Technical Appendix: Globalization, Returns to Accumulation and the World Distribution of Output

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1 Proofs of Proposition

In this section we will abstract from any reference to the index of the economy, except when strictly necessary, in order to save on notation.

Proposition 1 *In the absence of international trade, the relationship between output–per–worker and capital–per–worker is given by*

$$y_{it} = \Upsilon(\Gamma_t) k_{it}^{\alpha\varphi+\beta(1-\varphi)} \tag{1}$$

with $\Upsilon(\Gamma_t) \equiv \Theta_1^{\varphi}\Theta_2^{1-\varphi} \frac{(\alpha\varphi)^{\alpha\varphi}(\beta(1-\varphi))^{\beta(1-\varphi)}}{[\alpha\varphi+\beta(1-\varphi)]^{\alpha\varphi+\beta(1-\varphi)}} \frac{((1-\alpha)\varphi)^{(1-\alpha)\varphi}\left((1-\varphi)\frac{1-\beta}{1+x}\right)^{(1-\beta)(1-\varphi)}}{\left[(1-\alpha)\varphi+(1-\varphi)\frac{1-\beta}{1+x}\right]^{\varphi(1-\alpha)+(1-\varphi)(1-\beta)}} \Gamma_t^{1-\alpha\varphi-\beta(1-\varphi)}$

Proof of proposition 1: In the absence of trade $X_{1t} = X_{2t} = 0$ so that the problem the final good producer has to solve is

$$\max_{Z_{1t},Z_{2t}} Z_{1t}^{\varphi} Z_{2t}^{1-\varphi} - P_{1t}Z_{1t} - P_{2t}Z_{2t}$$

This yields the standard input demand functions $\varphi Y_t = P_{1t}Z_{1t}$ and $(1-\varphi)Y_t = P_{2t}Z_{2t}$.

Free entry on the final good market implies that $1 = \frac{P_{1t}^{\varphi}P_{2t}^{1-\varphi}}{\Phi}$ with $\Phi = \varphi^{\varphi}(1-\varphi)^{1-\varphi}$. Then, defining the relative price $p_t = P_{2t}/P_{1t}$, we have

$$P_{1t} = \Phi p_t^{\varphi-1} \text{ and } P_{2t} = \Phi p_t^{\varphi} \tag{2}$$

In the first intermediate good sector, the representative producer solves

$$\max_{K_{1t},K_{1t}} P_{1t}\Theta_1 K_{1t}^{\alpha}(\Gamma_t L_{1t})^{1-\alpha} - W_{1t}K_{1t} - q_t K_{1t}$$

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which leads to the standard input demand functions, $\alpha P_{1t}Z_{1t} = q_t K_{1t}$ and $(1-\alpha)P_{1t}Z_{1t} = W_{1t}L_{1t}$. Likewise, in sector 2, we have $\beta P_{2t}Z_{2t} = q_t K_{2t}$ and $(1-\beta)P_{2t}Z_{2t} = W_{2t}L_{2t}$. Finally, surplus maximization by the trade unions subject to labor demands leads to the wage setting rule $W_{2t} = \frac{\theta}{\theta-\beta}W_{1t} = (1+x)W_{1t}$.

In equilibrium, we have $K_{1t} + K_{2t} = K_t$ and $L_{1t} + L_{2t} = L_t$ such that solving the system composed of demand functions for intermediate good, capital and labor in each sector and making use of the wage setting rule, we easily get

$$K_{1t} = \frac{\alpha\varphi}{\alpha\varphi + \beta(1-\varphi)}K_t \qquad K_{2t} = \frac{\beta(1-\varphi)}{\alpha\varphi + \beta(1-\varphi)}K_t$$
$$L_{1t} = \frac{\varphi(1-\alpha)}{\varphi(1-\alpha) + (1-\varphi)\frac{1-\beta}{1+x}}L_t \qquad L_{2t} = \frac{(1-\varphi)\frac{1-\beta}{1+x}}{\varphi(1-\alpha) + (1-\varphi)\frac{1-\beta}{1+x}}L_t$$

Therefore, we can compute final output as

$$Y_t = \Upsilon(\Gamma_t)K_t^{\alpha\varphi+\beta(1-\varphi)}L_t^{1-\alpha\varphi-\beta(1-\varphi)} \tag{3}$$

where

$$\Upsilon(\Gamma_t) = \Theta_1^{\varphi}\Theta_2^{1-\varphi}\frac{(\alpha\varphi)^{\alpha\varphi}(\beta(1-\varphi))^{\beta(1-\varphi)}}{[\alpha\varphi+\beta(1-\varphi)]^{\alpha\varphi+\beta(1-\varphi)}}\frac{((1-\alpha)\varphi)^{(1-\alpha)\varphi}\left((1-\varphi)\frac{1-\beta}{1+x}\right)^{(1-\beta)(1-\varphi)}}{\left[(1-\alpha)\varphi+(1-\varphi)\frac{1-\beta}{1+x}\right]^{\varphi(1-\alpha)+(1-\varphi)(1-\beta)}}\Gamma_t^{1-\alpha\varphi-\beta(1-\varphi)}$$

such that output–per–worker $y_t = Y_t/L_t$ expresses, in terms of capital–per–worker $k_t = K_t/L_t$, as

$$y_t = \Upsilon(\Gamma_t)k_t^{\alpha\varphi+\beta(1-\varphi)} \tag{4}$$

Hence the dynamics of the economy — in intensive form — may be summarized by

$$(1+\gamma)(1+n)\frac{k_{t+1}}{\Gamma_{t+1}} = s\Upsilon\left(\frac{k_t}{\Gamma_t}\right)^{\alpha\varphi+\beta(1-\varphi)} + (1-\delta)\left(\frac{k_t}{\Gamma_t}\right)$$

where $\Upsilon = \Upsilon(\Gamma_t)/\Gamma_t^{1-\alpha\varphi-\beta(1-\varphi)}$ which admits

$$k^\star = \left(\frac{s\Upsilon}{(1+\gamma)(1+n)+\delta-1}\right)^{\frac{1}{1-\alpha\varphi-\beta(1-\varphi)}} = (\nu\Upsilon)^{\frac{1}{1-\alpha\varphi-\beta(1-\varphi)}}$$

as steady state.

Q.E.D □

Proposition 2 *In the autarky equilibrium, the private and social returns to capital are equalized and are independent of the size the labor market distorsion, x.*

Proof of proposition 2: In the autarkic economy, private (r_t^{A}) and social (z_t^{A}) returns to capital are the same. Indeed, the rental rate of capital, in terms of good 1, is given by

$$q_t = \alpha P_{1t} \frac{Z_{1t}}{K_{1t}} = \beta P_{2t} \frac{Z_{2t}}{K_{2t}}$$

In equilibrium, we have $P_{1t}Z_{1t} = \varphi Y_t$ and $P_{2t}Z_{2t} = (1-\varphi)Y_t$, therefore

$$q_t = \alpha\varphi \frac{Y_t}{K_{1t}} = \beta(1-\varphi)\frac{Y_t}{K_{2t}} = (\alpha\varphi + \beta(1-\varphi))\frac{Y_t}{K_t}$$

Hence, the returns to capital, in terms of the final good, are given by

$$r_t^{\text{A}} = z_t^{\text{A}} = q_t = (\alpha\varphi + \beta(1-\varphi))\frac{Y_t}{K_t} = (\alpha\varphi + \beta(1-\varphi))\Upsilon(\Gamma_t){k_t}^{\alpha\varphi+\beta(1-\varphi)-1}$$

Q.E.D □

Proposition 3 *In the absence of international trade, the steady state distribution of (log–) output–per–worker relative to a reference economy* (y_0)*, is given by*

$$\mu^{\widehat{\Upsilon}}(\widehat{y}) = \frac{1-\alpha\varphi-\beta(1-\varphi)}{\alpha\varphi+\beta(1-\varphi)}\mu^{\widehat{\nu}}\left(\frac{1-\alpha\varphi-\beta(1-\varphi)}{\alpha\varphi+\beta(1-\varphi)}\widehat{y}\right)$$

where $\mu^{\nu}(\cdot)$ *denotes the distribution of* $\widehat{\nu} \equiv \log(\nu) - \log(\nu_0)$.

Proof of Proposition 3: In the closed economy, the aggregate production function in economy i along a steady growth path is given by

$$y_i = \Upsilon(\Gamma)^{\frac{1}{1-\alpha\varphi-\beta(1-\varphi)}}\nu_i^{\frac{\alpha\varphi+\beta(1-\varphi)}{1-\alpha\varphi-\beta(1-\varphi)}}$$

where $\nu \equiv s/((1+\gamma)(1+n)-(1-\delta))$. Let us consider the (log–)difference between output per worker in economy i and in the big economy, $\widehat{y} = \log(y_i) - \log(y_0)$, where 0 denotes the big economy. Let us define $\widehat{\nu}_i = \log(\nu_i) - \log(\nu_0)$, we then have

$$\widehat{y}_i = g(\widehat{\nu}_i) = \frac{\alpha\varphi+\beta(1-\varphi)}{1-\alpha\varphi-\beta(1-\varphi)}\widehat{\nu}_i$$

Making use of the change of variable formula, and denoting by $\mu^{\widehat{\nu}}(\cdot)$ the distribution of $\widehat{\nu}$, we have

$$\mu^{\widehat{\Upsilon}}(\widehat{y}) = \frac{1-\alpha\varphi-\beta(1-\varphi)}{\alpha\varphi+\beta(1-\varphi)}\mu^{\nu}\left(\frac{1-\alpha\varphi-\beta(1-\varphi)}{\alpha\varphi+\beta(1-\varphi)}\widehat{y}\right)$$

Q.E.D □

Lemma 1 *For given international prices, there exists two levels of capital–per–worker denoted by $\underline{k}(p_t, \Gamma_t)$ and $\overline{k}(p_t, \Gamma_t)$, such that when endowed with a capital–per–worker below $\underline{k}(p_t, \Gamma_t)$, a small open economy specializes in the production of good 1, while it specializes in the production of good 2 when its capital–per–worker lies above $\overline{k}(p_t, \Gamma_t)$, with*

$$\begin{aligned}\underline{k}(p_t, \Gamma_t) &= \Gamma_t \left(p_t \frac{\Theta_2}{\Theta_1}\right)^{\frac{1}{\alpha-\beta}} \left(\frac{\beta}{\alpha}\right)^{\frac{\beta}{\alpha-\beta}} \left(\frac{1-\beta}{(1-\alpha)(1+x)}\right)^{\frac{1-\beta}{\alpha-\beta}}\\ \overline{k}(p_t, \Gamma_t) &= \Gamma_t \left(p_t \frac{\Theta_2}{\Theta_1}\right)^{\frac{1}{\alpha-\beta}} \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha-\beta}} \left(\frac{1-\beta}{(1-\alpha)(1+x)}\right)^{\frac{1-\alpha}{\alpha-\beta}}\end{aligned}$$

Proof of lemma 1: In the small open economy, each firm producing the final good takes the price of goods as given, such that final output is given by

$$Y_t = P_{1t}Z_{1t} + P_{2t}Z_{2t}$$

The intermediate goods producers problem may be rewritten as

$$\max_{K_{1t}, K_{2t}, L_{1t}, L_{2t}, Z_{1t}, Z_{2t}} P_{1t}Z_{1t} + P_{2t}Z_{2t} - q_tK_t - W_{1t}L_{1t} - W_{2t}L_{2t}$$

subject to

$$\begin{cases} Z_{1t} \leqslant \Theta_1 K_{1t}^{\alpha} (\Gamma_t L_{1t})^{1-\alpha} \\ Z_{2t} \leqslant \Theta_2 K_{2t}^{\beta} (\Gamma_t L_{2t})^{1-\beta} \\ K_{1t} + K_{2t} \leqslant K_t \\ L_{1t} + L_{2t} \leqslant L_t \\ L_{1t} \geqslant 0,\ L_{2t} \geqslant 0,\ K_{1t} \geqslant 0,\ K_{2t} \geqslant 0 \\ W_{2t} = (1+x)W_{1t} \end{cases}$$

Since technology is strictly increasing in inputs, the first four constraints ought to bind, such that the problem simplifies to

$$\max_{K_{1t}, K_t, L_{1t}, L_t} P_{1t}\Theta_1 K_{1t}^{\alpha}(\Gamma_t L_{1t})^{1-\alpha} + P_{2t}\Theta_2 (K_t - K_{1t})^{\beta} (\Gamma_t (L_t - L_{1t}))^{1-\beta} - q_t K_t - W_{1t}L_{1t} - (1+x)W_{1t}(L_t - L_{1t})$$

subject to $L_{1t} \geqslant 0,\ L_t \geqslant L_{1t},\ K_{1t} \geqslant 0,\ K_t \geqslant K_{1t}$ to which we associate the lagrange multipliers $\lambda_L^0, \lambda_L^1, \lambda_K^0, \lambda_K^1$. This leads to the following set of optimality conditions

$$\beta P_{2t}\Theta_2 \left(\frac{K_t - K_{1t}}{L_t - L_{1t}}\right)^{\beta-1} \Gamma_t^{1-\beta} = q_t \tag{5}$$

$$(1-\beta)P_{2t}\Theta_2 \left(\frac{K_t - K_{1t}}{L_t - L_{1t}}\right)^{\beta} \Gamma_t^{1-\beta} = (1+x)W_{1t} \tag{6}$$

$$\alpha P_{1t}\Theta_1 \left(\frac{K_{1t}}{L_{1t}}\right)^{\alpha-1} \Gamma_t^{1-\alpha} - \beta P_{2t}\Theta_2 \left(\frac{K_t - K_{1t}}{L_t - L_{1t}}\right)^{\beta-1} \Gamma_t^{1-\beta} = \lambda_{Kt}^1 - \lambda_{Kt}^0 \tag{7}$$

$$(1-\alpha)P_{1t}\Theta_1 \left(\frac{K_{1t}}{L_{1t}}\right)^{\alpha} \Gamma_t^{1-\alpha} - (1-\beta)P_{2t}\Theta_2 \left(\frac{K_t - K_{1t}}{L_t - L_{1t}}\right)^{\beta} \Gamma_t^{1-\beta} - xW_{1t} = \lambda_{Lt}^1 - \lambda_{Lt}^0 \tag{8}$$

An interior solution, for which $K_{1t}, K_{2t}, L_{1t}, L_{2t} > 0$ — which corresponds to a specialization phase — implies that K_{1t}, K_{2t}, L_{1t} and L_{2t} satisfy (using (6)–(8))

$$\frac{K_{2t}}{L_{2t}} = \frac{K_t - K_{1t}}{L_t - L_{1t}} = \frac{\beta(1-\alpha)(1+x)}{\alpha(1-\beta)} \frac{K_{1t}}{L_{1t}} \tag{9}$$

Let us first study the conditions under which an economy chooses to specialize in the production of type 1 intermediate good. In this case, $K_{1t} = K_t$ and $L_{1t} = L_t$, which implies that $\lambda^0_{Kt} = \lambda^0_{Lt} = 0$ and $\lambda^1_{Kt} \geqslant 0$ and $\lambda^1_{Lt} \geqslant 0$. Therefore, equations (6)–(8), evaluated along (9), satisfy

$$\alpha P_{1t}\Theta_1 \left(\frac{K_{1t}}{L_{1t}}\right)^{\alpha-1} \Gamma_t^{1-\alpha} - \beta P_{2t}\Theta_2 \left(\frac{\beta(1-\alpha)(1+x)}{\alpha(1-\beta)}\frac{K_{1t}}{L_{1t}}\right)^{\beta-1} \Gamma_t^{1-\beta} \geqslant 0$$
$$(1-\alpha)P_{1t}\Theta_1 \left(\frac{K_{1t}}{L_{1t}}\right)^{\alpha} \Gamma_t^{1-\alpha} - \frac{1-\beta}{1+x}P_{2t}\Theta_2 \left(\frac{\beta(1+x)(1-\alpha)}{\alpha(1-\beta)}\frac{K_{1t}}{L_{1t}}\right)^{\beta-1} \Gamma_t^{1-\beta} \geqslant 0$$

which triggers that

$$\frac{K_{1t}}{L_{1t}} = \frac{K_t}{L_t} \leqslant \underline{k}(p_t, \Gamma_t) \equiv \Gamma_t \left(p_t\frac{\Theta_2}{\Theta_1}\right)^{\frac{1}{\alpha-\beta}} \left(\frac{\beta}{\alpha}\right)^{\frac{\beta}{\alpha-\beta}} \left(\frac{1-\beta}{(1-\alpha)(1+x)}\right)^{\frac{1-\beta}{\alpha-\beta}}$$

where $p_t = P_{2t}/P_{1t}$.

Let us now study the conditions under which an economy chooses to specialize in the production of type 2 intermediate good. In this case, $K_{2t} = K_t$ and $L_{2t} = L_t$, which implies that $\lambda^1_{Kt} = \lambda^1_{Lt} = 0$ and $\lambda^0_{Kt} \geqslant 0$ and $\lambda^0_{Lt} \geqslant 0$. Therefore, equations (6)–(8), evaluated along (9), satisfy

$$\alpha P_{1t}\Theta_1 \left(\frac{\alpha(1-\beta)}{\beta(1-\alpha)(1+x)}\frac{K_t-K_{1t}}{L_t-L_{1t}}\right)^{\alpha-1} \Gamma_t^{1-\alpha} - \beta P_{2t}\Theta_2 \left(\frac{K_t-K_{1t}}{L_t-L_{1t}}\right)^{\beta-1} \Gamma_t^{1-\beta} \leqslant 0$$
$$(1-\alpha)P_{1t}\Theta_1 \left(\frac{\alpha(1-\beta)}{\beta(1-\alpha)(1+x)}\frac{K_t-K_{1t}}{L_t-L_{1t}}\right)^{\alpha} \Gamma_t^{1-\alpha} - \frac{1-\beta}{1+x}P_{2t}\Theta_2 \left(\frac{K_t-K_{1t}}{L_t-L_{1t}}\right)^{\beta-1} \Gamma_t^{1-\beta} \leqslant 0$$

which triggers that

$$\frac{K_t-K_{1t}}{L_t-L_{1t}} = \frac{K_t}{L_t} \geqslant \overline{k}(p_t, \Gamma_t) \equiv \Gamma_t \left(p_t\frac{\Theta_2}{\Theta_1}\right)^{\frac{1}{\alpha-\beta}} \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha-\beta}} \left(\frac{1-\beta}{(1-\alpha)(1+x)}\right)^{\frac{1-\alpha}{\alpha-\beta}}$$

Q.E.D □

Proposition 4 *Under free trade a country's level of output–per–worker is given by*

$$y_{it} = \begin{cases} \Phi p_t^{\varphi-1} A_{1t} k_{it}^{\alpha} \Gamma_t^{1-\alpha} & \text{if } k_{it} \leqslant \underline{k}(p_t, \Gamma_t) \\ A(p_t) k_{it} + B(p_t)\Gamma_t & \text{if } \underline{k}(p_t,\Gamma_t) \leqslant k_{it} \leqslant \overline{k}(p_t,\Gamma_t) \\ \Phi p_t^{\varphi} A_{2t} k_{it}^{\beta} \Gamma_t^{1-\beta} & \text{if } k_{it} \geqslant \overline{k}(p_t,\Gamma_t) \end{cases}$$

where $\Phi = \varphi^{\varphi}(1-\varphi)^{1-\varphi}$ *and*

$$A(p_t) = \Phi p_t^{\varphi-1} \frac{\alpha\left(1-\alpha-\frac{1-\beta}{1+x}\right)}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}} (p_t\Theta_2)^{\frac{1-\alpha}{\beta-\alpha}} \Theta_1^{\frac{1-\beta}{\alpha-\beta}} \left(\frac{\beta}{\alpha}\right)^{\frac{\beta(1-\alpha)}{\beta-\alpha}} \left(\frac{1-\beta}{(1+x)(1-\alpha)}\right)^{\frac{(1-\beta)(1-\alpha)}{\beta-\alpha}}$$

$$B(p_t) = \Phi p_t^{\varphi-1} \frac{(\beta-\alpha)(1-\alpha)}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}} (p_t\Theta_2)^{\frac{\alpha}{\alpha-\beta}} \Theta_1^{\frac{\beta}{\beta-\alpha}} \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha\beta}{\alpha-\beta}} \left(\frac{1-\beta}{(1+x)(1-\alpha)}\right)^{\frac{\alpha(1-\beta)}{\alpha-\beta}}$$

Proof of proposition 4: We have to study three cases, depending on the level of the capital per efficient unit of labor.

- $k_t \leqslant \underline{k}(p_t, \Gamma_t)$**:** In this case, the economy fully specializes in the production of type 1 intermediate good, we therefore have $y_t = P_{1t}z_{1t} = P_{1t}\Theta_1 k_t^{\alpha}\Gamma_t^{1-\alpha}$, where $y_t = Y_t/L_t$ and $z_{1t} = Z_{1t}/L_t$. Since P_{1t} is given by (2), we finally have $y_t = \Phi p_t^{\varphi-1}\Theta_1 k_t^{\alpha}\Gamma_t^{1-\alpha}$.

- $k_t \geqslant \overline{k}(p_t, \Gamma_t)$**:** In this case, the economy fully specializes in the production of type 2 intermediate good, we therefore have $y_t = P_{2t}z_{2t} = P_{2t}\Theta_2 k_t^{\beta}\Gamma_t^{1-\beta}$, where $z_{2t} = Z_{2t}/L_t$. Since P_{2t} is given by (2), we finally have $y_t = \Phi p_t^{\varphi}\Theta_2 k_t^{\beta}\Gamma_t^{1-\beta}$.

- $\underline{k}(p_t, \Gamma_t) \leqslant k \leqslant \overline{k}(p_t, \Gamma_t)$**:** In this case, the economy lies in the specialization process, and we have

  $$y_t = P_{1t}z_{1t} + P_{2t}z_{2t}$$

  We therefore have to solve the allocation of capital and labor problem. This implies solving the set of equations

  $$\alpha P_{1t}\Theta_1 \left(\frac{K_{1t}}{L_{1t}}\right)^{\alpha-1} \Gamma_t^{1-\alpha} = \beta P_{2t}\Theta_2 \left(\frac{K_t - K_{1t}}{L_t - L_{1t}}\right)^{\beta-1} \Gamma_t^{1-\beta} \tag{10}$$

  $$(1-\alpha) P_{1t}\Theta_1 \left(\frac{K_{1t}}{L_{1t}}\right)^{\alpha} \Gamma_t^{1-\alpha} = \frac{1-\beta}{1+x} P_{2t}\Theta_2 \left(\frac{K_t - K_{1t}}{L_t - L_{1t}}\right)^{\beta} \Gamma_t^{1-\beta} \tag{11}$$

  which implies that

  $$\frac{K_t - K_{1t}}{L_t - L_{1t}} = \frac{\beta(1-\alpha)(1+x)}{\alpha(1-\beta)} \frac{K_{1t}}{L_{1t}} \tag{12}$$

  $$\frac{K_{1t}}{L_{1t}} = \Gamma_t \left(p_t \frac{\Theta_2}{\Theta_1}\right)^{\frac{1}{\alpha-\beta}} \left(\frac{\beta}{\alpha}\right)^{\frac{\beta}{\alpha-\beta}} \left(\frac{1-\beta}{(1-\alpha)(1+x)}\right)^{\frac{1-\beta}{\alpha-\beta}} = \underline{k}(p_t, \Gamma_t) \tag{13}$$

  Let us then denote $\sigma_{Lt} = L_{1t}/L_t$ and $\sigma_{Kt} = K_{1t}/K_t$. Solving (12) and (13), we get

  $$\sigma_{Lt} = \frac{\beta(1-\alpha)}{\beta(1-\alpha) - \alpha\frac{1-\beta}{1+x}} - \frac{\alpha\frac{1-\beta}{1+x}}{\beta(1-\alpha) - \alpha\frac{1-\beta}{1+x}} \frac{k_t}{\underline{k}(p_t, \Gamma_t)} \tag{14}$$

  $$\sigma_{Kt} = \sigma_{Lt} \frac{\underline{k}(p_t, \Gamma_t)}{k_t} \tag{15}$$

We therefore easily get

$$\begin{aligned} P_{1t}z_{1t} &= P_{1t}\Theta_1\underline{k}(p_t)^{\alpha}\Gamma_t^{1-\alpha}\sigma_{Lt} \\ &= \frac{\beta(1-\alpha)}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}P_{1t}\Theta_1\underline{k}(p_t,\Gamma_t)^{\alpha}\Gamma_t^{1-\alpha} - \frac{\alpha\frac{1-\beta}{1+x}}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}P_{1t}\Theta_1\Gamma_t^{1-\alpha}\underline{k}(p_t,\Gamma_t)^{\alpha-1}k_t \end{aligned}$$

Likewise, straightforward calculation gives

$$1-\sigma_{Lt} = \frac{\alpha\frac{1-\beta}{1+x}}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}\left(\frac{k_t}{\underline{k}(p_t,\Gamma_t)}-1\right) \quad (16)$$

$$1-\sigma_{Kt} = \frac{\beta(1-\alpha)(1+x)}{\alpha(1-\beta)}\frac{\underline{k}(p_t,\Gamma_t)}{k_t}(1-\sigma_{Lt}) \quad (17)$$

We therefore easily get

$$\begin{aligned} P_{2t}z_{2t} &= P_{2t}\Theta_2\left(\frac{\beta(1-\alpha)(1+x)}{\alpha(1-\beta)}\underline{k}(p_t,\Gamma_t)\right)^{\beta}\Gamma_t^{1-\beta}(1-\sigma_{Lt}) \\ &= P_{2t}\Theta_2\left(\frac{\beta(1-\alpha)(1+x)}{\alpha(1-\beta)}\underline{k}(p_t,\Gamma_t)\right)^{\beta}\Gamma_t^{1-\beta}\frac{\alpha\frac{1-\beta}{1+x}}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}\left(\frac{k_t}{\underline{k}(p_t,\Gamma_t)}-1\right) \end{aligned}$$

Then, after simple although tedious algebra and making use of (2), we get

$$y_t = B(p_t)\Gamma_t + A(p_t)k_t \quad (18)$$

where

$$\begin{aligned} B(p_t) &= \Phi p_t^{\varphi-1}\frac{(\beta-\alpha)(1-\alpha)}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}(p_t\Theta_2)^{\frac{\alpha}{\alpha-\beta}}\Theta_1^{\frac{\beta}{\beta-\alpha}}\left(\frac{\beta}{\alpha}\right)^{\frac{\alpha\beta}{\alpha-\beta}}\left(\frac{1-\beta}{(1-\alpha)(1+x)}\right)^{\frac{\alpha(1-\beta)}{\alpha-\beta}} \\ A(p_t) &= \Phi p_t^{\varphi-1}\frac{\alpha\left(1-\alpha-\frac{1-\beta}{1+x}\right)}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}(p_t\Theta_2)^{\frac{1-\alpha}{\beta-\alpha}}\Theta_1^{\frac{1-\beta}{\alpha-\beta}}\left(\frac{\beta}{\alpha}\right)^{\frac{\beta(1-\alpha)}{\beta-\alpha}}\left(\frac{1-\beta}{(1-\alpha)(1+x)}\right)^{\frac{(1-\beta)(1-\alpha)}{\beta-\alpha}} \end{aligned}$$

Q.E.D □

Proposition 5 *If $x > 0$, then under free trade the social returns to capital are higher than the private return for all countries that do not fully specialize. Moreover, the difference between the private and social return to capital is increasing in x.*

Proof of proposition 5: Under free trade, the private returns to capital, (r_t^{FT}), are given by (see proof of lemma 1, equation (5))

$$r_t^{\text{FT}} = q_t = \beta P_{2t}\Theta_2 \left(\frac{K_t - K_{1t}}{L_t - L_{1t}}\right)^{\beta-1} \Gamma_t^{1-\beta}$$

which rewrites

$$r_t^{\text{FT}} = \beta P_{2t}\Theta_2 k_t^{\beta-1}\left(\frac{1-\sigma_{Kt}}{1-\sigma_{Lt}}\right)^{\beta-1}\Gamma_t^{1-\beta} = \beta P_{2t}\Theta_2\left(\frac{\beta(1-\alpha)(1+x)}{\alpha(1-\beta)}\underline{k}(p_t,\Gamma_t)\right)^{\beta-1}\Gamma_t^{1-\beta}$$

Plugging the definition of $\underline{k}(p_t,\Gamma_t)$ and that of P_{2t} in the latter equation, we get

$$r_t^{\text{FT}} = \beta\Phi p_t^{\varphi-1}(p_t\Theta_2)^{\frac{1-\alpha}{\beta-\alpha}}\Theta_1^{\frac{1-\beta}{\alpha-\beta}}\left(\frac{\beta}{\alpha}\right)^{-\frac{\alpha(1-\beta)}{\alpha-\beta}}\left(\frac{1-\beta}{(1-\alpha)(1+x)}\right)^{-\frac{(1-\alpha)(1-\beta)}{\alpha-\beta}}$$

Further from the optimal allocation of Z_1 and Z_2 in the big economy (autarkic world), we have

$$p_t = \frac{1-\varphi}{\varphi}\frac{Z_{1t}}{Z_{2t}}$$

Using the value of z_{1t} and z_{2t}, the relative price, p_t, evaluated at the steady growth path of the big economy (indexed by 0) is given by

$$p^\star = \frac{\Theta_1}{\Theta_2}\frac{\alpha^\alpha(\theta(1-\alpha))^{1-\alpha}}{\beta^\beta\left(\frac{1-\beta}{1+x}\right)^{1-\beta}}\left(\frac{\alpha\varphi+\beta(1-\varphi)}{(1-\alpha)\varphi+(1-\varphi)\frac{1-\beta}{1+x}}\right)^{\beta-\alpha}\left(\frac{k_{0t}}{\Gamma_t}\right)^{\alpha-\beta} \tag{19}$$

Plugging this expression in the definition of $\underline{k}(p,\Gamma)$, we can express the private return to capital (at the steady state of the big economy) as

$$\begin{aligned} r_t^{\text{FT}} &= \Theta_1^\varphi\Theta_2^{1-\varphi}\frac{(\alpha\varphi)^{\alpha\varphi}(\varphi(1-\alpha))^{\varphi(1-\alpha)}(\beta(1-\varphi))^{\beta(1-\varphi)}\left((1-\varphi)\frac{1-\beta}{1+x}\right)^{(1-\beta)(1-\varphi)}}{(\alpha\varphi+\beta(1-\varphi))^{\alpha\varphi+\beta(1-\varphi)}\left(\varphi(1-\alpha)+(1-\varphi)\frac{1-\beta}{1+x}\right)^{1-\alpha\varphi-\beta(1-\varphi)}}\cdots \\ &\quad \times(\alpha\varphi+\beta(1-\varphi)){k_{0t}}^{\alpha\varphi+\beta(1-\varphi)-1}\Gamma_t^{1-\alpha\varphi-\beta(1-\varphi)} \end{aligned}$$

or

$$r_t^{\text{FT}} = (\alpha\varphi+\beta(1-\varphi))\Upsilon(\Gamma_t){k_{0t}}^{\alpha\varphi+\beta(1-\varphi)-1} = r_t^{\text{A}}$$

We now consider the social return to capital, which is now obtained by deriving the aggregate production function when the economy produces both goods. Hence, we have $z_t^{\text{FT}} = A(p_t)$. Using the definition of A (see proposition 4) and the expression for $p^\star$, the social return to capital in the steady state of the big economy is given by

$$z_t^{\text{FT}} = \frac{1-\alpha-\frac{1-\beta}{1+x}}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}(\alpha\varphi+\beta(1-\varphi))B(\Gamma_t){k_t^\star}^{\alpha\varphi+\beta(1-\varphi)-1} = \frac{1-\alpha-\frac{1-\beta}{1+x}}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}z^{\text{A}}$$

It is then straightforward to verify that as long as $\alpha,\beta\in(0,1)$ and $x>0$ the multiplier term is greater than 1, such that $z^{\text{FT}} \geqslant z^{\text{A}}$.

Q.E.D □

Proposition 6 *Under free trade, regardless of the value of x, all economies possess a unique non–trivial steady state.*

Proof of Proposition 6: Given the form of the production function, the model admits 1, 3 or an infinity of non–trivial steady state (the trivial steady state being 0).

- Let us first consider the case where we have an infinity of steady state. This occurs when the ray k/ν overlies the linear part of the production function, which can only happen if and only if that the linear part reduced to $A(p)k$. This situation would then corresponds to $B(p) = 0$, which is impossible if $\beta > \alpha$.
- A second possibility of multiple equilibria is to have 3 steady states. But the form of the function triggers that a first steady state lies in the area where the economy is fully specialized in the production of type 1 intermediate goods, one in the zone of full specialization in type 2 intermediate goods, and one in the specialization area. This can only be the case if the linear part of the production function crosses the ray k/ν from below, which implies a negative value for $B(p)$ which is impossible.

Therefore, there exists a unique equilibrium.

Q.E.D $\square$

Lemma 2 *The relationship between the steady state level of relative output-per-worker in economy i, $\widehat{y}_i$, and the relative propensity for capital deepening, $\widehat{\nu}_i$ is given by*

$$\widehat{y}_i = \begin{cases} \log\left(\frac{(1-\alpha)\varphi+(1-\varphi)\frac{1-\beta}{1+x}}{1-\alpha}\right) + \frac{\alpha}{1-\alpha}(\widehat{\nu}_i - \underline{\widehat{\nu}}) & \text{if } \widehat{\nu}_i < \underline{\widehat{\nu}} \\ \log(\rho) - \log\left(1-(1-\rho)\exp(\widehat{\nu}_i)\right) & \text{if } \underline{\widehat{\nu}} \leqslant \widehat{\nu}_i \leqslant \overline{\widehat{\nu}} \\ \log\left(\frac{(1-\alpha)\varphi+(1-\varphi)\frac{1-\beta}{1+x}}{\frac{1-\beta}{1+x}}\right) + \frac{\beta}{1-\beta}(\widehat{\nu}_i - \overline{\widehat{\nu}}) & \text{if } \widehat{\nu}_i > \overline{\widehat{\nu}} \end{cases}$$

where $\underline{\widehat{\nu}} = \log\left(\frac{\alpha}{\alpha\varphi+\beta(1-\varphi)}\right)$ *and* $\overline{\widehat{\nu}} = \log\left(\frac{\beta}{\alpha\varphi+\beta(1-\varphi)}\right)$ *and* $\rho = \frac{(\beta-\alpha)\left(\varphi(1-\alpha)+(1-\varphi)\frac{1-\beta}{1+x}\right)}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}$

Proof of lemma 2: We first start by characterizing the two boundary values $\underline{\widehat{\nu}}$ and $\overline{\widehat{\nu}}$.

- Let us compute $\underline{\nu}(p_t,\Gamma_t) = \underline{k}(p_t,\Gamma_t)/\underline{y}(p_t,\Gamma_t)$. At this particular value, we have $y_{it} = P_{1t}z_{1it}$, such that

  $$\underline{\nu}(p_t,\Gamma_t) = \frac{p_t^{1-\varphi}}{\Phi}\frac{\underline{k}(p_t,\Gamma_t)^{1-\alpha}}{\Theta_1\Gamma_t^{1-\alpha}}$$

  Plugging into the previous equation the definition of $\underline{k}(p_t,\Gamma_t)$ and the definition of p_t along the steady growth path of the big economy (Equation (19)), we have

  $$\underline{\nu}(p_t,\Gamma_t) = \frac{\alpha}{\alpha\varphi+\beta(1-\varphi)}\frac{k_{0t}}{\Upsilon(\Gamma_t){k_{0t}}^{\alpha\varphi+\beta(1-\varphi)}} = \frac{\alpha}{\alpha\varphi+\beta(1-\varphi)}\frac{k_{0t}}{y_{0t}} = \frac{\alpha}{\alpha\varphi+\beta(1-\varphi)}\nu_0$$

  Hence, we have $\underline{\widehat{\nu}} \equiv \log(\underline{\nu}(p_t,\gamma_t)) - \log(\nu_0) = \log\left(\frac{\alpha}{\alpha\varphi+\beta(1-\varphi)}\right)$

- Let us now compute $\overline{\nu}(p_t,\Gamma_t) = \overline{k}(p_t,\Gamma_t)/\overline{y}(p_t,\Gamma_t)$. At this particular value, we have $y_{it} = P_{2t}z_{2it}$, such that

  $$\underline{\nu}(p_t,\Gamma_t) = \frac{p_t^{-\varphi}}{\Phi}\frac{\underline{k}(p_t,\Gamma_t)^{1-\beta}}{\Theta_2\Gamma_t^{1-\beta}}$$

  Plugging into the previous equation the definition of $\overline{k}(p_t,\Gamma_t)$ and the definition of p at the steady state of the big economy, we have

  $$\overline{\nu}(p_t,\Gamma_t) = \frac{\beta}{\alpha\varphi+\beta(1-\varphi)}\frac{k_{0,t}}{\Upsilon(\Gamma_t){k_{0,t}}^{\alpha\varphi+\beta(1-\varphi)}} = \frac{\beta}{\alpha\varphi+\beta(1-\varphi)}\frac{k_{0,t}}{y_{0,t}} = \frac{\beta}{\alpha\varphi+\beta(1-\varphi)}\nu_0$$

  Hence, we have $\overline{\widehat{\nu}} \equiv \log(\overline{\nu}(p_t,\Gamma_t)) - \log(\nu_0) = \log\left(\frac{\beta}{\alpha\varphi+\beta(1-\varphi)}\right)$

We then have to consider 3 cases:

$\nu_i < \underline{\nu}$: In this case $y = P_{1t}\Theta_1 k^\alpha \Gamma_t^{1-\alpha}$. Evaluating this along a steady growth path, we have $y_{it} = (\Phi p_t^{\varphi-1}\Theta_1)^{\frac{1}{1-\alpha}}\nu_i^{\frac{\alpha}{1-\alpha}}\Gamma_t$. Plugging the expression of the relative price along the steady growth path of the big economy (Equation (19)) in the output–per–worker in economy i, and remembering *(i)* the definition of $\Upsilon(\Gamma_t)$, *(ii)* the definition of output–per–worker in the big economy, we have

$$y_{it} = \left(\frac{\alpha\varphi+\beta(1-\varphi)}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\frac{(1-\alpha)+(1-\varphi)\frac{1-\beta}{1+x}}{1-\alpha}\left(\frac{\nu_i}{\nu_0}\right)^{\frac{\alpha}{1-\alpha}}y_{0t}$$

taking log on both sides, we have

$$\widehat{y}_{it} = \frac{\alpha}{1-\alpha}\log\left(\frac{\alpha\varphi+\beta(1-\varphi)}{\alpha}\right) + \log\left(\frac{(1-\alpha)\varphi\theta+(1-\beta)(1-\varphi)(\theta-\beta)}{\theta(1-\alpha)}\right) + \frac{\alpha}{1-\alpha}\widehat{\nu}_i$$

$\underline{\nu} \leqslant \nu_i \leqslant \overline{\nu}$: In this case $y_{it} = A(p_t)k_{it} + B(p_t)\Gamma_t$. Evaluating this along a steady growth path, where $k_{it} = \nu_i y_{it}$, we have $y_t = \frac{B(p_t)\Gamma_t}{1-A(p_t)\nu_i}$. Around the steady growth path of the big economy, we have

$$A(p_t) = \frac{(\alpha\varphi + \beta(1-\varphi))\left(1 - \alpha - \frac{1-\beta}{1+x}\right)}{\beta(1-\alpha) - \alpha\frac{1-\beta}{1+x}}\frac{y_{0t}}{k_{0t}} \text{ and } B(p_t)\Gamma_t = \frac{(\beta-\alpha)\left(\varphi(1-\alpha) + (1-\varphi)\frac{1-\beta}{1+x}\right)}{\beta(1-\alpha) - \alpha\frac{1-\beta}{1+x}}y_{0t}$$

Remembering that $\nu_0 = \frac{k_{0t}}{y_{0t}}$, we then get $y_{it} = \frac{\rho y_{0t}}{1-(1-\rho)\frac{\nu}{\nu_0}}$ where $\rho = \frac{(\beta-\alpha)\left(\varphi(1-\alpha)+(1-\varphi)\frac{1-\beta}{1+x}\right)}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}$ such that $\widehat{y} = \log(\rho) - \log\left(1 - (1-\rho)\exp(\widehat{\nu}_i)\right)$.

$\nu_i > \overline{\nu}$: In this case $y_t = P_{2t}\Theta_2 k_t^{\beta}\Gamma_t^{1-\beta}$. Evaluating this along a steady growth path, we have $y_{it} = (\Phi p_t^{\varphi}\Theta_2)^{\frac{1}{1-\beta}}\nu_i^{\frac{\beta}{1-\beta}}\Gamma_t$. Then, using previous results for $\nu_i < \underline{\nu}$, we then have, around the steady growth path of the big economy,

$$y_{it} = \left(\frac{\alpha\varphi + \beta(1-\varphi)}{\beta}\right)^{\frac{\beta}{1-\beta}} \frac{\varphi(1-\alpha) + (1-\varphi)\frac{1-\beta}{1+x}}{\frac{1-\beta}{1+x}} \left(\frac{\nu_i}{\nu_0}\right)^{\frac{\beta}{1-\beta}} y_{0t}$$

taking log on both sides, we have

$$\widehat{y}_i = \frac{\beta}{1-\beta}\log\left(\frac{\alpha\varphi + \beta(1-\varphi)}{\beta}\right) + \log\left(\frac{\varphi(1-\alpha) + (1-\varphi)\frac{1-\beta}{1+x}}{\frac{1-\beta}{1+x}}\right) + \frac{\beta}{1-\beta}\widehat{\nu}_i$$

Q.E.D □

Proposition 7 *The steady state distribution of output–per–worker, relative to the reference economy, $\widehat{y}$, is given by*

$$\mu^{\widehat{Y}}(\widehat{y})=\begin{cases}\frac{1-\alpha}{\alpha}\mu_{\widehat{\nu}}\left(\frac{1-\alpha}{\alpha}(\widehat{y}-\underline{\widehat{y}})-\log\left(\frac{\alpha}{\alpha\varphi+\beta(1-\varphi)}\right)\right) & \text{if } \widehat{y}<\underline{\widehat{y}}\\[2ex] \dfrac{(1-\rho)\exp(\widehat{y})}{1-(1-\rho)\exp(\widehat{y})}\mu_{\widehat{\nu}}\left(\log\left(\dfrac{1-(1-\rho)\exp(\widehat{y})}{\rho}\right)\right) & \text{if } \underline{\widehat{y}}\leqslant\widehat{y}\leqslant\overline{\widehat{y}}\\[2ex] \frac{1-\beta}{\beta}\mu_{\widehat{\nu}}\left(\frac{1-\beta}{\beta}(\widehat{y}-\overline{\widehat{y}})-\log\left(\frac{\beta}{\alpha\varphi+\beta(1-\varphi)}\right)\right) & \text{if } \widehat{y}>\overline{\widehat{y}}\end{cases}$$

where $\mu_{\widehat{\nu}}(\cdot)$ is the distribution of $\widehat{\nu}=\log(\nu)-\log(\nu_0)$, $\rho\equiv\frac{(\alpha\varphi+\beta(1-\varphi))\left(1-\alpha-\frac{1-\beta}{1+x}\right)}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}$,

$$\underline{\widehat{y}}=\log\left(\frac{\varphi(1-\alpha)+(1-\varphi)\frac{1-\beta}{1+x}}{1-\alpha}\right)\ \textit{and}\ \overline{\widehat{y}}=\log\left(\frac{\varphi(1-\alpha)+(1-\varphi)\frac{1-\beta}{1+x}}{\frac{1-\beta}{1+x}}\right)$$

Proof of proposition 7: We start by computing the threshold values, $\underline{y}(p)$ and $\overline{y}(p)$, for the distribution. These values are simply obtained by plugging the values for $\underline{k}(p)$ and $\overline{k}(p)$ in the relevant production functions. We therefore have

$$\underline{y}(p_t,\Gamma_t)=P_{1t}\Theta_1\underline{k}(p_t,\Gamma_t)^{\alpha}\Gamma_t^{1-\alpha}\text{ and }\overline{y}(p_t,\Gamma_t)=P_{2t}\Theta_2\overline{k}(p_t,\Gamma_t)^{\beta}\Gamma_t^{1-\beta}$$

using the definition of $\underline{k}(p_t,\Gamma_t)$, $\overline{k}(p_t,\Gamma_t)$, P_{1t} and P_{2t}, we get

$$\underline{y}(p_t,\Gamma_t)=\Phi\Gamma_t p_t^{\varphi-1+\frac{\alpha}{\alpha-\beta}}\Theta_1^{\frac{\beta}{\beta-\alpha}}\Theta_2^{\frac{\alpha}{\alpha-\beta}}\left(\frac{\beta}{\alpha}\right)^{\frac{\alpha\beta}{\alpha-\beta}}\left(\frac{1-\beta}{(1-\alpha)(1+x)}\right)^{\frac{\alpha(1-\beta)}{\alpha-\beta}}$$

and

$$\overline{y}(p_t,\Gamma_t)=\Phi\Gamma_t p_t^{\varphi-1+\frac{\beta}{\alpha-\beta}}\Theta_1^{\frac{\beta}{\beta-\alpha}}\Theta_2^{\frac{\alpha}{\alpha-\beta}}\left(\frac{\beta}{\alpha}\right)^{\frac{\alpha\beta}{\alpha-\beta}}\left(\frac{1-\beta}{(1-\alpha)(1+x)}\right)^{\frac{\beta(1-\alpha)}{\alpha-\beta}}$$

We now determine the shape of the distribution along a steady growth path, that is when $k_{it}=\nu_i y_{it}$. In this case, the relative price is given by expression (19). Hence,denoting by y_{0t} the output level along a steady growth path in the big economy, we can reexpress the thresholds as

$$\underline{y}(p_t,\Gamma_t)=\frac{\varphi(1-\alpha)+(1-\varphi)\frac{1-\beta}{1+x}}{1-\alpha}y_{0,t}\text{ and }\overline{y}(p_t,\Gamma_t)=\frac{\varphi(1-\alpha)+(1-\varphi)\frac{1-\beta}{1+x}}{\frac{1-\beta}{1+x}}y_{0,t}$$

We now study the distribution of $\widehat{y}=\log(y_t)-\log(y_{0,t})$. The direct application of the change of variable formula on the relationship reported in lemma 2 yields

$$\mu^{\widehat{Y}}(\widehat{y})=\begin{cases}\frac{1-\alpha}{\alpha}\mu_{\widehat{\nu}}\left(\frac{1-\alpha}{\alpha}(\widehat{y}-\underline{\widehat{y}})-\log\left(\frac{\alpha}{\alpha\varphi+\beta(1-\varphi)}\right)\right) & \text{if } \widehat{y}<\underline{\widehat{y}}\\[2ex] \dfrac{(1-\rho)\exp(\widehat{y})}{1-(1-\rho)\exp(\widehat{y})}\mu_{\widehat{\nu}}\left(\log\left(\dfrac{1-(1-\rho)\exp(\widehat{y})}{\rho}\right)\right) & \text{if } \underline{\widehat{y}}\leqslant\widehat{y}\leqslant\overline{\widehat{y}}\\[2ex] \frac{1-\beta}{\beta}\mu_{\widehat{\nu}}\left(\frac{1-\beta}{\beta}(\widehat{y}-\overline{\widehat{y}})-\log\left(\frac{\beta}{\alpha\varphi+\beta(1-\varphi)}\right)\right) & \text{if } \widehat{y}>\overline{\widehat{y}}\end{cases}$$

where $\mu_{\widehat{\nu}}(\cdot)$ is the distribution of $\widehat{\nu}=\log(\nu)-\log(\nu_0)$, $\rho\equiv\frac{(\alpha\varphi+\beta(1-\varphi))\left(1-\alpha-\frac{1-\beta}{1+x}\right)}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}$,

$$\underline{\widehat{y}}\equiv\log\left(\frac{\underline{y}(p)}{y_0}\right)=\log\left(\frac{\varphi(1-\alpha)+(1-\varphi)\frac{1-\beta}{1+x}}{1-\alpha}\right)\text{ and }\overline{\widehat{y}}\equiv\log\left(\frac{\overline{y}(p)}{y_b}\right)=\log\left(\frac{\varphi(1-\alpha)+(1-\varphi)\frac{1-\beta}{1+x}}{\frac{1-\beta}{1+x}}\right)$$

Q.E.D □

Proposition 8 *If $x > 0$, the first order effect of free trade is to increase the sensitivity of $\widehat{y}_i = \log(y_i) - \log(y_0)$ with respect to $\widehat{\nu}_i = \log(\nu_i) - \log(\nu_0)$. However, if x=0, free trade has no first order effect on the mapping from $\widehat{\nu}_i$ to $\widehat{y}_i$.*

Proof of proposition 8 Let us first consider the autarkic case. In this situation, the aggregate production function in economy i in the steady state is given by $y_{it} = \Upsilon(\Gamma_t)^{\frac{1}{1-\alpha\varphi-\beta(1-\varphi)}} \nu_i^{\frac{\alpha\varphi-\beta(1-\varphi)}{1-\alpha\varphi-\beta(1-\varphi)}}$. Let us consider the (log–)difference between output per worker in economy i and in the big economy, $\widehat{y}_i = \log(y)_i - \log(y_0)$. Let us define $\widehat{\nu}_i = \log(\nu_i) - \log(\nu_0)$, we then have $\widehat{y}_i = \frac{\alpha\varphi+\beta(1-\varphi)}{1-\alpha\varphi-\beta(1-\varphi)}\widehat{\nu}_i$, which is independent from the distortion induced by the existence of trade union.

Let us now consider the case of an open economy, and use the relationship established in lemma 2. We compute the sensitivity of the dispersion in the level of output to the dispersion in the long run propensity to accumulate capital. Three cases should be considered

$\widehat{\nu}_i < \underline{\widehat{\nu}}$: The sensitivity is given by $\frac{\partial \widehat{y}_i}{\partial \widehat{\nu}_i} = \frac{\alpha}{1-\alpha}$ is unaffected by the trade union markup.

$\widehat{\nu}_i > \overline{\overline{\widehat{\nu}}}$: The sensitivity is given by $\frac{\partial \widehat{y}_i}{\partial \widehat{\nu}_i} = \frac{\beta}{1-\beta}$ is unaffected by the trade union markup.

$\underline{\widehat{\nu}} < \widehat{\nu}_i < \overline{\overline{\widehat{\nu}}}$: The sensitivity is given by $\frac{\partial \widehat{y}_i}{\partial \widehat{\nu}_i} = \frac{\rho\exp(\widehat{\nu}_i)}{1-\rho\exp(\widehat{\nu}_i)}$ where $\rho \equiv \frac{(\alpha\varphi+\beta(1-\varphi))\left(1-\alpha-\frac{1-\beta}{1+x}\right)}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}$. Note that $\frac{\partial^2 \widehat{y}}{\partial\widehat{\nu}\partial\rho} = \frac{\exp(\widehat{\nu})}{(1-\rho\exp(\widehat{\nu}))^2} > 0$ such that the sensitivity of $\widehat{y}$ to $\widehat{\nu}$ is an increasing function of ρ. Then, note that

$$\frac{\partial\rho}{\partial x} = \frac{(1-\alpha)(1-\beta)(\alpha\varphi+\beta(1-\varphi))}{(\beta-\alpha)(1+x)^2\left(\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}\right)^2} > 0$$

Hence, in an open economy, the larger the trade union distorsion, the greater the sensitivity of $\widehat{y}_i$ to $\widehat{\nu}_i$.

Q.E.D □

Corollary 1 *If the distribution of* $\widehat{\nu}_i$ *is concentrated around zero, then the first order effect of free trade on the cross–country distribution of output–per–worker is nil when* $x = 0$*. In contrast, it leads to an increase in dispersion when* $x > 0$*.*

Proof of Corollary 1: Let us recall that within a close economy, the dispersion of log output per worker is determined by $\widehat{y}_i = \frac{\alpha\varphi+\beta(1-\varphi)}{1-\alpha\varphi+\beta(1-\varphi)}\widehat{\nu}_i$ while, when we open trade, it changes to $\widehat{y} = \log(\rho) - \log\left(1-(1-\rho)\exp(\widehat{\nu}_i)\right)$ which can be approximated, around the steady state of the big economy as $\widehat{y}_i \simeq \frac{\rho}{1-\rho}\widehat{\nu}_i$ where $\rho \equiv \frac{(\alpha\varphi+\beta(1-\varphi))\left(1-\alpha-\frac{1-\beta}{1+x}\right)}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}$. Note that absent of trade union (x=0), $\rho = \alpha\varphi + \beta(1-\varphi)$, such that we regain the dispersion in the close economy. Conversely, we saw in the proof of Proposition 8 that $\partial\rho/\partial x > 0$, such that $\frac{\partial\widehat{y}}{\partial\widehat{\nu}}\Big|_{x>0} > \frac{\partial\widehat{y}}{\partial\widehat{\nu}}\Big|_{x=0}$

Q.E.D □

Lemma 3 *Consider two observable random variables,* Y_1 *and* Y_2*, which are both transformations of a random variables* ν*, where the first transformation is linear and results in the variable* $Y_1 = \alpha\nu$*,* $\alpha > 0$*, while the second transformation is non–linear (continuous and differentiable) and results in the variable* $Y_2 = g(\nu)$*,* $g'(\cdot) > 0$*. If the distribution of* Y_1 *is uni–modal, then a necessary condition for the distribution of* Y_2 *to be bi–modal is that* $g(\cdot)$ *not be a convex function.*

Proof of lemma 3: Since Y_1 is unimodal and is a linear transformation of ν, it has to be the case that ν is also unimodal. From the change of variable formula, we know that the distribution of ν is given by $\mu_{Y_2}(y_2) = \frac{\mu_\nu(g^{-1}(y_2))}{|g'(g^{-1}(y_2))|}$. Since $g'(\cdot) \geqslant 0$, this reduces to $\mu_{Y_2}(y_2) = \frac{\mu_\nu(g^{-1}(y_2))}{g'(g^{-1}(y_2))}$. A necessary condition for the existence of a least two modes in μ_{Y_2} is that there exists y_{20}, such that μ_{Y_2} is decreasing at the left of y_{20} and increasing above it. Therefore, it has to be the case that for any $\delta > 0$, $\varepsilon > 0$, with $\delta < \varepsilon$, $g'(g^{-1}(y_{20} - \delta)) \leqslant g'(g^{-1}(y_{20}))$ and $g'(g^{-1}(y_{20} + \delta)) \leqslant g'(g^{-1}(y_{20} - \delta))$. Hence, it has to be the case that $\frac{g'(g^{-1}(y_{20})) - g'(g^{-1}(y_{20}-\delta))}{\delta} \geqslant 0$ and $\frac{g'(g^{-1}(y_{20}+\delta)) - g'(g^{-1}(y_{20}-\delta))}{\delta} \leqslant 0$. Q.E.D □

Proposition 9 *If x=0, the opening of trade in intermediate goods cannot generate the emergence of a bi–modality in the cross–country distribution of output–per–worker. Conversely, if $x > 0$ then the opening up of trade may cause the distribution to exhibit bi–modality.*

Proof of Proposition 9: Lemma 2 established the functional $g(\cdot)$ that relates $\widehat{\nu}_i$ to $\widehat{y}_i$. Differentiating $g(\cdot)$, we get

$$g'(\widehat{\nu}_i) = \begin{cases} \frac{\alpha}{1-\alpha} & \text{if } \widehat{\nu}_i < \underline{\widehat{\nu}} \\ \frac{\rho\exp(\widehat{\nu}_i)}{1-\rho\exp(\widehat{\nu}_i)} & \text{if } \underline{\widehat{\nu}} \leqslant \widehat{\nu}_i \leqslant \overline{\widehat{\nu}} \\ \frac{\beta}{1-\beta} & \text{if } \widehat{\nu}_i > \overline{\widehat{\nu}} \end{cases}$$

with $\rho \equiv \frac{(\alpha\varphi+\beta(1-\varphi))\left(1-\alpha-\frac{1-\beta}{1+x}\right)}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}$. First, note that, whatever $x \geqslant 0$, for $\underline{\widehat{\nu}} \leqslant \widehat{\nu} \leqslant \overline{\widehat{\nu}}$, we have $g''(\widehat{\nu}) = \frac{\rho\exp(\widehat{\nu})}{(1-\rho\exp(\widehat{\nu}))^2} \geqslant 0$

- When $x = 0$, $\rho = \alpha\varphi + \beta(1-\varphi)$, implying that $g'(\underline{\widehat{\nu}}) = \alpha/(1-\alpha)$ and $g'(\overline{\widehat{\nu}}) = \beta/(1-\beta)$. Hence, $g''(\cdot) \geqslant 0$ over the whole support of $\widehat{\nu}$. From lemma 3, we now that this rules out bi–modality.
- When $x > 0$, we have $g'(\underline{\widehat{\nu}}) = \frac{\alpha}{1-\alpha}\frac{1-\alpha-\frac{1-\beta}{1+x}}{\beta-\alpha}$ and $g'(\underline{\widehat{\nu}}) = \frac{\beta}{1-\beta}\frac{1-\alpha-\frac{1-\beta}{1+x}}{\frac{\beta-\alpha}{1+x}}$. But, since $\beta \in (0,1)$, $\frac{1-\alpha-\frac{1-\beta}{1+x}}{\beta-\alpha} > 1$, such that $g'(\cdot)$ is increasing for $\nu \in (-\infty, \overline{\widehat{\nu}})$. Conversely, as soon as $x > 0$, $\frac{1-\alpha-\frac{1-\beta}{1+x}}{\frac{\beta-\alpha}{1+x}} < 1$, which implies that $\lim_{\widehat{\nu}\uparrow\overline{\widehat{\nu}}} g'(\widehat{\nu}) > \lim_{\widehat{\nu}\downarrow\overline{\widehat{\nu}}} g'(\widehat{\nu})$ Therefore, $g'(\cdot)$ is decreasing over some range of values for ν, which creates the possibility of bi–modality.

Q.E.D □

2 Allowing for International Capital Flows

In this section, we discuss the implications of relaxing the assumption of no capital mobility in the model. More precisely, we document the extent to which our main results are robustness to allowing for trade on financial capital markets.

Let us first consider the case of a perfectly frictionless international financial markets. In this case, the returns to capital are equalized across countries and the location of capital is independent of the countries propensity to save. Therefore, in the absence of trade in intermediate goods — and in the absence of differences in Ω_i — the level of output–per–worker is identical across countries. In contrast, when trade in intermediate goods is allowed, the cross country distribution of output–per–worker is indeterminate since there are two mechanisms for equalizing the returns to capital across country: through trade or through international capital flow. Hence, in the extreme case of perfect international capital markets, the model has no clear predictions on how the opening up of trade will affect the cross-country distribution of output. This is a rather unsatisfactory result. In order to have a better sense of how our results can be extended in the presence of international capital flows, it is useful to consider the limiting behavior of a model with imperfections in international capital market.

To this end, let us consider the case where domestic firms face a risk premium on borrowing in the international market which is proportional to the country's debt–to–GDP ratio, and let us examine the outcome when this risk premium tends to zero. More precisely, let us assume that the cost of capital in country i, q_i, is equal to the cost of capital in the large reference economy (the US) plus a risk premium which is proportional to the country's debt to GDP ratio as given by

$$q_i = q_0 + \rho \left(\frac{k_i - a_i}{y_i} \right)$$

where ρ is the gradient of the risk premium, a_i is the wealth–per–worker in country i and hence $(k_i - a_i)$ is the amount of international debt–per–worker in country i. Through the accumulation equation, the wealth–per–worker along a steady growth path is given by $a_i = \nu_i y_i$ and therefore the determination of the domestic cost of capital can alternatively be written as

$$q_i = q_0 + \rho \left(\frac{k_i}{y_i} - \nu_i \right)$$

Given this equation for the determination of cost of capital, a country level of capital–per–worker and output–per–worker is determined by equating the international cost of capital to the domestic return on capital. In the absence of international trade in intermediates, the limiting outcome as ρ goes to zero will have all countries producing the same amount of output–per–worker since this is the only way the returns to capital can be equalized across countries. Hence, in this case — and assuming no differences in Ω_i — the cross country distribution of output–per–worker is concentrated at a single point.

If we now open up trade in intermediates, the determination of output–per–worker in country i depends on ν_i. In particular, if $\underline{\widehat{\nu}} \leqslant \widehat{\nu}_i \leqslant \overline{\widehat{\nu}}$ then the determination of output–per–worker remains the same as in the absence international capital flows since the returns to capital are equalized (recall that within the incomplete specialization area, the social returns are constant). As a matter of fact, in the presence of international capital flows, it is easy to verify that in the limit as ρ goes to zero, the determination of $\widehat{y}_i$

is given by

$$\widehat{y}_i = \begin{cases} \log\left(\frac{(1-\alpha)\varphi+(1-\varphi)\frac{1-\beta}{1+x}}{1-\alpha}\right) & \text{if } \widehat{\nu} < \underline{\widehat{\nu}} \\ \log\left(\frac{(\beta-\alpha)(\varphi(1-\alpha)+(1-\varphi)\frac{1-\beta}{1+x}}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}\right) - \log\left(1 - \frac{(\alpha\varphi+\beta(1-\varphi))(1-\alpha-\frac{1-\beta}{1+x})}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}\exp(\widehat{\nu}_i)\right) & \text{if } \underline{\widehat{\nu}} \leqslant \widehat{\nu} \leqslant \overline{\widehat{\nu}} \\ \log\left(\frac{(1-\alpha)\varphi+(1-\varphi)\frac{1-\beta}{1+x}}{\frac{1-\beta}{1+x}}\right) & \text{if } \widehat{\nu} > \overline{\widehat{\nu}} \end{cases}$$

As can be seen from the mapping between $\widehat{y}_i$ and $\widehat{\nu}_i$, in the limiting case where ρ tends to zero, the opening up of trade in intermediates causes an increase in dispersion in output–per–worker. This is because, for countries with $\widehat{\nu}_i \in [\underline{\widehat{\nu}}, \overline{\widehat{\nu}}]$, output–per–worker is no longer equalized but instead becomes an increasing function of ν_i. Furthermore, in addition to this increase in dispersion for the countries with $\widehat{\nu}_i \in [\underline{\widehat{\nu}}, \overline{\widehat{\nu}}]$, the countries with either $\widehat{\nu}_i < \underline{\widehat{\nu}}$ or $\widehat{\nu}_i > \overline{\widehat{\nu}}$ will bunch at two points in the distribution of y_i. Indeed, let us consider the case of a country with a low propensity to capital accumulation ($\widehat{\nu}_i < \underline{\widehat{\nu}}$). In the limiting case where ρ tends to zero, the only way its return on capital can equalize the world return on capital is to accumulate up to the point its capital–output ratio reaches $\underline{\nu}$, such that its $\widehat{\nu}_i = \underline{\widehat{\nu}}$. This phenomenon likely gives rise to bi-modality. Therefore, the main results presented in the paper survive the introduction of international capital flows as long as the international capital market is not perfectly frictionless.[1]

3 Allowing for Endogenous World Prices

In the main body of the paper, we have assumed that under free trade the world price for intermediate goods correspond to the autarky prices of these goods in the reference economy. Our defense for this assumption is that the reference economy used in our empirical work is the US economy and since the US economy is extremely large economically this may constitute a good approximation. However, it is clearly an approximation. Therefore in this section we discuss how our results must be modified and rephrased when this assumption is relaxed. It is rather easy to derive the mapping between $\widehat{\nu}_i$ and $\widehat{y}_i$ for the case where world prices under free trade do not correspond to the reference economy's autarky prices. To do so, let us denote by $\nu^\star$ the value of $\widehat{\nu}$ such that a country with $\widehat{\nu}_i = \nu^\star$ does not trade in equilibrium since the world prices of intermediates are equal to its autarky prices. Then, the mapping between $\widehat{\nu}_i$ and $\widehat{y}_i$ becomes

$$\widehat{y}_i = \begin{cases} \log\left(\frac{(1-\alpha)\varphi+(1-\varphi)\frac{1-\beta}{1+x}}{1-\alpha}\right) + \frac{\alpha}{1-\alpha}(\widehat{\nu}_i - \nu^\star - \underline{\nu}) & \text{if } (\widehat{\nu}_i - \nu^\star) < \underline{\widehat{\nu}} \\ \log\left(\frac{(\beta-\alpha)(\varphi(1-\alpha)+(1-\varphi)\frac{1-\beta}{1+x}}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}\right) - \log\left(1 - \frac{(\alpha\varphi+\beta(1-\varphi))(1-\alpha-\frac{1-\beta}{1+x})}{\beta(1-\alpha)-\alpha\frac{1-\beta}{1+x}}\exp(\widehat{\nu}_i - \nu^\star)\right) & \text{if } \underline{\widehat{\nu}} \leqslant (\widehat{\nu}_i - \nu^\star) \leqslant \overline{\widehat{\nu}} \\ \log\left(\frac{(1-\alpha)\varphi+(1-\varphi)\frac{1-\beta}{1+x}}{\frac{1-\beta}{1+x}}\right) + \frac{\beta}{1-\beta}(\widehat{\nu}_i - \nu^\star - \overline{\nu}) & \text{if } (\widehat{\nu}_i - \nu^\star) > \overline{\widehat{\nu}} \end{cases}$$

The previous equation makes it clear that the presence $\nu^\star \neq 0$ simply causes a translation of our original mapping between $\widehat{\nu}_i$ and $\widehat{y}_i$. However, the problem with this mapping is that we do not know the value of

[1] In the presence of international capital flows, there are two distinct mechanisms which cause an increase in the dispersion of y_i. There is an increase due to increased dispersion of capital–output ratios and, if $x > 0$, there is increased dispersion due to an increase in the return to capital. The model of Ventura [1997] is an alternative way of emphasizing the first mechanism, while the model presented in the main body of this paper emphasizes the second mechanism. As the empirical section of the paper has shown, the data are more supportive of the second mechanism.

$\nu^{\star}$. Nonetheless, we can still make a conditional statement regarding how the opening of trade will affect the distribution of $\widehat{y}_i$. In particular, in this more general case, Corollary 1 should simply be rephrased as follows

If the distribution of $\widehat{\nu}_i$ is concentrated around $\nu^{\star}$, then the first order effect of free trade on the cross–country distribution of $\widehat{y}_i$ is nil when $x = 0$. In contrast, it leads to an increase in dispersion when $x > 0$.

This extended version of Corollary 1 clarifies that our main results hinge on the notion that $\nu^{\star}$ be not too different from the mean of $\widehat{\nu}_i$.[2] In other words, the key condition for our results on the effect of free trade to hold is that the average capital–output ratio across countries must not be substantially different the capital–output ratio across the world.[3]

[2] In fact, the result can be strengthened slightly by noting that what is key for our results is that $\nu^{\star}$ not be substantially greater than the mean of $\widehat{\nu}_i$.

[3] Based on our calculation using the World Penn tables, this condition appears satisfied.

4 Fixed Effects

Table 1: Fixed Effects

	OLS		IV1		IV1		IV2		IV3	
	60–78	78–98	60–78	78–98	60–78	78–98	60–78	78–98	60–78	78–98
$\widehat{y}_0$	-0.059	-0.062	-0.056	-0.059	-0.054	-0.059	-0.053	-0.061	-0.075	-0.084
	(0.004)	(0.005)	(0.004)	(0.005)	(0.005)	(0.006)	(0.005)	(0.006)	(0.010)	(0.012)
$\widehat{\nu}$	0.021	0.044	0.030	0.053	0.030	0.053	0.019	0.042	0.015	0.063
	(0.003)	(0.004)	(0.005)	(0.007)	(0.006)	(0.009)	(0.004)	(0.008)	(0.010)	(0.012)
$\widehat{H}$	–	–	–	–	0.006	0.010	0.010	0.018	0.008	0.008
					(0.005)	(0.008)	(0.005)	(0.007)	(0.006)	(0.010)
F_{EXCL}	–	–	0.000	0.002	0.000	0.000	0.000	0.023	0.002	0.006
$\mathcal{Q}(\widehat{\nu})$	38.546	[0.000]	16.828	[0.000]	13.079	[0.000]	10.038	[0.002]	10.154	[0.001]

Note: Standard errors in parenthesis, p–values in brackets. The set of instruments corresponds to the IV1, IV2 and IV3 sets discussed in the body text.

Table 2: Fixed Effects and Openness

	OP<med(OP)		OP⩾med(OP)		ΔOP<med(ΔOP)		ΔOP⩾med(ΔOP)	
	60–78	78–98	60–78	78–98	60–78	78–98	60–78	78–98
$\widehat{y}_0$	-0.053	-0.057	-0.042	-0.045	-0.050	-0.057	-0.052	-0.057
	(0.008)	(0.009)	(0.007)	(0.009)	(0.008)	(0.009)	(0.005)	(0.006)
$\widehat{\nu}$	0.034	0.045	0.025	0.055	0.044	0.065	0.017	0.044
	(0.009)	(0.007)	(0.007)	(0.011)	(0.010)	(0.012)	(0.005)	(0.007)
F_{EXCL}	0.003	0.000	0.0240	0.157	0.052	0.005	0.000	0.029
$\mathcal{Q}(\widehat{\nu})$	2.219	[0.136]	14.466	[0.000]	6.934	[0.008]	13.790	[0.000]

Note: Standard errors in parenthesis, p–values in brackets. The set of instruments is composed of the average (c/y) over the sub–sample and the average growth rate of population over the 15 first periods of the sub–sample.