + \frac{1-\rho}{4\alpha(1-\alpha)}$$

since $\alpha \in (0, 1)$ and $\rho < 1$ the second term is positive. Note that in a neighborhood of $\alpha = 0.5$, $4\alpha(1 - \alpha) \simeq 1$, it is sufficient that $\varphi > 1$.
Q.E.D. □

A.6 Proofs of Propositions

Proof (Proof of proposition 1): Given that the denominator is positive, $\alpha \in (0, 1)$, $\varpi \in (0, 1)$, $\varphi > 0$ and $\rho > 0$, the sign of the coefficient in front of $z_{\cdot t}$ in the solution is entirely determined by the sign of $\sigma\rho - 1$. The result then follows.
Q.E.D. □

Proof (Proof of proposition 2): Let us now consider the expenditures ratio

$$\Theta_t = \frac{C_{11t}^y + P_{2t}^y C_{12t}^y}{C_{21t}^y + P_{2t}^y C_{22t}^y}$$

its log–linear approximation is given by

$$\vartheta_t = \alpha c_{11t}^y + (1-\alpha)(p_{2t}^y + c_{12t}^y) - (1-\alpha)c_{21t}^y - \alpha(p_{2t}^y + c_{22t}^y)$$

Making use of (61)–(62), this rewrites as

$$\vartheta_t = 2\alpha(c_{11t}^y - c_{22t}^y) + (1-2\alpha)p_{2t}^y - (y_{1t} - y_{2t})$$

Plugging the solution of the log–linear version of the model, one gets

$$\frac{\partial \vartheta_t}{\partial y_{1t}} = \frac{(1-2\alpha)(1-\rho-\varpi(1-\sigma\rho))}{\rho - 4\alpha(1-\alpha)(\rho-\varphi+\varpi\varphi(1-\sigma\rho))}$$

Let us then consider the case where $\rho\sigma = 1$, this reduces to

$$\frac{\partial \vartheta_t}{\partial y_{1t}} = \frac{(1-2\alpha)(1-\rho)}{(1-4\alpha(1-\alpha))\rho + 4\alpha(1-\alpha)\varphi}$$

Since we established in Result 1 that the denominator is positive, the sign of the latter derivative is given by the sign of

$$(1-2\alpha)(1-\rho)$$

The result follows.
Q.E.D. □

Proof (Proof of proposition 3): As noted in the text, share holdings are constant to a first order approximation and the budget constraints write

$$\begin{aligned} \text{Home: } & S_{11}^y y_{1t} + S_{12}^y P_{2t}^y y_{2t} = C_{11t}^y + P_{2t}^y C_{12t}^y \\ \text{Abroad: } & S_{21}^y y_{1t} + S_{22}^y P_{2t}^y y_{2t} = C_{21t}^y + P_{2t}^y C_{22t}^y \end{aligned}$$

Computing the ratio of the two budget constraints, we get

$$\Theta_t = \frac{S_{11}^y y_{1t} + S_{12}^y P_{2t}^y y_{2t}}{S_{21}^y y_{1t} + S_{22}^y P_{2t}^y y_{2t}}$$

which admits the log–linear version

$$\vartheta_t = \frac{S_{11}^y y_1}{S_{11}^y y_1 + S_{12}^y P_2^y y_2} y_{1t} + \frac{S_{12}^y P_2^y y_2}{S_{11}^y y_1 + S_{12}^y P_2^y y_2}(p_{2t}^y + y_{2t}) - \frac{S_{21}^y y_1}{S_{21}^y y_1 + S_{22}^y P_2^y y_2} y_{1t} - \frac{S_{22}^y P_2^y y_2}{S_{21}^y y_1 + S_{22}^y P_2^y y_2}(p_{2t}^y + y_{2t})$$

Making use of equilibrium on equity markets

$$\begin{aligned} S_{11}^y + S_{21}^y &= 1 \\ S_{12}^y + S_{22}^y &= 1 \end{aligned}$$

and assuming symmetry ($y_1 = y_2 = y$, $p_2^y = 1$ and $S_{11}^y = S_{22}^y = s$) this reduces to

$$\vartheta_t = (1-2s)(p_{2t}^y + y_{2t} - y_{1t})$$

then

$$\frac{\partial \vartheta_t}{\partial y_{1t}} = (1-2s)\left(\frac{\partial p_{2t}^y}{\partial y_{1t}} - 1\right)$$

From Proposition 2, it is clear then in the case of complementary goods ($\rho < 1$) and home bias ($\alpha > 0.5$)

$$\frac{\partial \vartheta_t}{\partial y_{1t}} < 0$$

Assuming that traded goods are highly substitutable ($\varphi > \rho + (1 - \rho)/4\alpha(1 - \alpha)$), we have from lemma 2 that $\partial p^y_{2t}/\partial y_{1t} < 1$. Then the condition for ϑ_t to decrease with y_{1t} is that , s, the share of domestic traded firms held by domestic agents be lower than 0.5.

Q.E.D. □