Withering Government Spending Multipliers
Technical Appendix

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1 Additional regressions

Unfortunately, the Philadelphia real–time data database does not report nominal government expenditures. We therefore build a (unsatisfactory) nominal expenditures variables to investigate misperceptions. Nominal government expenditures are then given by the product of the real government expenditures and the GDP deflator.

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Table 1: Forecasting regressions (Real Government Expenditures, $\mu(t|t+1)$)

Cst.	R_t	$\Delta S\&P$	ΔY	π	F	D.W.	R^2
1966Q1–2002Q4							
0.000	-0.000	-0.001	-0.013	-0.044	0.13	2.10	0.00
(0.001)	(0.059)	(0.005)	(0.046)	(0.076)	[0.971]		
0.000	-0.020	-0.001	-0.006	–	0.07	2.08	0.00
(0.001)	(0.048)	(0.004)	(0.044)		[0.977]		
0.000	-0.018	-0.001	–	–	0.09	2.08	0.00
(0.001)	(0.045)	(0.004)			[0.913]		
0.000	-0.017	–	–	–	0.14	2.08	0.00
(0.001)	(0.045)				[0.705]		
-0.000	–	-0.001	–	–	0.02	2.07	0.00
(0.000)		(0.004)			[0.891]		
-0.000	–	–	-0.001	–	0.00	2.07	0.00
(0.000)			(0.042)		[0.990]		
0.000	–	–	–	-0.035	0.39	2.09	0.00
(0.001)				(0.055)	[0.532]		
1966Q1–1980Q4							
-0.002	-0.030	-0.017	0.042	0.069	1.73	1.90	0.13
(0.002)	(0.115)	(0.007)	(0.058)	(0.135)	[0.157]		
-0.002	0.010	-0.017	0.031	–	2.27	1.90	0.12
(0.002)	(0.084)	(0.007)	(0.054)		[0.091]		
-0.001	-0.012	-0.018	–	–	3.32	1.92	0.12
(0.001)	(0.074)	(0.007)			[0.043]		
-0.002	0.045	–	–	–	0.35	1.90	0.01
(0.001)	(0.075)				[0.555]		
-0.001	–	-0.017	–	–	6.73	1.91	0.12
(0.001)		(0.006)			[0.012]		
-0.001	–	–	0.033	–	0.44	1.90	0.01
(0.001)			(0.050)		[0.510]		
-0.002	–	–	–	0.037	0.20	1.92	0.00
(0.001)				(0.084)	[0.658]		
1981Q1–2002Q4							
0.001	-0.185	0.009	-0.059	0.356	2.02	2.22	0.10
(0.001)	(0.086)	(0.006)	(0.074)	(0.171)	[0.099]		
0.002	-0.046	0.009	-0.089	–	1.41	2.27	0.05
(0.001)	(0.055)	(0.006)	(0.073)		[0.246]		
0.001	-0.033	0.009	–	–	1.43	2.27	0.03
(0.001)	(0.054)	(0.006)			[0.244]		
0.001	-0.040	–	–	–	0.53	2.29	0.01
(0.001)	(0.055)				[0.470]		
0.000	–	0.009	–	–	2.53	2.26	0.03
(0.000)		(0.006)			[0.115]		
0.001	–	–	-0.074	–	1.01	2.28	0.01
(0.001)			(0.073)		[0.317]		
-0.000	–	–	–	0.081	0.58	2.26	0.01
(0.001)				(0.106)	[0.448]		

Note: R =Federal fund rate, $\Delta S\&P$ = changes in the $S\&P$ stock market index, ΔY = changes in GDP, π inflation rate (GDP Deflator). Standard deviations in parenthesis. F denotes the joint significance test of the regressors, associated p–value into brackets.

Table 2: Forecasting regressions (Nominal Government Expenditures, $\mu(t|T)$)

Cst.	R_t	$\Delta S\&P$	ΔY	π	F	D.W.	R^2
1966Q1–2002Q4							
-0.002	0.144	0.004	-0.038	-0.046	0.33	2.05	0.01
(0.003)	(0.159)	(0.012)	(0.123)	(0.204)	[0.858]		
-0.002	0.123	0.004	-0.031	–	0.42	2.04	0.01
(0.003)	(0.128)	(0.012)	(0.118)		[0.736]		
-0.002	0.133	0.004	–	–	0.61	2.03	0.01
(0.002)	(0.122)	(0.012)			[0.546]		
-0.002	0.126	–	–	–	1.10	2.04	0.01
(0.002)	(0.120)				[0.297]		
-0.000	–	0.002	–	–	0.03	2.02	0.00
(0.001)		(0.012)			[0.856]		
0.000	–	–	-0.065	–	0.33	2.05	0.00
(0.001)			(0.113)		[0.568]		
-0.001	–	–	–	0.080	0.29	2.02	0.00
(0.002)				(0.148)	[0.593]		
1966Q1–1980Q4							
-0.011	-0.278	-0.030	0.063	0.854	2.51	1.98	0.18
(0.005)	(0.300)	(0.018)	(0.152)	(0.352)	[0.052]		
-0.007	0.218	-0.029	-0.075	–	1.77	1.94	0.09
(0.005)	(0.229)	(0.018)	(0.147)		[0.164]		
-0.008	0.271	-0.027	–	–	2.58	1.92	0.09
(0.004)	(0.203)	(0.018)			[0.085]		
-0.009	0.359	–	–	–	3.15	1.86	0.05
(0.004)	(0.197)				[0.081]		
-0.003	–	-0.034	–	–	3.59	1.86	0.06
(0.001)		(0.017)			[0.063]		
-0.002	–	–	-0.127	–	0.89	1.84	0.02
(0.002)			(0.134)		[0.349]		
-0.012	–	–	–	0.617	7.41	1.92	0.13
(0.003)				(0.212)	[0.009]		
1981Q1–2002Q4							
-0.000	-0.021	0.022	0.081	0.199	0.56	2.29	0.03
(0.003)	(0.240)	(0.016)	(0.205)	(0.477)	[0.691]		
0.000	0.056	0.022	0.064	–	0.70	2.29	0.03
(0.003)	(0.150)	(0.016)	(0.200)		[0.553]		
0.001	0.048	0.022	–	–	1.01	2.30	0.02
(0.003)	(0.146)	(0.016)			[0.367]		
0.001	0.032	–	–	–	0.05	2.32	0.00
(0.003)	(0.147)				[0.830]		
0.001	–	0.022	–	–	1.95	2.30	0.02
(0.001)		(0.016)			[0.166]		
0.001	–	–	0.059	–	0.09	2.31	0.00
(0.002)			(0.196)		[0.764]		
0.001	–	–	–	0.104	0.13	2.32	0.00
(0.002)				(0.286)	[0.718]		

Note: R =Federal fund rate, $\Delta S\&P$ = changes in the $S\&P$ stock market index. Standard deviations in parenthesis. F denotes the joint significance test of the regressors, associated p–value into brackets.

Table 3: Forecasting regressions (Nominal Government Expenditures, $\mu(t|t+1)$)

Cst.	R_t	$\Delta S\&P$	ΔY	π	F	D.W.	R^2
1966Q1–2002Q4							
-0.000	0.010	-0.004	-0.009	-0.017	0.17	2.10	0.00
(0.001)	(0.064)	(0.005)	(0.049)	(0.082)	[0.953]		
-0.000	0.002	-0.004	-0.007	–	0.21	2.09	0.00
(0.001)	(0.051)	(0.005)	(0.047)		[0.887]		
-0.000	0.005	-0.004	–	–	0.31	2.09	0.00
(0.001)	(0.049)	(0.005)			[0.732]		
-0.000	0.011	–	–	–	0.05	2.08	0.00
(0.001)	(0.048)				[0.826]		
-0.000	–	-0.004	–	–	0.62	2.09	0.00
(0.000)		(0.005)			[0.432]		
-0.000	–	–	-0.007	–	0.02	2.08	0.00
(0.000)			(0.045)		[0.876]		
-0.000	–	–	–	0.004	0.00	2.07	0.00
(0.001)				(0.059)	[0.946]		
1966Q1–1980Q4							
-0.002	-0.051	-0.019	0.032	0.122	1.84	2.03	0.13
(0.002)	(0.129)	(0.008)	(0.065)	(0.151)	[0.134]		
-0.002	0.020	-0.019	0.012	–	2.31	2.02	0.12
(0.002)	(0.094)	(0.008)	(0.060)		[0.086]		
-0.001	0.011	-0.020	–	–	3.51	2.03	0.12
(0.002)	(0.083)	(0.007)			[0.037]		
-0.002	0.074	–	–	–	0.78	1.97	0.01
(0.002)	(0.084)				[0.382]		
-0.001	–	-0.020	–	–	7.12	2.03	0.12
(0.001)		(0.007)			[0.010]		
-0.001	–	–	0.013	–	0.05	1.97	0.00
(0.001)			(0.056)		[0.819]		
-0.002	–	–	–	0.086	0.84	2.00	0.01
(0.001)				(0.094)	[0.363]		
1981Q1–2002Q4							
0.001	-0.159	0.006	-0.028	0.359	1.25	2.12	0.06
(0.001)	(0.093)	(0.006)	(0.079)	(0.185)	[0.297]		
0.001	-0.018	0.006	-0.058	–	0.49	2.16	0.02
(0.001)	(0.059)	(0.006)	(0.079)		[0.688]		
0.001	-0.010	0.006	–	–	0.48	2.15	0.01
(0.001)	(0.058)	(0.006)			[0.621]		
0.001	-0.014	–	–	–	0.06	2.18	0.00
(0.001)	(0.058)				[0.806]		
0.000	–	0.006	–	–	0.94	2.15	0.01
(0.000)		(0.006)			[0.335]		
0.001	–	–	-0.051	–	0.43	2.19	0.01
(0.001)			(0.077)		[0.511]		
-0.000	–	–	–	0.117	1.07	2.16	0.01
(0.001)				(0.112)	[0.304]		

Note: R =Federal fund rate, $\Delta S\&P$ = changes in the $S\&P$ stock market index. Standard deviations in parenthesis. F denotes the joint significance test of the regressors, associated p–value into brackets.

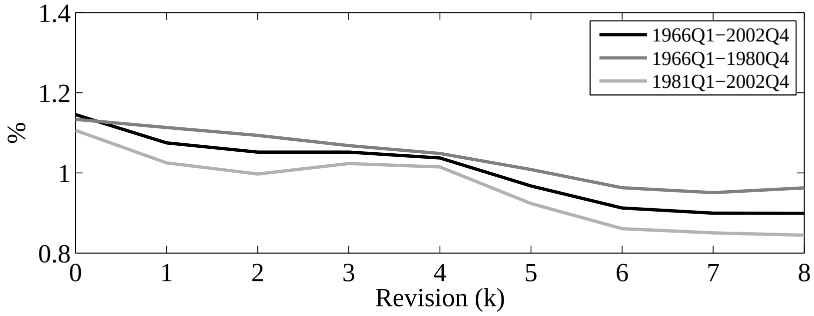
Table 4: Properties of Misperceived Changes in Government Expenditures

	$g(t\|T) - g(t\|t)$			$g(t\|t+1) - g(t\|t)$		
	σ_μ	σ_μ/σ_g	ρ	σ_μ	σ_μ/σ_g	ρ
Real Government Spendings						
1966Q1–2002Q4	1.15	1.13	0.03	0.42	0.41	-0.04
1966Q1–1979Q4	1.13	0.99	0.17	0.41	0.36	0.04
1980Q1–2002Q4	1.11	1.22	-0.16	0.41	0.45	-0.15
Nominal Government Spendings						
1966Q1–2002Q4	1.12	0.99	-0.01	0.45	0.40	-0.04
1966Q1–1979Q4	1.09	0.88	0.12	0.46	0.37	0.01
1980Q1–2002Q4	1.11	1.12	-0.16	0.44	0.44	-0.10

Note: σ_μ denotes the standard deviation of the measurement error, σ_μ/σ_g denotes the ratio of the standard deviation of the measurement error to the standard deviation of $g(t|T)$, and ρ is the first order autocorrelation of the measurement error.

Figure 1: Standard deviation of $g(t|T) - g(t|t+k)$

(a) Real Government Spendings



(b) Nominal Government Spendings

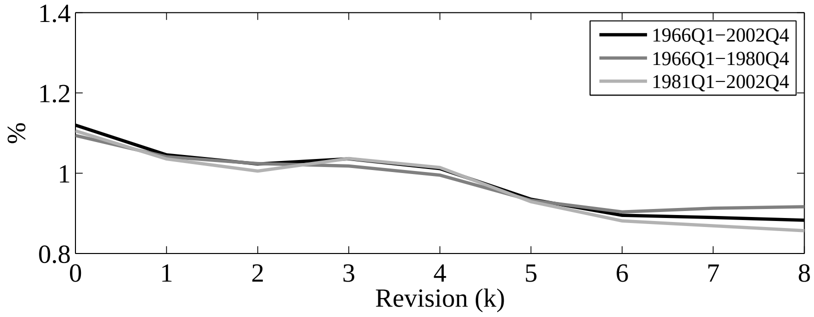
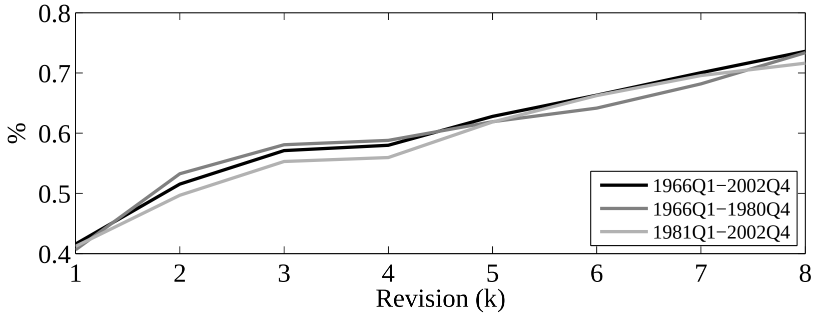


Figure 2: Standard deviation of $g(t|t+k) - g(t|t)$

(a) Real Government Spendings



(b) Nominal Government Spendings

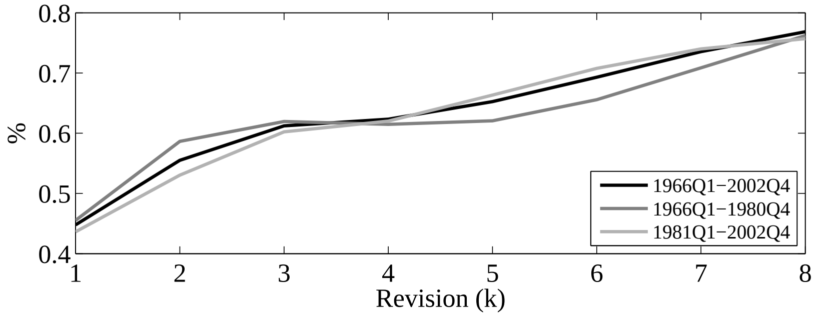
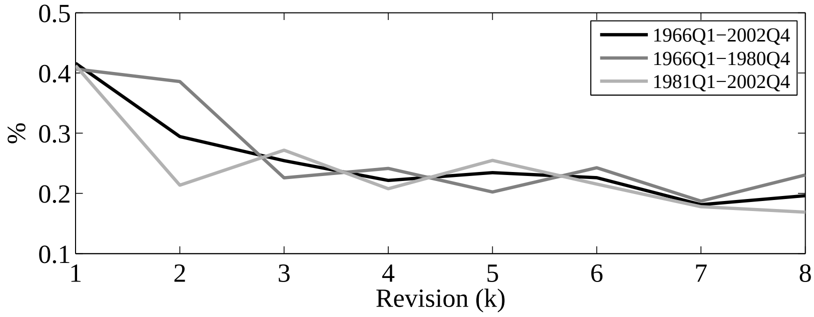
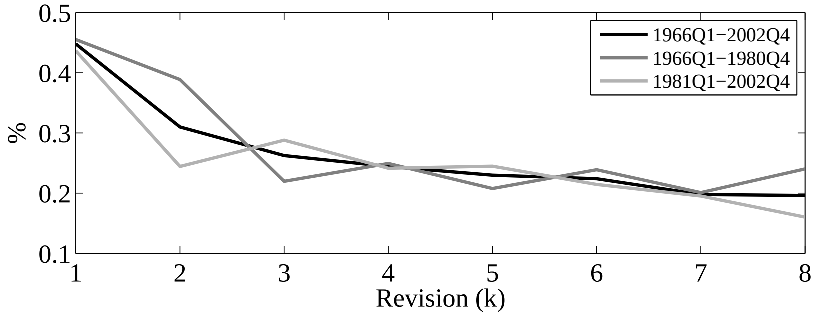


Figure 3: Standard deviation of $g(t|t+k) - g(t|t+k-1)$

(a) Real Government Spendings



(b) Nominal Government Spendings



2 Model

First–order conditions of the Household's program

$$
\begin{aligned}
c_t: &\quad \frac{1}{c_t - bc_{t-1}} - \beta b \mathbb{E}_t\left[\frac{1}{c_{t+1} - bc_t}\right] = \Lambda_t P_t && (1)\\
h_t: &\quad \nu_h h_t^{\sigma_h} = \Lambda_t P_t w_t && (2)\\
u_t: &\quad r_{k,t} = z'(u_t) && (3)\\
M_t: &\quad \nu_m \left(\frac{M_t}{P_t}\right)^{-\sigma_m} = \Lambda_t P_t \frac{R_t - 1}{R_t} && (4)\\
B_t^{\mathrm{D}}: &\quad \Lambda_t = \beta R_t \mathbb{E}_t \Lambda_{t+1} && (5)\\
B_t^{\mathrm{F}}: &\quad \Lambda_t\left(1 + \chi\left(\frac{e_t B_t^{\mathrm{F}}}{P_t}\right)\right) = \beta R_t^{\star} \mathbb{E}_t \frac{e_{t+1}}{e_t}\Lambda_{t+1} && (6)\\
k_{t+1}: &\quad q_t = \beta \mathbb{E}_t\left[\Lambda_{t+1} P_{t+1} r_{k,t+1} + q_{t+1}(1-\delta)\right] && (7)\\
i_t: &\quad P_t \Lambda_t = q_t\left(1 - \Phi\left(\frac{i_t}{i_{t-1}}\right) - \frac{i_t}{i_{t-1}}\Phi'\left(\frac{i_t}{i_{t-1}}\right)\right)\\
&\quad + \beta \mathbb{E}_t\left[\frac{\Lambda_{t+1} P_{t+1}}{\Lambda_t P_t} q_{t+1} \frac{i_{t+1}}{i_t}\Phi'\left(\frac{i_{t+1}}{i_t}\right)\right] && (8)
\end{aligned}
$$

where Λ_t and q_t denote the Lagrange multiplier to, respectively, the budget constraint and the capital accumulation.

Equilibrium Let us define the following variables: $\lambda_t = \Lambda_t P_t$, $p_t^{\star} = e_{t-1} P_t^{\star}/P_{t-1}$, $\pi_t = P_t/P_{t-1}$, $m_t = M_t/P_t$, $b_t = e_t B_t^{\mathrm{F}}/P_t$, $\Delta_t = e_t/e_{t-1}$.

The equilibrium involves the following equations

$$
\begin{aligned}
&\lambda_t = \frac{1}{c_t - bc_{t-1}} - \beta b \mathbb{E}_t\left[\frac{1}{c_{t+1} - bc_t}\right] && (9)\\
&\nu_h h_t^{\sigma_h} = \lambda_t w_t && (10)\\
&r_{k,t} = z'(u_t) && (11)\\
&\nu_m m_t^{-\sigma_m} = \lambda_t \frac{R_t - 1}{R_t} && (12)\\
&\lambda_t = q_t\left(1 - \varphi\left(\frac{i_t}{k_t} - \delta\right)\right) && (13)\\
&y_t = c_t + i_t + g_t + z(u_t)k_t + \frac{\chi}{2} b_t^2 && (14)\\
&x_t = x_{d,t} + x_{d,t}^{\star} && (15)\\
&x_t = a_t (u_t k_t)^{\alpha} h_t^{1-\alpha} && (16)
\end{aligned}
$$

$$\alpha s_t x_t = r_{k,t} u_t k_t \tag{17}$$

$$(1-\alpha) s_t x_t = w_t h_t \tag{18}$$

$$x_{d,t} = \left(\frac{P_{xt}}{P_t}\right)^{\frac{1}{\rho-1}} \omega y_t \tag{19}$$

$$x^{\star}_{d,t} = \left(\frac{P_{xt}}{e_t P^{\star}_t}\right)^{\frac{1}{\rho-1}} (1-\omega^{\star}) y^{\star}_t \tag{20}$$

$$x_{f,t} = \left(\frac{e_t P^{\star}_{xt}}{P_t}\right)^{\frac{1}{\rho-1}} (1-\omega) y_t \tag{21}$$

$$\left(\omega p_{x,t}^{\frac{\rho}{\rho-1}} + (1-\omega)\left(\frac{\Delta_t p^{\star}_{x,t}}{\pi_t}\right)^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}} = 1 \tag{22}$$

$$\widehat{R}_t = \rho_r \widehat{R}_{t-1} + (1-\rho_r)[\gamma_\pi \widehat{\pi}_t + \gamma_y \widehat{y}_t + \gamma_{\Delta e} \widehat{\Delta e}_t] \tag{23}$$

$$k_{t+1} = i_t - \frac{\varphi}{2}\left(\frac{i_t}{k_t} - \delta\right)^2 k_t + (1-\delta) k_t \tag{24}$$

$$b_t = R^{\star}_t \Delta_t \frac{b_{t-1}}{\pi_t} + p_{xt} x_t - y_t \tag{25}$$

$$\lambda_t = \beta R_t \mathbb{E}_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{26}$$

$$\lambda_t \left(1 + \chi b_t\right) = \beta R^{\star}_t \mathbb{E}_t \Delta_{t+1} \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{27}$$

$$q_t = \beta \mathbb{E}_t \left[\lambda_{t+1} r_{k,t+1} + q_{t+1}\left(1 - \delta + \frac{\varphi}{2}\left(\left(\frac{i_{t+1}}{k_{t+1}}\right)^2 - \sigma^2\right)\right)\right] \tag{28}$$

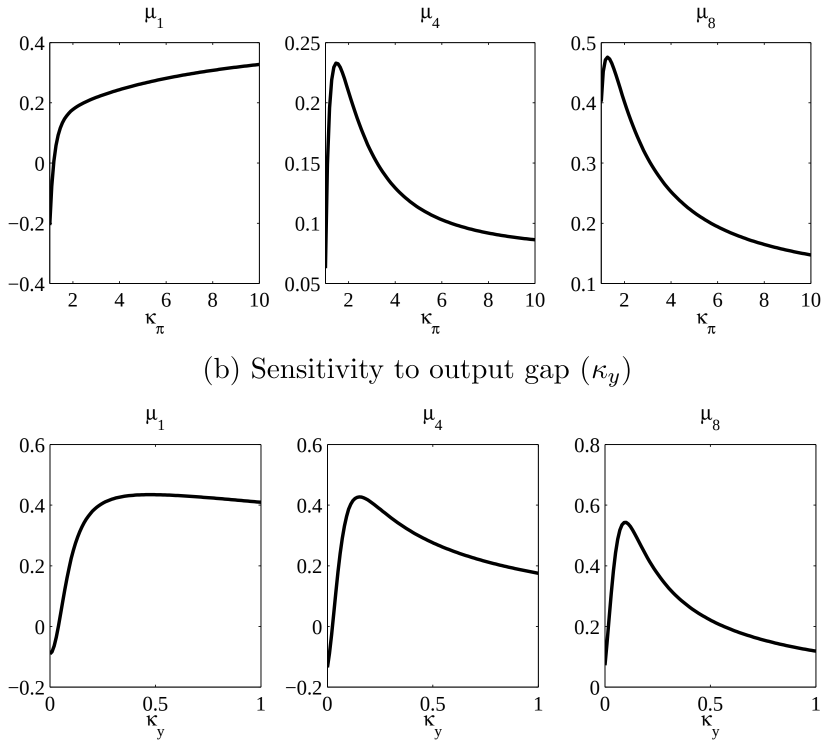
$$p^{\star}_{x,t} = p^{\star}_t \tag{29}$$

$$\frac{1}{y^{\star}_t} = \beta R^{\star}_t \mathbb{E}_t \left[\frac{1}{y^{\star}_{t+1} \pi^{\star}_{t+1}}\right] \tag{30}$$

and the log–linearized Phillips curve.

Figure 4: Multiplier in the post–1980 era: Sensitivity to Monetary Policy
(a) Sensitivity to inflation (κ_π)

(b) Sensitivity to output gap (κ_y)



In order to assess the empirical relevance of a break in the persistence of canadian government expenditures in the post 1980 era, we regress the log of government expenditures on its lagged values, a constant, a trend and the interaction between each of these variables and a dummy variable which equals 0 in the pre and 1 in the post–1980 period

$$\log(G_t) = \alpha_1 + \beta_1 t + \rho_1 \log(G_{t-1}) + \alpha_2 \mathbb{I}_{(t>1980Q4)} + \beta_2 \mathbb{I}_{(t>1980Q4)} \times t + \rho_2 \mathbb{I}_{(t>1980Q4)} \times \log(G_{t-1}) + u_t$$

The Fisher test for the joint significance of the interaction terms ($H_0 : \alpha_2 = \beta_2 = \rho_2 = 0$) has a value F=2.39 and is distributed as a Fisher (3,161), which leads us to reject the null with a probability value of 0.07. We therefore consider two distinct processes for government spendings. In the pre–1980 era, the persistence and the standard deviation of the innovation are respectively 0.96 and 2.09%. In the post 1980 era, the corresponding estimates are 0.78 and 1.54%.

Table 5: Volatility of measurement errors (Model)

	$g(t\|T) - g(t\|t)$	$g(t\|t+1) - g(t\|t)$
Pre–1980	0.605	0.182
Post–1980	0.846	0.178
% Change	39.83	-1.92