Booms and Banking Crises
Supplementary Appendix
Not Intended for Publication

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A Comparative statics

This section reports some useful comparative statics that shed light on the dynamics at work in the model.

Result 1 *The corporate loan rate is a decreasing function of the capital stock, and an increasing function of the technology shock.*

This can be easily seen from the optimal behavior of the firm which states

$$R_t = z_t f_k(k_t) + 1 - \delta$$

The result follows from the existence of marginal decreasing returns to scale with respect to the capital stock and the fact that the technology shock affects positively the production efficiency.

Proposition 1 *In a stable trade equilibrium*[1]*, the interbank market increases with the corporate loan rate*

$$\frac{\mathrm{d}\rho_t}{\mathrm{d}R_t} > 0$$

An increase in the corporate loan rate makes it more attractive for banks to borrow on the interbank market and lend to the firms. This implies that more banks are willing to borrow (positive effect on the extensive margin) and that each individual bank is willing to leverage more (positive effect on the intensive margin, See corollary 1).

Proof: The interbank market can be easily obtained from the market clearing condition

$$G(\rho_t, R_t) = \left(1 - \mu\left(\frac{\rho_t}{R_t}\right)\right)\phi_t - \mu\left(\frac{\rho_t}{R_t}\right) = 0 \tag{1}$$

where $G(\rho_t, R_t)$ is the net demand function for loans on the interbank market. Note that we restrict our attention to the stable Walrasian equilibrium. A sufficient condition for the equilibrium to be stable in the sense of Walrasian tatônnement is

$$\frac{\partial G_(\rho_t; R)}{\partial \rho_t} < 0 \Longleftrightarrow \mu'\left(\frac{\rho_t}{R_t}\right)\frac{1+\phi_t}{R_t} - \left(1 - \mu\left(\frac{\rho_t}{R_t}\right)\right)\frac{\mathrm{d}\phi_t}{\mathrm{d}\rho_t} > 0$$

Total differentiation of the market clearing condition 1 then yields

$$\underbrace{\left[\mu'\left(\frac{\rho_t}{R_t}\right)\frac{1+\phi_t}{R_t} - \left(1 - \mu\left(\frac{\rho_t}{R_t}\right)\right)\frac{\mathrm{d}\phi_t}{\mathrm{d}\rho_t}\right]}_{>0}\mathrm{d}\rho_t = \underbrace{\mu'\left(\frac{\rho_t}{R_t}\right)\frac{1+\phi_t}{R_t}\frac{\rho_t}{R_t}}_{>0}\mathrm{d}R_t$$

Q.E.D. ■

[1] By this we mean an equilibrium in which trade takes place on the interbank market, such that there exists an equilibrium which is stable in the sense of a Walrasian tatônement process.

Corollary 1 *In a stable trade equilibrium, the individual interbank funding ratio increases with the corporate loan rate.*

Proof: Recall that, from the incentive constraint, we have

$$\phi_t = \frac{\rho_t - \gamma}{\theta\gamma} \iff \frac{\mathrm{d}\phi_t}{\mathrm{d}R_t} = \frac{1}{\gamma\theta}\frac{\mathrm{d}\rho_t}{\mathrm{d}R_t} > 0$$

Q.E.D. ■

Proposition 2 *In a stable trade equilibrium, the cutoff $\overline{p}_t$ increases with the corporate loan rate*

$$\frac{\mathrm{d}\overline{p}_t}{\mathrm{d}R_t} > 0$$

Proof: The cutoff is obtained from the participation constraint as $\overline{p}_t = \rho_t/R_t$, which implies that

$$\frac{\mathrm{d}\overline{p}_t}{\mathrm{d}R_t} = \frac{1}{R_t}\frac{\mathrm{d}\rho_t}{\mathrm{d}R_t} - \frac{\rho_t}{R_t^2}$$

Using proposition 1, this rewrites

$$\frac{\mathrm{d}\overline{p}_t}{\mathrm{d}R_t} = \frac{\rho_t}{R_t^2}\left(\frac{\mu'\left(\frac{\rho_t}{R_t}\right)\frac{1+\phi_t}{R_t}}{\mu'\left(\frac{\rho_t}{R_t}\right)\frac{1+\phi_t}{R_t} - \left(1-\mu\left(\frac{\rho_t}{R_t}\right)\right)\frac{\mathrm{d}\phi_t}{\mathrm{d}\rho_t}} - 1\right) = \frac{\rho_t}{R_t^2}\left(\frac{\left(1-\mu\left(\frac{\rho_t}{R_t}\right)\right)\frac{\mathrm{d}\phi_t}{\mathrm{d}\rho_t}}{\mu'\left(\frac{\rho_t}{R_t}\right)\frac{1+\phi_t}{R_t} - \left(1-\mu\left(\frac{\rho_t}{R_t}\right)\right)\frac{\mathrm{d}\phi_t}{\mathrm{d}\rho_t}}\right)$$

Using Corollary 1, we have $\mathrm{d}\phi_t/\mathrm{d}R_t > 0$ and hence $\mathrm{d}\overline{p}_t/\mathrm{d}R_t > 0$.
Q.E.D. ■

Proposition 3 *In a stable trade equilibrium, the rate on equity/deposit increases with the corporate loan rate*

$$\frac{\mathrm{d}r_t}{\mathrm{d}R_t} > 0$$

Proof: In normal times, the rate on deposits is given by

$$r_t = \frac{R_t}{1-\mu(\overline{p}_t)}\int_{\overline{p}_t}^{1} p\mathrm{d}\mu(p)$$

Hence

$$\frac{\mathrm{d}r_t}{\mathrm{d}R_t} = \frac{\int_{\overline{p}_t}^{1} p\mathrm{d}\mu(p)}{1-\mu(\overline{p}_t)} + \frac{\mu'(\overline{p}_t)\int_{\overline{p}_t}^{1} p\mathrm{d}\mu(p) - \overline{p}_t\mu'(\overline{p}_t)(1-\mu(\overline{p}_t))}{(1-\mu(\overline{p}_t))^2}\frac{\mathrm{d}\overline{p}_t}{\mathrm{d}R_t}$$

Using the definition of r_t, this rewrites

$$\frac{\mathrm{d}r_t}{\mathrm{d}R_t} = \frac{r_t}{R_t} + \frac{\mu'(\overline{p}_t)}{1-\mu(\overline{p}_t)}\left(\frac{r_t}{R_t} - \overline{p}_t\right)$$

Since $\overline{p}_t = \rho_t/R_t$, we have

$$\frac{\mathrm{d}r_t}{\mathrm{d}R_t} = \frac{r_t}{R_t} + \frac{\mu'(\overline{p}_t)}{1-\mu(\overline{p}_t)}\left(\frac{r_t - \rho_t}{R_t}\right)\frac{\mathrm{d}\overline{p}_t}{\mathrm{d}R_t}$$

Given that[2] $r_t > \rho_t$ and $\mathrm{d}\overline{p}_t/\mathrm{d}R_t > 0$, the result follows.
Q.E.D. ■

Lemma 1 *In a stable trade equilibrium, the spread between the corporate loan and equity/deposit rates increases with the cutoff.*

Proof: In normal times, a measure of the spread is given by

$$\Psi_t \equiv \frac{R_t}{r_t} = \frac{1-\mu(\overline{p}_t)}{\int_{\overline{p}_t}^1 p\mathrm{d}\mu(p)}$$

Hence, totally differentiating and using the definition of Ψ_t

$$\frac{\mathrm{d}\Psi_t}{\mathrm{d}\overline{p}_t} = \frac{\mu'(\overline{p}_t)}{\int_{\overline{p}_t}^1 p\mathrm{d}\mu(p)}\left(\frac{R_t}{r_t}\overline{p}_t - 1\right) = \frac{\mu'(\overline{p}_t)}{\int_{\overline{p}_t}^1 p\mathrm{d}\mu(p)}\left(\frac{\rho_t}{r_t} - 1\right) < 0$$

Q.E.D. ■

Corollary 2 *In a stable trade equilibrium, we have*

$$\frac{\mathrm{d}\rho_t}{\mathrm{d}A_t} < 0, \quad \frac{\mathrm{d}r_t}{\mathrm{d}A_t} < 0, \quad \frac{\mathrm{d}\phi_t}{\mathrm{d}A_t} < 0, \quad \frac{\mathrm{d}\overline{p}_t}{\mathrm{d}A_t} < 0, \quad \frac{\mathrm{d}\Psi_t}{\mathrm{d}A_t} > 0,$$

$$\frac{\mathrm{d}\rho_t}{\mathrm{d}z_t} > 0, \quad \frac{\mathrm{d}r_t}{\mathrm{d}z_t} > 0, \quad \frac{\mathrm{d}\phi_t}{\mathrm{d}z_t} > 0, \quad \frac{\mathrm{d}\overline{p}_t}{\mathrm{d}z_t} > 0, \quad \frac{\mathrm{d}\Psi_t}{\mathrm{d}z_t} < 0$$

Proof: This result follows from the fact that in normal times, $A_t = k_t$ and from Result 1 in the following propositions, corollaries and lemmas.
Q.E.D. ■

This corollary tells us that, in normal times, the various interest rates in the economy decrease as agents accumulate assets, as $k_t = A_t$. Positive technology shocks balance the effects of accumulation.

Proposition 4 *During a banking crisis credit increases with the level of assets*

$$\frac{\mathrm{d}k_t}{\mathrm{d}A_t} > 0$$

Proof: In an banking crisis, we have $k_t = A_t\left(1-\mu\left(\frac{\gamma}{R_t}\right)\right)$. Total differentiation of the last expression yields

$$\mathrm{d}k_t = \left(1-\mu\left(\frac{\gamma}{R_t}\right)\right)\mathrm{d}A_t + \frac{\gamma}{R_t^2}A_t\mu'\left(\frac{\gamma}{R_t}\right)\underbrace{\frac{\partial R_t}{\partial k_t}}_{<0}\mathrm{d}k_t$$

[2] Recall that $r_t(p) = pR_t(1+\phi_t) - \rho_t\phi_t$. Let us define $p = \overline{p}_t + \varepsilon$, with $\varepsilon > 0$, then $r_t(p) = \rho_t + \varepsilon R_t(1+\phi_t) > \rho_t$. Since r_t is obtained from the mass of bank with ability $p > \overline{p}_t$, we have $r_t > \rho_t$.

Hence

$$\frac{\mathrm{d}k_t}{\mathrm{d}A_t} = \frac{1-\mu\left(\frac{\gamma}{R_t}\right)}{1-\frac{\gamma}{R_t^2}A_t\mu'\left(\frac{\gamma}{R_t}\right)\frac{\partial R_t}{\partial k_t}} \geqslant 0$$

Q.E.D. ■

Proposition 5 *During an banking crisis credit, the rate on equity increases with the corporate loan rate*

$$\frac{\mathrm{d}r_t}{\mathrm{d}R_t} > 0$$

<u>Proof:</u> In an banking crisis, the rate on equity is given by

$$r_t = \gamma\mu\left(\frac{\gamma}{R_t}\right) + R_t\int_{\frac{\gamma}{R_t}}^{1} p\,\mathrm{d}\mu(p)$$

Totally differentiating this last expression, we get

$$\begin{aligned}\mathrm{d}r_t &= -\left(\frac{\gamma}{R_t}\right)^2\mu'\left(\frac{\gamma}{R_t}\right)\mathrm{d}R_t + \int_{\frac{\gamma}{R_t}}^{1} p\,\mathrm{d}\mu(p)\mathrm{d}R_t + \left(\frac{\gamma}{R_t}\right)^2\mu'\left(\frac{\gamma}{R_t}\right)\mathrm{d}R_t \\ &= \int_{\frac{\gamma}{R_t}}^{1} p\,\mathrm{d}\mu(p)\mathrm{d}R_t\end{aligned}$$

Hence

$$\frac{\mathrm{d}r_t}{\mathrm{d}R_t} = \int_{\frac{\gamma}{R_t}}^{1} p\,\mathrm{d}\mu(p) \geqslant 0$$

Q.E.D. ■

Proposition 6 *During an banking crisis credit, the spread, Ψ_t, between rate on corporate loans and the rate on equity increases with the corporate loan rate*

$$\frac{\mathrm{d}\Psi_t}{\mathrm{d}R_t} > 0$$

<u>Proof:</u> We have

$$\Psi_t = \frac{R_t}{r_t} \Longrightarrow \mathrm{d}\Psi_t = \frac{\mathrm{d}R_t}{r_t} - \frac{R_t}{r_t^2}\mathrm{d}r_t$$

such that

$$\mathrm{d}\Psi_t = \frac{\mathrm{d}R_t}{r_t}\left(1-\frac{R_t}{r_t}\frac{\mathrm{d}r_t}{\mathrm{d}R_t}\right) = \frac{\mathrm{d}R_t}{r_t}\left(1-\frac{R_t}{r_t}\int_{\frac{\gamma}{R_t}}^{1} p\,\mathrm{d}\mu(p)\right)$$

From the expression of the equity rate, we have

$$\int_{\frac{\gamma}{R_t}}^{1} p\,\mathrm{d}\mu(p) = \frac{r_t}{R_t} - \frac{\gamma}{R_t}\mu\left(\frac{\gamma}{R_t}\right)$$

such that

$$\frac{\mathrm{d}\Psi_t}{\mathrm{d}R_t} = \frac{\gamma}{r_t}\mu\left(\frac{\gamma}{R_t}\right) > 0$$

Q.E.D. ■

Corollary 3 *During an banking crisis, we have*

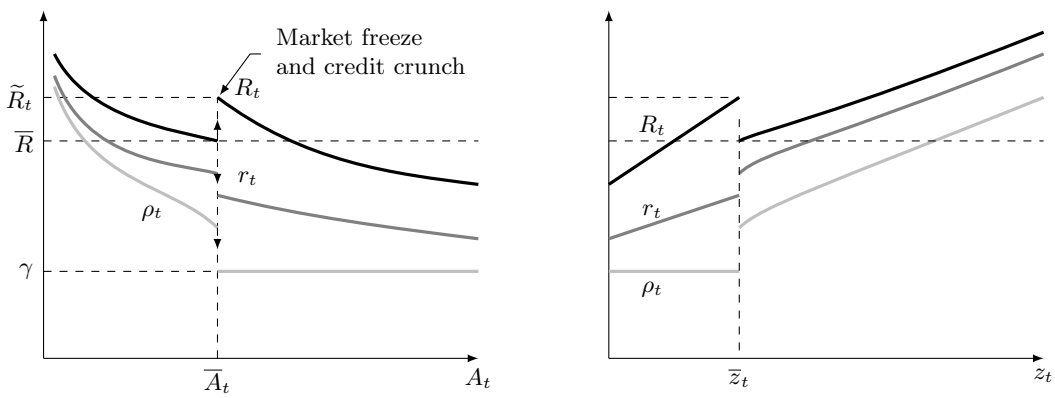
$$\frac{\mathrm{d}\rho_t}{\mathrm{d}A_t} = 0, \quad \frac{\mathrm{d}r_t}{\mathrm{d}A_t} < 0, \quad \frac{\mathrm{d}\phi_t}{\mathrm{d}A_t} = 0, \quad \frac{\mathrm{d}\Psi_t}{\mathrm{d}A_t} < 0,$$

$$\frac{\mathrm{d}\rho_t}{\mathrm{d}z_t} = 0, \quad \frac{\mathrm{d}r_t}{\mathrm{d}z_t} > 0, \quad \frac{\mathrm{d}\phi_t}{\mathrm{d}z_t} = 0, \quad \frac{\mathrm{d}\Psi_t}{\mathrm{d}z_t} > 0$$

Proof: Results on ρ_t and ϕ_t originate in the collapse of the interbank market. Results on r_t and ψ_t are direct consequences of Propositions 4, 5, 6 and Result 1.
Q.E.D. ■

Figure A.1 reports the relationship between interest rates and the state variables of the system. In particular, it shows that, in the no–trade equilibrium ($A_t > \overline{A}_t$ or $z_t < \overline{z}_t$), R_t may well be above $\overline{R}$. This is the case, for example, for $\tilde{R}_t$. But it is important to keep in mind that this rate is irrelevant for the existence of interbank trade because –in the absence of coordination failures– the only rate that matters in that respect is the one that would prevail in the equilibrium with trade if it existed as the interbank market freezes if and only if an equilibrium with trade cannot be sustained in the first place. It follows that condition $R_t < \overline{R}$ is neither necessary nor sufficient for the interbank market to freeze.

Figure A.1: Interest rates



(a) Assets (A_t) as endogenous source of crisis (b) Productivity (z_t) as exogenous source of crisis

B Details on the Solutions

B.1 Allocations

B.1.1 The Competitive General Equilibrium

We present the formal definition of a competitive general equilibrium, in its recursive form, in the case with endogenous labor supply— the case with inelastic labor supply can be simply obtained as a restriction of the current model.

The state variables for a particular individual's optimization problem at time t are *(i)* the individual asset holdings a_t, *(ii)* the aggregate asset holdings A_t, and *(iii)* the realization of the technology shocks z_t. In the sequel, we denote by $\Gamma(A_t, z_t)$ the perceived law of motion of aggregate assets in the economy and by $R(A_t, z_t)$, $r(A_t, z_t)$, $\rho(A_t, z_t)$ and $w(A_t, z_t)$ the pricing functions for corporate loans, deposits, interbank loans and labor, respectively; we also denote by $\pi(A_t, z_t)$ and $\chi(A_t, z_t)$ the profit and rebate functions. All these functions are functions of aggregate assets (A_t) and productivity (z_t), which are both taken as given by the household and capture the presence of externalities. The household's recursive optimization problem writes as

$$V(a_t, A_t, z_t) = \max_{a_{t+1}, c_t, n_t} u(c_t - v(n_t)) + \beta \mathbb{E}_t V(a_{t+1}, A_{t+1}, z_{t+1})$$

subject to

$$\begin{aligned} a_{t+1} + c_t &= r(A_t, z_t)a_t + w(A_t, z_t)n_t + \pi(A_t, z_t) + \chi(A_t, z_t) \\ A_{t+1} &= \Gamma(A_t, z_t) \end{aligned}$$

where, in the calibrated version of the model, $u(x) = x^{1-\sigma}/(1-\sigma)$ and $v(n) = n^{1+\nu}/(1+\nu)$. The solution to this problem is a set of decision rules $a(a_t, A_t, z_t)$, $n(a_t, A_t, z_t)$, and $c(a_t, A_t, z_t)$.

The firm's problem is simply given by $\max_{k_t, h_t} z_t F(k_t, h_t) + (1-\delta)k_t - w(A_t, z_t)h_t - R(A_t, z_t)k_t$, which leads to the decision rules $k(A_t, z_t)$ and $h(A_t, z_t)$. Finally, the solution of the banks' problem leads to aggregate loans $\ell(A_t, z_t)$ and $\phi(A_t, z_t) = (\rho(A_t, z_t) - \gamma)/\gamma\theta$, where only aggregate assets enter the solution due to the linearity of the problem. In the recursive rational expectation equilibrium, actual and perceived law of motions coincide.

Definition 1 (Recursive competitive general equilibrium) *A recursive competitive equilibrium is a sequence of prices defined by the pricing functions $R(A_t, z_t)$, $r(A_t, z_t)$, $\rho(A_t, z_t)$ and $w(A_t, z_t)$, a perceived law of motion for aggregate assets $\Gamma(A_t, z_t)$ and a set of decision rules $\{c(a_t, A_t, z_t), a'(a_t, A_t, z_t), k(A_t, z_t), h(A_t, z_t), \phi(A_t, z_t), \ell(A_t, z_t)\}$ with the value function $V(a_t, A_t, z_t)$ such that*

1. *$\{c(a_t, A_t, z_t), n(a_t, A_t, z_t), a'(a_t, A_t, z_t)\}$ and $V(a_t, A_t, z_t)$ solve the household's recursive optimization problem taking $R(A_t, z_t)$, $r(A_t, z_t)$, $w(A_t, z_t)$, $\pi(A_t, z_t)$, $\chi(A_t, z_t)$ and $\Gamma(A_t, z_t)$ as given.*

2. *$\{k(A_t, z_t), h(A_t, z_t)\}$ solve the firm's optimization problem taking $R(A_t, z_t)$, $w(A_t, z_t)$ as given.*

3. *$\phi(A_t, z_t)$ solves the bank's optimization problem taking $R(A_t, z_t)$, $r(A_t, z_t)$ and $\rho(A_t, z_t)$ as given. Aggregate loans are*

$$\ell(A_t, z_t) = \begin{cases} A_t & \text{if } A_t \leqslant \overline{A}(z_t) \\ \left(1 - \mu\left(\frac{\gamma}{R(A_t, z_t)}\right)\right) A_t & \text{otherwise} \end{cases}$$

4. *The perceived law of motion for aggregate assets is consistent with the actual law of motion: $a'(a_t, A_t, z_t) = \Gamma(A_t, z_t)$.*

5. *Wages satisfy $w(A_t, z_t) = z_t F_h(k(A_t, z_t), h(A_t, z_t)$, and the corporate loan rate satisfies $R(A_t, z_t) = z_t F_k(k(A_t, z_t), h(A_t, z_t)) + 1 - \delta$, the deposit rate satisfies*

$$r(A_t, z_t) = \begin{cases} R(A_t, z_t) \int_{\overline{p}(A_t, z_t)}^{1} p \frac{\mathrm{d}\mu(p)}{1 - \mu(\overline{p}(A_t, z_t))} & \text{if } A_t \leqslant \overline{A}(z_t) \\ R(A_t, z_t) \left(\frac{\gamma}{R(A_t, z_t)} \mu\left(\frac{\gamma}{R(A_t, z_t)}\right) + \int_{\frac{\gamma}{R(A_t, z_t)}}^{1} p \, \mathrm{d}\mu(p)\right) & \text{otherwise.} \end{cases}$$

where $\overline{p}(A_t, z_t) \equiv \rho(A_t, z_t)/R(A_t, z_t)$

6. *The aggregate intermediation cost rebated to the household is given by $\chi(A_t, z_t) = (R(A_t, z_t) - r(A_t, z_t))A_t - (R(A_t, z_t) - \gamma)(A_t - k(A_t, z_t))$, and the firms' profits are equal to zero, $\pi(A_t, z_t) = 0$.*

7. *Goods, labor, capital and interbank markets clear:*

$$\begin{aligned} c(A_t, A_t, z_t) + a'(A_t, A_t, z_t) &= zF(k(A_t, z_t), h(A_t, z_t)) + (\gamma + \delta - 1)(A_t - k_t) + (1 - \delta)A \\ h(A_t, z_t) &= n(A_t, A_t, z_t) \\ k(A_t, z_t) &= \ell(A_t, z_t) \\ \mu\left(\frac{\rho(A_t, z_t)}{R(A_t, z_t)}\right) &= \left(1 - \mu\left(\frac{\rho(A_t, z_t)}{R(A_t, z_t)}\right)\right) \frac{\rho(A_t, z_t) - \gamma}{\gamma\theta} \end{aligned}$$

8. *The banking sector's absorption capacity is given by*

$$\overline{A}(z_t) = \left(\frac{1 - \alpha}{\vartheta}\right)^{\frac{1}{v}} \left(\frac{\alpha}{\overline{R} + \delta - 1}\right)^{\frac{v + \alpha}{v(1 - \alpha)}} z_t^{\frac{1 + v}{v(1 - \alpha)}}$$

B.1.2 The Constrained Efficient Allocation

We present the formal definition of a constrained efficient allocation, in its recursive form, as decided by a benevolent central planner (CP) who maximizes the household's utility with respect to assets, consumption, labor, and capital, subject to the same set of allocations as the decentralized equilibrium supports, but who is not subject to the externalities. The only —but key— difference with respect to the decentralized equilibrium is that the central planner internalizes all the effects of her saving decisions. She understands that her savings, A_t, affects the equilibrium retail loan market interest rate, R_t (see relation (8) in the paper); that in turn R_t affects the equilibrium interbank market rate, ρ_t (see relation (5) in the paper); that together R_t and ρ_t determine banks' efficiency, $\overline{p}_t$ (see relation (PC) in the paper); and that bank efficiency ultimately affects the return on her savings, r_t (see relation (6) in the paper). She also understands that the cost of financial intermediation, χ_t, is rebated to her lump sum.

The state variables for the CP's optimization problem at time t are the aggregate asset holdings, A_t, and the TFP level, z_t. The CP's resource constraint is obtained by aggregating the individual household's budget constraint:

$$A_{t+1} + c_t = r_t A_t + w_t h_t + \chi_t + \pi_t.$$

Substituting the expression of profits and financial intermediation costs, we obtain

$$A_{t+1}+c_t = r_t A_t + w_t h_t + \underbrace{(R_t - r_t)A_t - (R_t - \gamma)(A_t - k_t)}_{\chi_t} + \underbrace{z_t F(k_t, h_t) + (1-\delta)k_t - w_t h_t - R_t k_t}_{\pi_t},$$

which simplifies to

$$A_{t+1} + c_t = z_t F(k_t, h_t) + (1-\delta)k_t + \gamma(A_t - k_t)$$

where k_t is given by

$$k_t = \begin{cases} A_t & \text{if } A_t \leqslant \overline{A}(z_t) \\ A_t\left(1 - \mu\left(\frac{\gamma}{R_t}\right)\right) & \text{otherwise,} \end{cases}$$

R_t is given by $R_t = z_t F_k(k_t, h_t) + 1 - \delta$ and is under the control of the CP (where $F_k(k_t, h_t)$ denotes the first derivative of $F(k_t, h_t)$ with respect to k_t), and

$$\overline{A}(z_t) = \left(\frac{1-\alpha}{\vartheta}\right)^{\frac{1}{\upsilon}} \left(\frac{\alpha}{\overline{R} + \delta - 1}\right)^{\frac{\upsilon+\alpha}{\upsilon(1-\alpha)}} z_t^{\frac{1+\upsilon}{\upsilon(1-\alpha)}}.$$

Definition 2 (Recursive constrained efficient allocation) *A recursive constrained efficient allocation is defined by a set of decision rules* $\{c(A_t, z_t), h(A_t, z_t), k(A_t, z_t), A'(A_t, z_t)\}$ *with the value function* $V^{CP}(A_t, z_t)$ *that solve the recursive optimization problem*

$$V^{CP}(A_t, z_t) = \max_{A_{t+1}, c_t, h_t, k_t} u(c_t - v(h_t)) + \beta \mathbb{E}_t V^{CP}(A_{t+1}, z_{t+1})$$

subject to

$$A_{t+1} + c_t = z_t F(k_t, h_t) + (1-\delta)k_t + \gamma(A_t - k_t)$$
$$k_t = \begin{cases} A_t & \text{if } A_t \leqslant \overline{A}(z_t) \\ A_t \left(1 - \mu\left(\frac{\gamma}{z_t F_k(k_t,h_t)+1-\delta}\right)\right) & \text{otherwise} \end{cases}$$

B.1.3 No Saving Glut Externality Allocation

We present the formal definition of the no saving glut externality allocation, in its recursive form. This allocation is the optimal allocation of an agent, referred to as CP2, who maximizes the household's utility with respect to assets, consumption, labor, and capital, subject to the same set of allocations as the decentralized equilibrium supports and to the same rebate externality as the private household, but who is not subject to the saving glut externality. Hence, the only —but key— difference with respect to the decentralized equilibrium is that CP2 understands that her savings, A_t, affects the equilibrium retail loan market interest rate, R_t (see relation (8) in the paper); that in turn R_t affects the equilibrium interbank market rate, ρ_t (see relation (5) in the paper); that together R_t and ρ_t determine banks' efficiency, $\overline{p}_t$ (see relation (PC) in the paper); and that bank efficiency ultimately affects the return on her savings, r_t (see relation (6) in the paper). However, CP2 does not realize that the cost of financial intermediation, χ_t, is eventually rebated to her lump sum. The allocation described here is therefore not constrained efficient, and departs from the one described in Section B.1.2.

Solving CP2's optimization problem amounts to solving a particular version of the central planner's problem described in Section B.1.2, in which the central planner would take the rebate as given; hence we pose $\chi_t = \overline{\chi}_t$. The state variables for the CP2's optimization problem at time t are the aggregate asset holdings, A_t, and the TFP level, z_t. The CP2's resource constraint is obtained by aggregating the individual household's budget constraint:

$$A_{t+1} + c_t = r_t A_t + w_t h_t + \pi_t + \overline{\chi}_t.$$

Substituting the expression of profits, we obtain

$$A_{t+1} + c_t = r_t A_t + w_t h_t + \underbrace{z_t F(k_t, h_t) + (1-\delta)k_t - w_t h_t - R_t k_t}_{\pi_t} + \overline{\chi}_t,$$

which simplifies to

$$A_{t+1} + c_t = r_t A_t + z_t F(k_t, h_t) + (1-\delta)k_t - R_t k_t + \overline{\chi}_t$$

where k_t, r_t are given by

$$k_t = \begin{cases} A_t & \text{if } A_t \leqslant \overline{A}_t(z_t) \\ A_t\left(1-\mu\left(\frac{\gamma}{R_t}\right)\right) & \text{otherwise} \end{cases}$$

$$r_t = \begin{cases} R_t \int_{\frac{\rho_t}{R_t}}^{1} p \frac{\mathrm{d}\mu(p)}{1-\mu\left(\frac{\rho_t}{R_t}\right)} & \text{if } A_t \leqslant \overline{A}_t(z_t) \\ R_t\left(\frac{\gamma}{R_T}\mu\left(\frac{\gamma}{R_t}\right) + \int_{\frac{\gamma}{R_t}}^{1} p\,\mathrm{d}\mu\left(p\right)\right) & \text{otherwise,} \end{cases}$$

where R_t and ρ_t are under the control of CP2 and satisfy

$$R_t = z_t F_k(k_t, h_t) + 1 - \delta$$
$$\mu\left(\frac{\rho_t}{R_t}\right) = \left(1-\mu\left(\frac{\rho_t}{R_t}\right)\right)\frac{\rho_t - \gamma}{\gamma\theta},$$

and

$$\overline{A}(z_t) = \left(\frac{1-\alpha}{\vartheta}\right)^{\frac{1}{\upsilon}}\left(\frac{\alpha}{\overline{R}+\delta-1}\right)^{\frac{\upsilon+\alpha}{\upsilon(1-\alpha)}} z_t^{\frac{1+\upsilon}{\upsilon(1-\alpha)}}.$$

Definition 3 (Recursive no saving glut externality allocation) *A recursive no savings glut allocation is defined by a set of decision rules* $\{c(A_t,z_t), h(A_t,z_t), k(A_t,z_t), A'(A_t,z_t), R(A_t,z_t), r(A_t,z_t), \rho(A_t,z_t)\}$ *with the value function* $V^{CP2}(A_t,z_t)$ *such that:*

1. $\{c(A_t,z_t), h(A_t,z_t), k(A_t,z_t), A'(A_t,z_t), R(A_t,z_t), r(A_t,z_t), \rho(A_t,z_t)\}$ *and* $V^{CP2}(A_t,z_t)$ *solve the recursive optimization problem*

$$V^{CP2}(A_t,z_t) = \max_{A_{t+1},c_t,h_t,k_t} u(c_t - v(h_t)) + \beta\mathbb{E}_t V^{CP2}(A_{t+1},z_{t+1})$$

subject to

$$A_{t+1} + c_t = r_t A_t + z_t F(k_t,h_t) + (1-\delta)k_t - R_t k_t + \overline{\chi}_t$$
$$k_t = \begin{cases} A_t & \textit{if } A_t \leqslant \overline{A}_t(z_t) \\ A_t\left(1-\mu\left(\frac{\gamma}{R_t}\right)\right) & \textit{otherwise} \end{cases}$$
$$r_t = \begin{cases} R_t \int_{\frac{\rho_t}{R_t}}^{1} p \frac{\mathrm{d}\mu(p)}{1-\mu\left(\frac{\rho_t}{R_t}\right)} & \textit{if } A_t \leqslant \overline{A}(z_t) \\ R_t\left(\frac{\gamma}{R_t}\mu\left(\frac{\gamma}{R_t}\right) + \int_{\frac{\gamma}{R_t}}^{1} p\,\mathrm{d}\mu\left(p\right)\right) & \textit{otherwise.} \end{cases}$$
$$R_t = z_t F_k(k_t,h_t) + 1 - \delta$$
$$\mu\left(\frac{\rho_t}{R_t}\right) = \left(1-\mu\left(\frac{\rho_t}{R_t}\right)\right)\frac{\rho_t-\gamma}{\gamma\theta}$$

2. $\overline{\chi}_t$ *is consistent with the definition of the financial intermediation costs, and satisfies:*

$$\overline{\chi}_t = (R(A_t,z_t) - r(A_t,z_t))A_t - (R(A_t,z_t)-\gamma)(A_t - k_t(A_t,z_t))$$

B.2 Tauchen–Hussey (1991)

Tauchen and Hussey (1991) provides a simple way to obtain a discrete approximation of an AR(1) process the form

$$z_{t+1} = \rho z_t + (1-\rho)\overline{z} + \varepsilon_{t+1}$$

where $\varepsilon_{t+1} \rightsquigarrow \mathscr{N}(0, \sigma^2)$. This implies that

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left\{-\frac{1}{2}\left(\frac{z_{t+1} - \rho z_t - (1-\rho)\overline{z}}{\sigma}\right)^2\right\} \mathrm{d}z_{t+1} = \int f(z_{t+1}|z_t)\mathrm{d}z_{t+1} = 1.$$

Tauchen and Hussey propose to replace the integral by

$$\int \Phi(z_{t+1}; z_t, \overline{s}) f(z_{t+1}|\overline{z})\mathrm{d}z_{t+1} \equiv \int \frac{f(z_{t+1}|z_t)}{f(z_{t+1}|\overline{z})} f(z_{t+1}|\overline{z})\mathrm{d}z_{t+1} = 1$$

where $f(z_{t+1}|\overline{z})$ denotes the density of z_{t+1} conditional on the fact that $z_t = \overline{z}$ (in fact the unconditional density function), which in our case implies that

$$\Phi(z_{t+1}; z_t, \overline{z}) \equiv \frac{f(z_{t+1}|z_t)}{f(z_{t+1}|\overline{s})} = \exp\left\{-\frac{1}{2}\left[\left(\frac{z_{t+1} - \rho z_t - (1-\rho)\overline{z}}{\sigma}\right)^2 - \left(\frac{z_{t+1} - \overline{z}}{\sigma}\right)^2\right]\right\}$$

Using the change of variable $\zeta_t = (z_t - \overline{z})/(\sigma\sqrt{2})$, this rewrites

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left\{-\left((\zeta_{t+1} - \rho\zeta_t)^2 - \zeta_{t+1}^2\right)\right\} \exp\left(-\zeta_{t+1}^2\right) \mathrm{d}\zeta_{t+1}$$

for which we can use a Gauss–Hermite quadrature. Given the quadrature nodes ζ_i and weights ω_i, $i = 1, \ldots, n$, the quadrature leads to the formula

$$\frac{1}{\sqrt{\pi}} \sum_{j=1}^{n} \omega_j \Phi(\zeta_j; \zeta_i; \overline{x}) \simeq 1$$

and the quantity $\omega_j \Phi(\zeta_j; \zeta_i; \overline{x})/\sqrt{\pi}$ is an "estimate" $\pi_{ij} \equiv \mathrm{Prob}(z_{t+1} = z_j | z_t = z_i)$ of the transition probability from state i to state j. Since, in general, $\sum_{j=1}^{n} \pi_{ij} = 1$ will not hold exactly, Tauchen and Hussey therefore propose the following modification:

$$\pi_{ij} = \frac{\omega_j \Phi(z_j; z_i; \overline{s})}{\sqrt{\pi}\varsigma_i}$$

where $\varsigma_i = \frac{1}{\sqrt{\pi}} \sum_{j=1}^{n} \omega_j \Phi(z_j; z_i; \overline{x})$.

B.3 Computation of Impulse Response Functions

The Impulse Response Functions (IRF) as in Koop, Pesaran and Potter (1996). In case of an arbitrary shock of magnitude Δ, given state variables at time 0, an IRF for variable x at horizon h is defined as:

$$I_x(h, \Delta, 0) = E[x_{t+h}|\Delta; \mathcal{I}_0] - E_0[x_{t+h}|\mathcal{I}_0]$$

where $\mathcal{I}_0$ is the information set. The conditional expectations, involved in IRF computations, are calculated using Monte-Carlo integration. Then IRF are obtained as follows:

Step 1: We set the horizon H of IRF. The economy is placed at the steady state level of assets, A_1, (or physical capital, k_1 in the RBC model) attained when TFP is at its long-run average level. We draw N random vector u_h, $h = 1, \ldots, H$, from a Uniform (0,1), and obtain a $(N \times H)$ matrix, U, of random numbers that will be used to simulate the Markov chain.

Step 2: Starting from the long–run average TFP level ($\log(z_1) = 0$ in our case), we simulate N time series of length (H) for TFP using the transition matrix P and the matrix of random numbers U. Starting from the initial condition on assets (or physical capital in the RBC model) and using the approximate solution of the model and the simulated TFP, we obtain time series for the main aggregates and evaluate them in logarithm. These time series are denoted $x_h^n(A_0, z_h)$ for $n = 1, \ldots, N$ and $h = 1, \ldots, H$.

Step 3: We replicate the computation of step 2, where the initial TFP level is given by the long–run average + 1 standard deviation, to for the time series $\widetilde{z}_h(\Delta)$. The so–obtained time series, which correspond the shocked time series, are denoted $x_h^n(A_0, \widetilde{z}_h(\Delta))$ for $n = 1, \ldots, N$ and $h = 1, \ldots, H$.

Step 4: We form the averages for each simulated data:

$$\begin{aligned} \bar{x}_{h,0}^N &= \frac{1}{N}\sum_{n=1}^{N} x_n^h(A_0, z_h) \quad , h = 0, \ldots, H \\ \bar{x}_{h,\Delta,0}^N &= \frac{1}{N}\sum_{n=1}^{N} x_n^h(A_0, \widetilde{z}_h(\Delta)) \quad , h = 0, \ldots, H \end{aligned} \tag{2}$$

Step 5: We compute the IRF as the difference between the two averages :

$$I_x^N(h, \Delta, 0) = \bar{x}_{h,\Delta,0}^N - \bar{x}_{h,0}^N \quad h = 0, \ldots, H$$

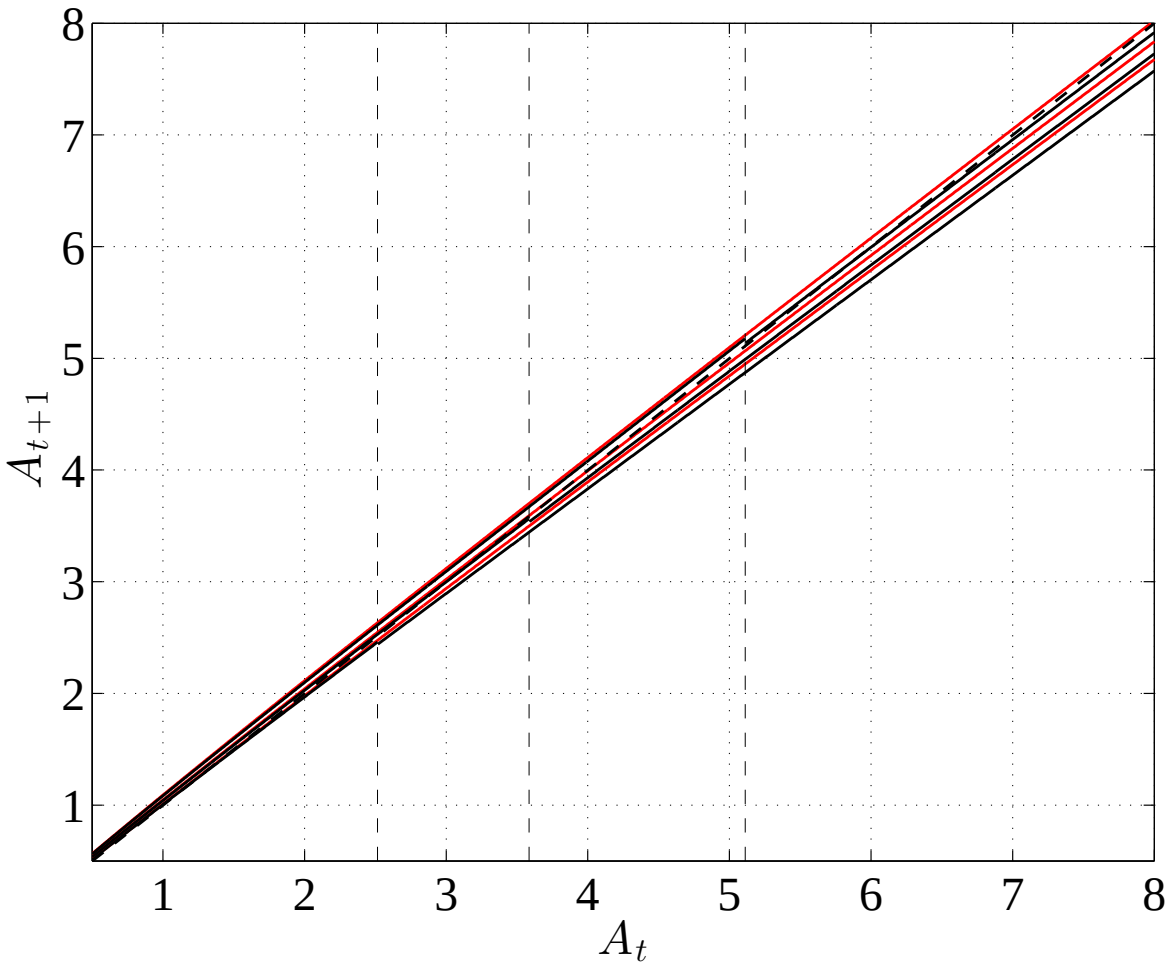
Thus for N large, we have:

$$\lim_{N\to\infty} I_x^N(h, \Delta, 0) = I_x(h, \Delta, 0) \quad h = 0, \ldots, H$$

B.4 Optimal Decision Rules

Figure B.2 below shows the optimal decision rules for the minimum, the average, and the maximal values of TFP in our grid (black lines). The dashed vertical lines indicate the levels of the absorption capacity, $\overline{A}_t$. For comparison purposes, we also report the optimal decisions, which would prevail in the absence of friction (RBC model).

Figure B.2: Decision Rules (Calibrated Version)



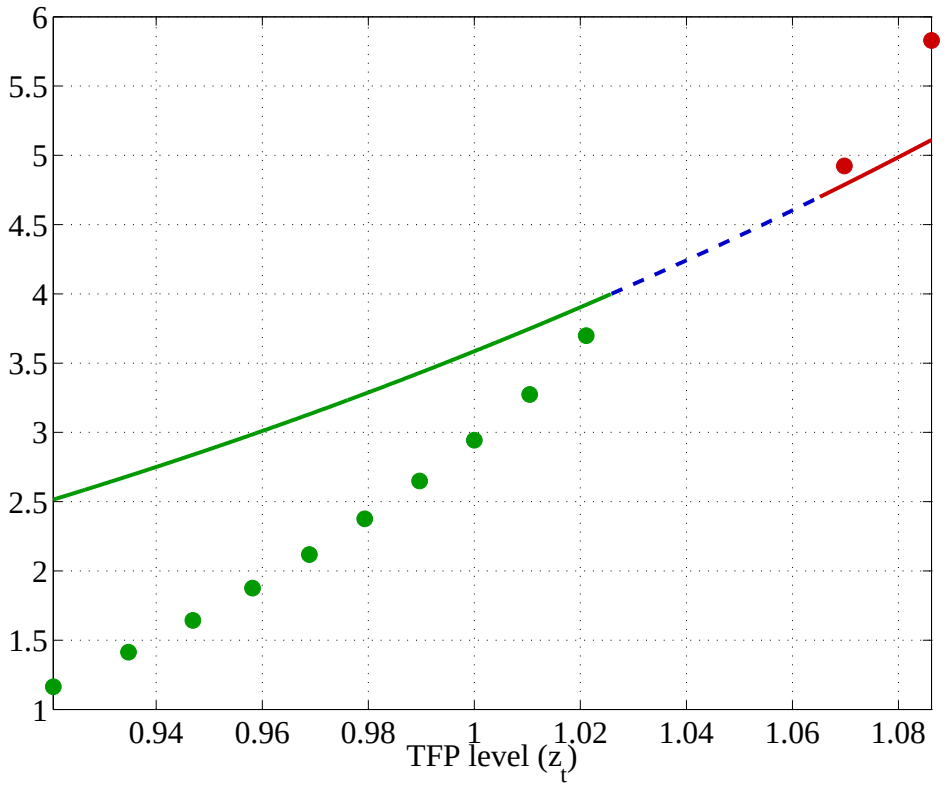
Note: We report the decision rules for the minimum, average and maximal values of TFP in our grid. The dashed vertical lines indicates the level of the absorption capacity. Dark line: Decentralized Equilibrium. Red Line: First Best Allocation.

B.5 Stochastic Steady States

The aim of this section is to get a sense of the properties of the model at its stochastic steady state. For each value of z in our grid, we compute numerically the steady state level of assets that corresponds to the intersection of the decision rule associated to z with the 45° line. Figure B.3 shows how these steady state values, $A(z)$, vary as z increases (green and red points). To know whether a steady state corresponds to normal or to crisis times, we also report the level of banks' absorption capacity $\overline{A}(z)$ associated to each z (line).

Three different types of stochastic steady states coexist in our model. For low values of z, the economy is always in a normal times regime. In this case, the household keeps her assets low so as to smooth her consumption profile, implying that $A(z) \leqslant \overline{A}(z)$ (green points). In contrast, for very high values of z, the household tends to accumulate assets disproportionately, $A(z) > \overline{A}(z)$, and the economy is in crisis (red points). Finally, for intermediate values of z (here values in–between 1.05 and 1.08), the economy alternates normal and crisis times. This happens when the level of $A(z)$ that would prevail in a normal times regime is above $\overline{A}(z)$ (and therefore cannot be sustained as a normal times steady state) while, at the same time, the level of $A(z)$ that would prevail in a crisis times regime is below $\overline{A}(z)$ (and therefore cannot be sustained as a crisis times steady state). Since none of these situations is sustainable, the economy switches from normal to crisis times in the steady state. Note that the existence of such a region opens the door to the possibility of endogenous cycle —a possibility we investigate in Section D.1.

Figure B.3: Stochastic steady states (Calibrated Version)



$\overline{A}$ in normal times; $\overline{A}$ in crisis times; $\overline{A}$ for which there is no steady states, but endogenous cycles; steady state level of A_t in normal times; steady state level of A_t in crisis times;

Note: This figure reports the steady state values of asset holdings, A, corresponding to each TFP level in the grid (bullet points) alongside with the level of the absorption capacity ($\overline{A}$). Colors indicate whether the economy is in normal times (green) or crisis times (red) in the steady state, or if no steady state value but rather an endogenous cycle exists (dashed blue).

C Supplementary Tables and Figures

C.1 Logistic Regressions

Table 1 reports the full results of the Logit regressions, summarized in Table 2 in the paper.

Table 1: Prediction of recessions

	Banking crisis	Recession		
	ST model (15)	all	financial	severe
$L.\Delta\log(loans/P)$	1.544	-3.963^{**}	-2.253	-0.672
	(2.081)	(1.606)	(2.975)	(2.901)
$L2.\Delta\log(loans/P)$	8.571^{***}	2.568	10.363^{***}	1.026
	(2.403)	(1.608)	(2.413)	(2.849)
$L3.\Delta\log(loans/P)$	3.114	1.527	5.428^{**}	5.648^{**}
	(2.347)	(1.583)	(2.382)	(2.788)
$L4.\Delta\log(loans/P)$	2.555^{*}	3.772^{**}	0.748	6.566^{*}
	(1.441)	(1.770)	(2.466)	(3.348)
$L5.\Delta\log(loans/P)$	3.539^{**}	-3.696^{***}	1.105	-5.081
	(1.555)	(1.372)	(2.306)	(3.189)
$L1/5.\Delta\log(GDP/P)$	yes	yes	yes	yes
Country fixed effects	yes	yes	yes	yes
N. obs	1,272	1,272	1,272	880
N. countries	14	14	14	12
N. events	78	196	41	54
Sum of lag coefficients	19.322^{***}	0.208	15.391^{***}	7.486
Standard error	4.329	3.057	5.736	6.684
Pseudo R^2	0.123	0.053	0.127	0.152

Note: All models include country fixed effects and the lagged growth rates of real GDP per capita. First column: replication of Schularick and Taylor (2012, page 1052, Table 6, model (15)); the dependent variable is a dummy equal to one in the country/year where a banking crisis breaks out. Other models: the dependent variable is a dummy equal to one in the country/year where a recession starts. Last column: as in Table 1 of the paper a severe recession is a recession associated with a peak-to-trough growth rate of real GDP per capita in the bottom three deciles of the distribution; Denmark and Germany are excluded because of the fixed effects. Variable $\Delta\log(x)$ is the growth rate of x. Standard errors in parentheses. *,**,***: Significant at the 10%, 5%, 1% thresholds.

Table 2 reports the results of the same Logit regressions as in Table 1, when one uses physical capital as explanatory variable, instead of credit. The series of capital is computed based on the series of investment–to–GDP ratio in the Schularik and Taylor (2012) data set.

Table 2: Prediction of recessions: Capital versus Credit

		Recession		
	Banking Crises	all	financial	severe
Capital Only				
Sum of lag credit coefficients of Capital	23.01***	7.33	28.98***	17.01
N. obs	1,098	1,098	901	839
N. countries	14	14	14	12
Pseudo R^2	0.084	0.058	0.094	0.123
Capital and Credit				
Sum of lag credit coefficients of Capital	20.22**	4.75	23.13	22.75
Sum of lag credit coefficients of Credit	20.15***	1.68	18.12**	-3.69
N. obs	995	995	840	690
N. countries	14	14	14	12
Pseudo R^2	0.148	0.074	0.178	0.207

Note: All models include country fixed effects and the lagged growth rates of real credit and real GDP per capita (the coefficients are reported in Section D.1 of the companion technical appendix). First column: replication of Schularick and Taylor (2012, page 1052, Table 6, model (15)); the dependent variable is a dummy equal to one in the country/year where a banking crisis breaks out. Other models: the dependent variable is a dummy equal to one in the country/year where a recession starts. Last column: as in Table 1 in the main text a severe recession is a recession associated with a peak-to-trough growth rate of real GDP per capita in the bottom three deciles of the distribution; Denmark and Germany are excluded because of the fixed effects. Standard errors in parentheses. The correlation between credit growth and capital growth is 0.29. *,**,***: Significant at the 10%, 5%, 1% thresholds.

C.2 Estimates of the Technology Shock Process

C.2.1 Methodology

Technology is represented by the following Cobb–Douglas production function

$$Y_t = z_t K_t^{\alpha} (\Psi_t h_t)^{1-\alpha} \text{ with } \alpha \in (0,1)$$

where $\Psi_t = \psi \Psi_{t-1}$ denotes the exogenous growth component. Total factor productivity, TFP_t, in the model is given by

$$\log(TFP_t) = \log(Y_t) - \alpha \log(K_t) - (1-\alpha)\log(h_t). \tag{3}$$

Thus, z_t in the model corresponds to stationary component of TFP. To estimate parameter ρ_z in equation (1) of the paper, we proceed as follows:

1. We derive the level of TFP, TFP_t, either from (3) using the series of physical capital and hours worked, or directly from the PennWorld Table (PWT) 0.8;
2. We de–trend $log(TFP_t)$ and derive its cyclical component;
3. We estimate ρ_z in the AR(1) process of TFP using the cyclical component of $log(TFP_t)$.

C.2.2 Measurement

The measurement of TFP poses several challenges related to the measurement of the capital stock and the measurement of hours worked. There exists several measures of hours worked for the United States. A common practice in the Business Cycle Literature is to rely on hours worked in the nonfarm business sector (NFBS) data published by the Bureau of Labor statistics. These data are available at the quarterly frequency from 1948 to 2013. Annual data are obtained by averaging quarterly data for each year. Although these data are commonly used in the macroeconomic literature, it can easily be argued that they only cover a limited share of total hours worked by excluding some sectors. We therefore also consider total annual hours worked in the economy as reported by the Conference Board Total Economy Database (downloadable from `http://www.conference-board.org/data/economydatabase/`), which gathers annual data for the period 1950-2013.

The capital stock is built using the law of motion of capital from the model and feeding it with real investment data. The level of the initial capital stock is set such that the economy is on a balanced growth path in that particular year, implying that $K_{1950} = \frac{k}{y} Y_{1950}$ where k/y denotes the capital output ratio of the economy implied by our model ($k/y =$

$\alpha\beta/(\psi-\beta(1-\delta)) = 2.1$).[3] Our measure of investment is the gross private domestic investment (GPDI) as published by the Bureau of Economic Analysis. Note that obtaining real data raises the issue of deflating. As a way to check for the sensitivity of the results, we consider two deflating methods. First the GPDI is deflated using the implicit price deflator of GDP. We then consider a version where it is deflated using its own chained price index (index=100 in 2009). Note that the GDP —also obtained from NIPA— used in the calculation of TFP is then deflated accordingly. As an alternative measure of the physical capital stock, we also used the capital stock at constant national prices for United States as reported by the PWT 8.0 (code RKNANPUSA666NRUG).

We then build several alternative measurements of TFP depending of the measure of hours worked, the capital stock and the deflating of nominal investment and GDP.

C.2.3 Results

Table 3 reports the various measurements. Beyond these measures, and as a way to assess our

Table 3: Measurement of TFP

	TFP_1	TFP_2	TFP_3	TFP_4	TFP_5	TFP_6	TFP_7	TFP_8
Hours Worked:								
NFBS	×		×		×		×	
TED		×		×		×		×
Physical Capital:								
Model based	×	×	×	×				
PWT					×	×	×	×
Deflating:								
Implicit Deflator	×	×			×	×		
Chained			×	×			×	×

results, we also report estimates obtained using the Fernald's measure of TFP (denoted TFP_F and TFP_{Fu} for a measure of TFP adjusted for utilization) downloadable from `http://www.frbsf.org/economic-research/economists/jfernald/quarterly_tfp.xls`, and multifactor productivity measures for the private business sector (TFP_{PBS}) and the nonfarm business sector (TFP_{NFBS}) as reported by the Bureau of Labor Statistics (downloadable from `http://www.bls.gov/mfp/`).

We use three alternative ways of removing the trend in (log-) TFP: *i)* a linear trend, *ii)* a

[3] As a way to investigate the sensitivity of the results to initial conditions, we also built the capital stock starting from 1929, rather than 1950, and then use the so–obtained capital data from 1950 on. But we found no material difference in the results.

quadratic trend and *iii)* using the Hodrick–Prescott filter with $\lambda = 100$ (as used in several papers in the sudden stop literature). The first detrending procedure implies a constant exogenous rate of growth and is therefore consistent with the theoretical model. The quadratic trend is less consistent, but more flexible than the linear trend. The HP trend, which is even more flexible, helps us account for the change in the long–term US productivity growth trend in the seventies. The detrended TFP, z_t, is then assumed to follow an AR(1) of the form

$$z_t = \rho_z z_{t-1} + \varepsilon_t \text{ with } \varepsilon_t \rightsquigarrow N(0, \sigma_z) \tag{4}$$

Tables 4–6 report estimates of the persistence parameter, ρ_z, and the standard deviation, σ_z, of the innovation of the AR(1)process for the various specifications of the trend and various samples. We first consider the case where TFP is linearly detrended as it corresponds to the trend in the model. For all our measures of TFP, the persistence parameter lies systematically in the range 0.88–0.92 in the largest sample (1950–2008). These estimates are consistent across samples, although the persistence slightly decreases and hovers around 0.85 for most specifications. Compared to direct measures provided by the PennWorld Table, Fernald or the BLS, the persistence of the process is slightly lower in our case and for all time periods. In the particular sample 1980–2008, and only in that particular case, the persistence decreases significantly to reach 0.55 on average. Figure C.4 shows that this only happens when the sample starts around 1980, but goes back to higher persistence later on. Using a quadratic trend barely affects these results.

Finally, Table 6 reports the results, when one estimates (4) using the HP–filtered series of log TFP. It is then clear that the estimates of ρ_z falls significantly, and hovers around 0.4 for both the model–consistent and the direct measures of TFP. Note however that the HP trend is not consistent with the form of growh we consider in the model. In our sensitivity analysis (Table 6 in the paper), we experiment with $\rho_z = 0.44$, to investigate the role of persistence for our results.

Table 4: TFP: Estimated AR(1) Process, Linear Trend

	TFP_1	TFP_2	TFP_3	TFP_4	TFP_5	TFP_6	TFP_7	TFP_8	TFP_{PWT}	TFP_F	TFP_{Fu}	TFP_{PBS}	TFP_{NBFS}
1950-2008													
ρ_z	0.908	0.887	0.895	0.883	0.902	0.879	0.904	0.880	0.884	0.891	0.929	0.915	0.905
	(0.055)	(0.060)	(0.053)	(0.058)	(0.048)	(0.053)	(0.049)	(0.054)	(0.062)	(0.053)	(0.039)	(0.045)	(0.051)
σ_z	1.200	1.307	1.170	1.300	1.026	1.119	1.030	1.134	1.435	1.744	1.336	1.665	1.713
1960-2008													
ρ_z	0.865	0.841	0.763	0.749	0.858	0.826	0.860	0.827	0.915	0.759	0.865	0.803	0.812
	(0.063)	(0.068)	(0.075)	(0.080)	(0.063)	(0.068)	(0.062)	(0.068)	(0.060)	(0.076)	(0.059)	(0.065)	(0.068)
σ_z	1.193	1.267	1.097	1.188	1.033	1.081	1.022	1.074	1.368	1.652	1.342	1.574	1.627
1970-2008													
ρ_z	0.888	0.858	0.788	0.739	0.895	0.847	0.893	0.844	0.861	0.737	0.851	0.812	0.832
	(0.073)	(0.081)	(0.101)	(0.113)	(0.074)	(0.084)	(0.074)	(0.085)	(0.075)	(0.110)	(0.086)	(0.096)	(0.093)
σ_z	1.181	1.231	1.116	1.175	1.010	1.032	1.024	1.051	1.306	1.717	1.349	1.611	1.667
1980-2008													
ρ_z	0.583	0.528	0.519	0.535	0.759	0.618	0.762	0.627	0.651	0.651	0.787	0.710	0.693
	(0.152)	(0.187)	(0.173)	(0.209)	(0.119)	(0.158)	(0.118)	(0.158)	(0.167)	(0.143)	(0.103)	(0.123)	(0.120)
σ_z	0.915	0.998	0.878	0.983	0.871	0.882	0.866	0.881	1.045	1.456	1.355	1.287	1.313
1990-2008													
ρ_z	0.698	0.857	0.664	0.814	0.776	0.778	0.784	0.809	0.890	0.518	0.741	0.634	0.636
	(0.200)	(0.253)	(0.201)	(0.255)	(0.152)	(0.228)	(0.151)	(0.225)	(0.216)	(0.238)	(0.164)	(0.180)	(0.179)
σ_z	0.925	0.932	0.871	0.866	0.888	0.823	0.878	0.811	0.937	1.306	1.503	1.075	1.018

Note: Standard deviation into parenthesis.

Figure C.4: Persistence Parameters: Sensitivity to Starting Date of Sample (Linear Detrending)

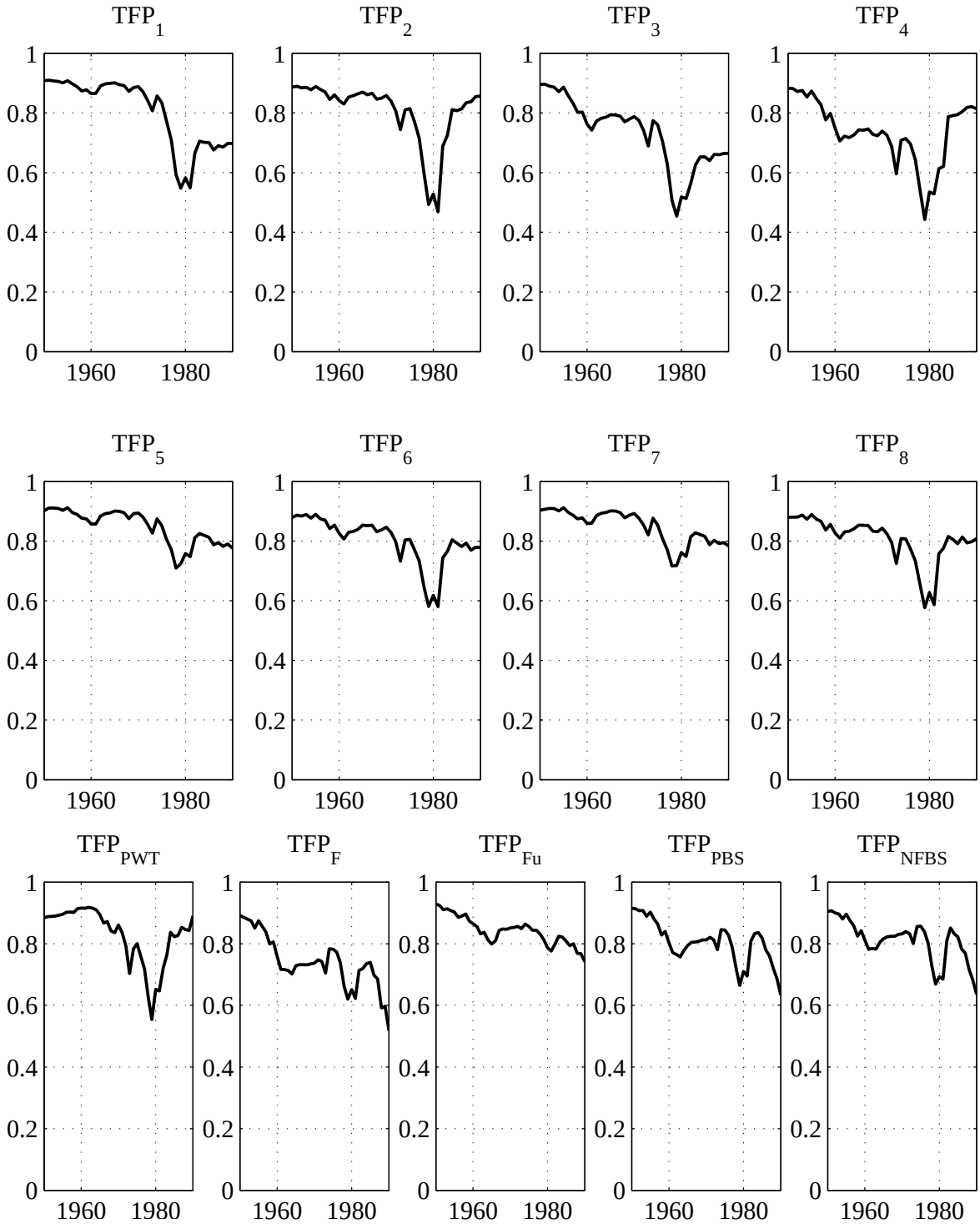


Table 5: TFP: Estimated AR(1) Process, Quadratic Trend

	TFP_1	TFP_2	TFP_3	TFP_4	TFP_5	TFP_6	TFP_7	TFP_8	TFP_{PWT}	TFP_F	TFP_{Fu}	TFP_{PBS}	TFP_{NBFS}
1950-2008													
ρ_z	0.909	0.885	0.884	0.849	0.915	0.883	0.916	0.882	0.836	0.857	0.917	0.902	0.893
	(0.055)	(0.061)	(0.062)	(0.069)	(0.052)	(0.058)	(0.052)	(0.059)	(0.067)	(0.068)	(0.054)	(0.059)	(0.062)
σ_z	1.201	1.302	1.156	1.263	1.029	1.110	1.033	1.123	1.417	1.703	1.288	1.640	1.703
1960-2008													
ρ_z	0.799	0.802	0.752	0.755	0.812	0.804	0.814	0.805	0.770	0.773	0.879	0.821	0.822
	(0.066)	(0.071)	(0.075)	(0.080)	(0.063)	(0.069)	(0.063)	(0.069)	(0.085)	(0.077)	(0.062)	(0.067)	(0.069)
σ_z	1.164	1.258	1.092	1.189	1.012	1.075	1.002	1.069	1.332	1.661	1.336	1.587	1.634
1970-2008													
ρ_z	0.641	0.713	0.561	0.625	0.630	0.627	0.619	0.618	0.752	0.501	0.704	0.569	0.565
	(0.133)	(0.138)	(0.141)	(0.148)	(0.127)	(0.147)	(0.129)	(0.148)	(0.132)	(0.135)	(0.113)	(0.121)	(0.119)
σ_z	1.126	1.239	1.064	1.177	0.941	1.014	0.952	1.030	1.314	1.606	1.317	1.488	1.520
1980-2008													
ρ_z	0.540	0.528	0.517	0.422	0.703	0.618	0.707	0.629	0.651	0.520	0.751	0.600	0.519
	(0.179)	(0.187)	(0.175)	(0.195)	(0.149)	(0.178)	(0.149)	(0.177)	(0.167)	(0.192)	(0.154)	(0.170)	(0.181)
σ_z	0.931	0.998	0.882	0.917	0.876	0.903	0.872	0.902	1.045	1.466	1.393	1.285	1.287
1990-2008													
ρ_z	0.698	0.515	0.656	0.447	0.783	0.682	0.793	0.703	0.519	0.530	0.778	0.638	0.629
	(0.207)	(0.234)	(0.210)	(0.243)	(0.189)	(0.206)	(0.186)	(0.202)	(0.201)	(0.258)	(0.191)	(0.250)	(0.257)
σ_z	0.935	0.779	0.882	0.709	0.932	0.777	0.921	0.763	0.775	1.338	1.569	1.148	1.091

Note: Standard deviation into parenthesis.

Figure C.5: Persistence Parameters: Sensitivity to Starting Date of Sample (Quadratic Detrending)

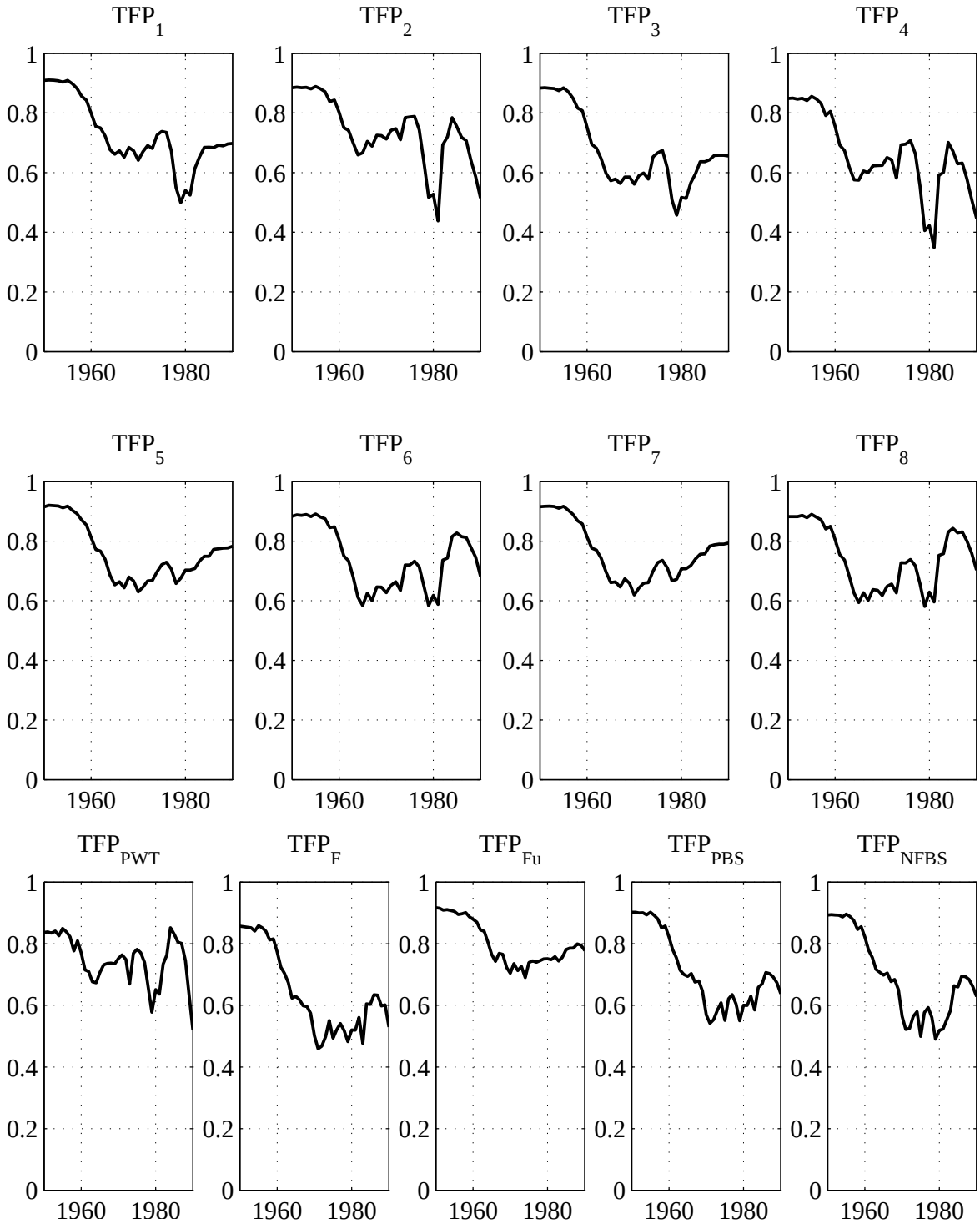
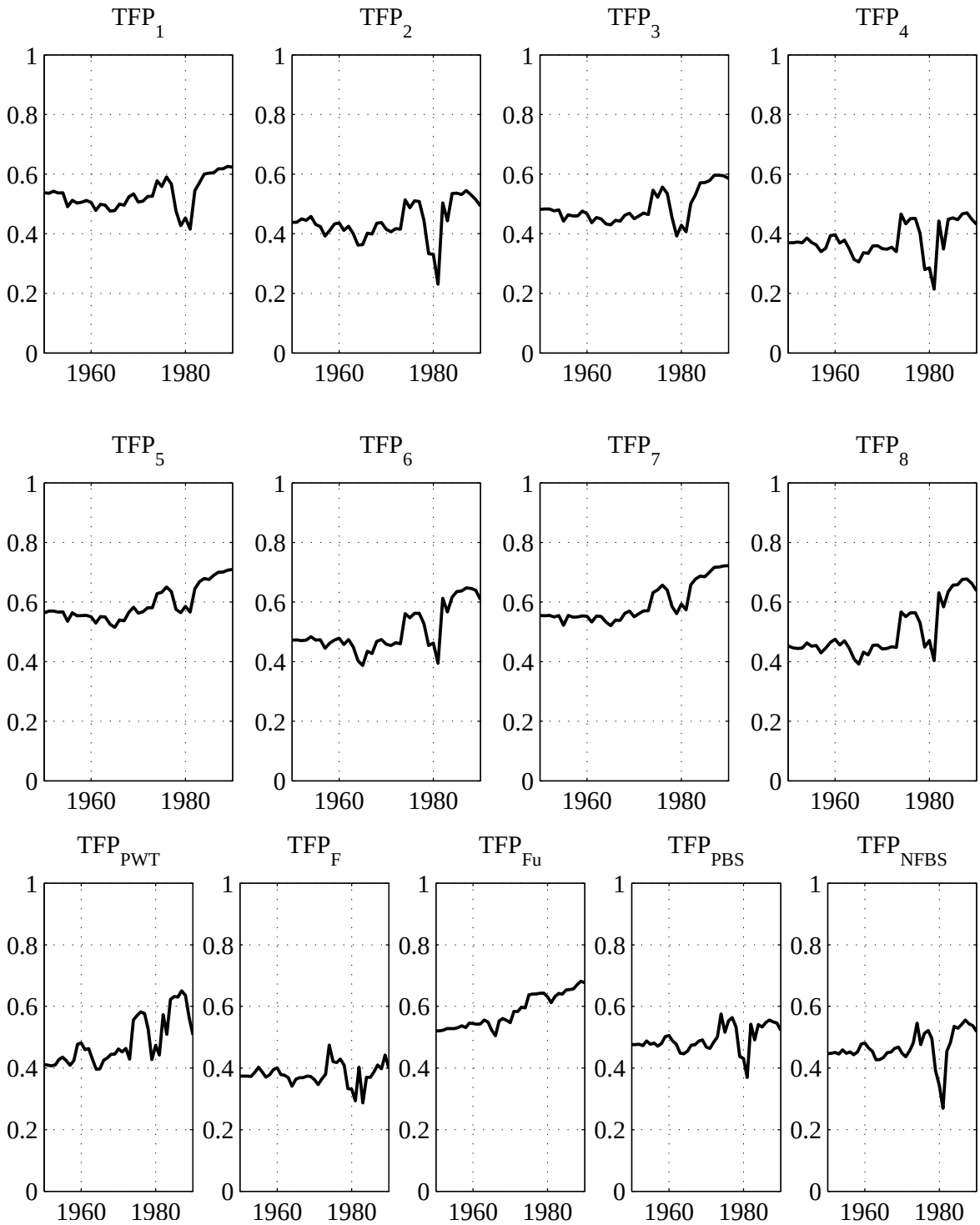


Table 6: TFP: Estimated AR(1) Process, Hodrick-Prescott Trend

	TFP_1	TFP_2	TFP_3	TFP_4	TFP_5	TFP_6	TFP_7	TFP_8	TFP_{PWT}	TFP_F	TFP_{Fu}	TFP_{PBS}	TFP_{NBFS}
1950-2008													
ρ_z	0.538	0.438	0.482	0.370	0.563	0.472	0.555	0.453	0.412	0.374	0.521	0.476	0.447
	(0.115)	(0.123)	(0.118)	(0.127)	(0.112)	(0.120)	(0.113)	(0.122)	(0.123)	(0.129)	(0.122)	(0.120)	(0.122)
σ_z	0.981	1.058	0.960	1.035	0.845	0.916	0.849	0.925	1.171	1.422	1.072	1.358	1.405
1960-2008													
ρ_z	0.505	0.436	0.467	0.397	0.551	0.479	0.552	0.475	0.482	0.401	0.544	0.505	0.481
	(0.127)	(0.136)	(0.128)	(0.136)	(0.123)	(0.133)	(0.123)	(0.134)	(0.131)	(0.140)	(0.133)	(0.129)	(0.131)
σ_z	0.958	1.016	0.928	0.987	0.839	0.885	0.829	0.879	1.107	1.425	1.126	1.348	1.386
1970-2008													
ρ_z	0.507	0.414	0.450	0.350	0.563	0.458	0.550	0.443	0.462	0.362	0.547	0.468	0.448
	(0.141)	(0.152)	(0.146)	(0.156)	(0.137)	(0.151)	(0.138)	(0.152)	(0.149)	(0.157)	(0.148)	(0.145)	(0.145)
σ_z	0.993	1.026	0.971	1.003	0.860	0.885	0.870	0.898	1.100	1.488	1.179	1.391	1.412
1980-2008													
ρ_z	0.453	0.331	0.429	0.286	0.585	0.462	0.593	0.471	0.474	0.331	0.631	0.431	0.343
	(0.182)	(0.197)	(0.183)	(0.200)	(0.166)	(0.187)	(0.166)	(0.187)	(0.182)	(0.204)	(0.164)	(0.185)	(0.188)
σ_z	0.865	0.883	0.833	0.844	0.798	0.809	0.794	0.807	0.930	1.317	1.231	1.167	1.163
1990-2008													
ρ_z	0.623	0.493	0.585	0.432	0.710	0.608	0.722	0.638	0.508	0.398	0.676	0.522	0.518
	(0.216)	(0.276)	(0.217)	(0.280)	(0.192)	(0.239)	(0.190)	(0.237)	(0.244)	(0.270)	(0.205)	(0.244)	(0.248)
σ_z	0.863	0.776	0.819	0.717	0.819	0.715	0.808	0.702	0.758	1.216	1.372	1.017	0.967

Note: Standard deviation into parenthesis.

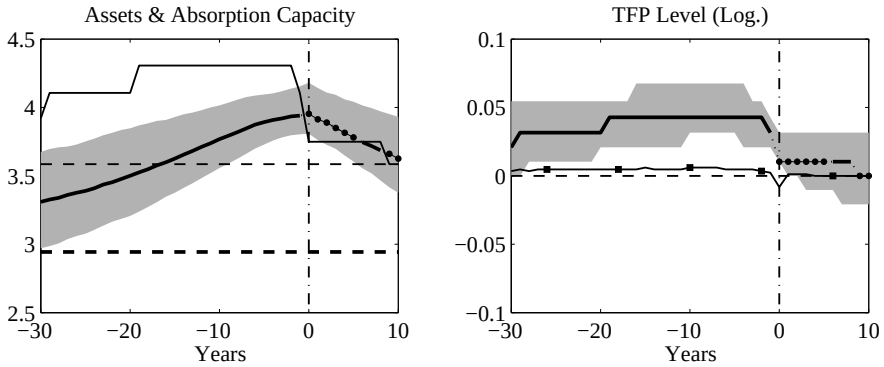
Figure C.6: Persistence Parameters: Sensitivity to Starting Date of Sample (HP Detrending)



C.3 Typical Path to Financial Recessions Conditional on TFP being Above Trend

Figure C.7 reports the typical path to crisis for asset holdings and the (log) technology shock, along with their 68% confidence interval (shaded area), in the baseline model, conditional on TFP being above trend at the time the recession starts (*i.e.* the crisis starts).

Figure C.7: Typical path to financial recessions

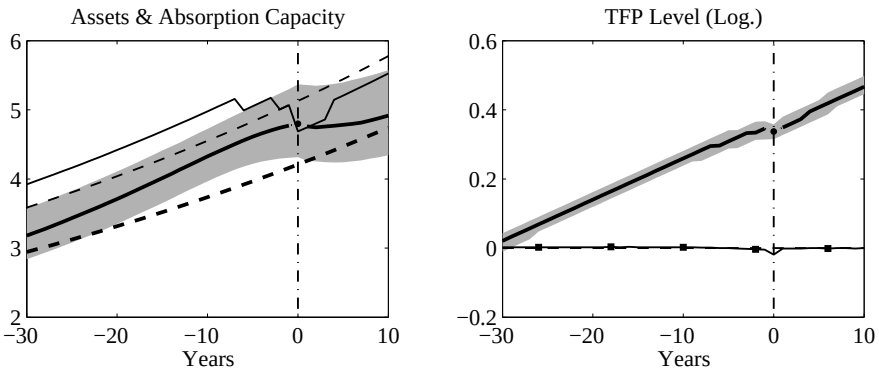


Dynamics in normal times, Dynamics during a crisis, Dynamics of $\overline{A}_t$, Average across simulations, Dynamics in the no savings glut externality economy, 68% Confidence band around typical path, Underlying TFP innovations (ε_t). For presentation purposes, the reported series have been deflated for the underlying exogenous growth trend.

C.4 (Non–Deflated) Typical Path to Financial Recessions

Figure C.8 reports the typical path to financial recessions for asset holdings and the (log) technology shock, along with their 68% confidence interval (shaded area), in the baseline model. This typical path only differs from the one presented in Figure 5 in the paper in that it was not deflated for the underlying exogenous growth trend in TFP.

Figure C.8: Typical path to financial recessions



Dynamics in normal times, Dynamics during a crisis, Dynamics of $\overline{A}_t$, Average across simulations, Dynamics in the no savings glut externality economy, 68% Confidence band around typical path, Underlying TFP innovations (ε_t).

C.5 Response to a Large Negative Shock

The economy is started from the average steady state and is subjected to a 7% drop in total factor productivity in the initial period. Like other DSGE models featuring financial frictions, a banking crisis breaks out *on impact*.

Figure C.9: Response to a large drop in TFP (I)

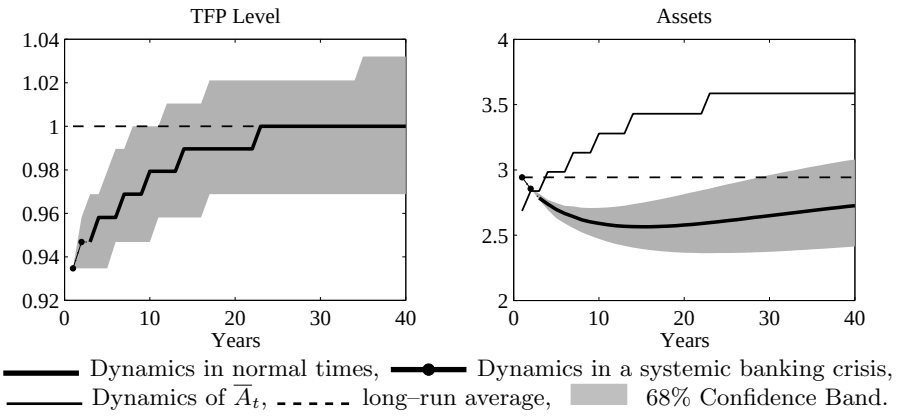
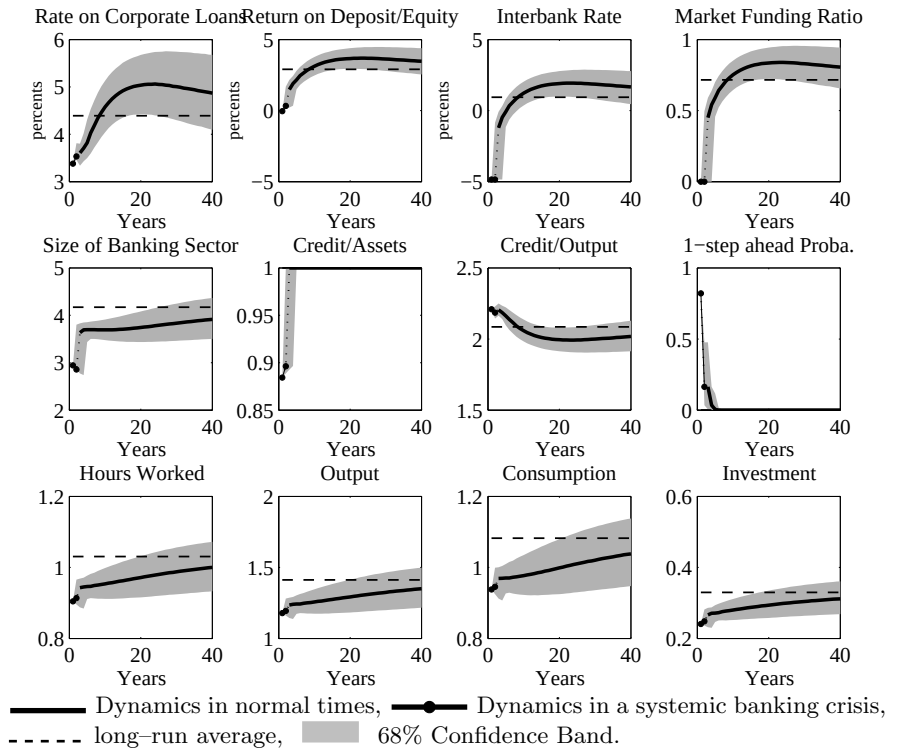


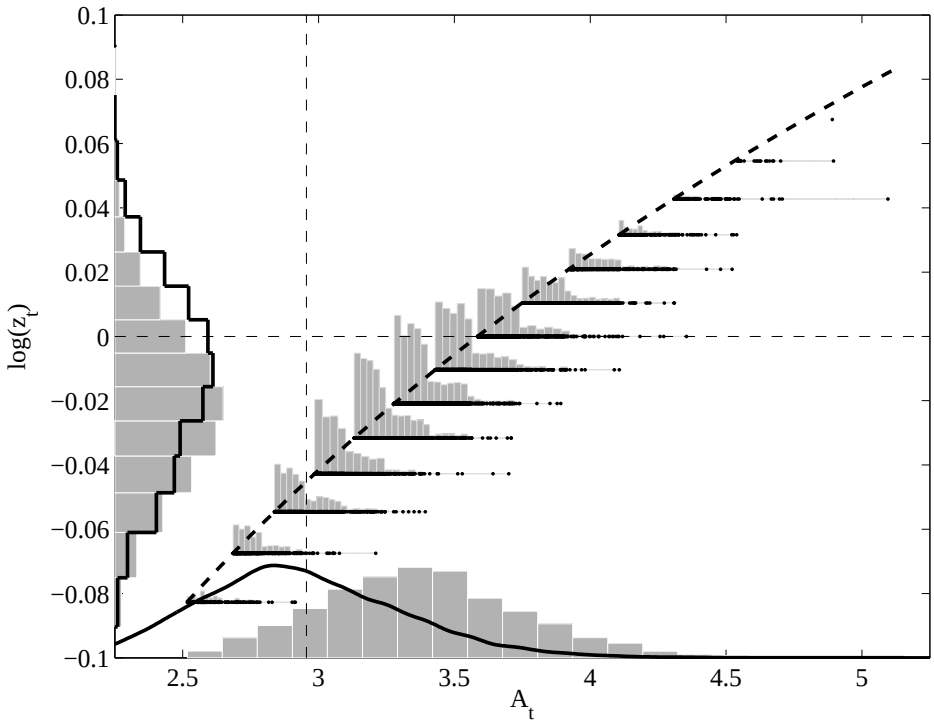
Figure C.10: Response to a large drop in TFP (II)



C.6 Ergodic Distribution of Assets and Productivity at the Beginning of Financial and Other Recessions

Given that various combinations of z_t and A_t may prompt crises, a question that arises is how typical the "typical path" is. To answer this question we present in Figure C.11 below the whole ergodic distributions of z_t and A_t at the times when financial recessions (bars) or other recessions (black plain lines) start. These distributions are based on a 500,000 period simulation of the model.

Figure C.11: Ergodic distribution of assets and productivity at the beginning of financial and other recessions



Distributions at the beginning of other recessions, Distributions at the beginning of financial recessions, Absorption capacity frontier ($\overline{z}_t$), Crisis realizations, long run averages.

The figure features two types of distributions: the marginal distributions of A_t and z_t on the x– and y–axes, respectively; and the distribution of A_t *conditional* on the level of z_t, along the absorption capacity frontier. The banking sector's absorption capacity frontier, $\overline{z}_t$, is defined in Proposition 3 in the paper and represented by the dashed line in the figure. It increases with A_t, as a higher level of TFP is required for the banking sector to absorb larger savings. In the figure, crises are represented by the dots, and break out when, given A_t, $z_t < \overline{z}_t$ (*i.e.*, z_t is below the absorption capacity frontier), or equivalently when, given z_t, $A_t > \overline{A}_t$ (*i.e.*, A_t is on the right side of the absorption capacity frontier).

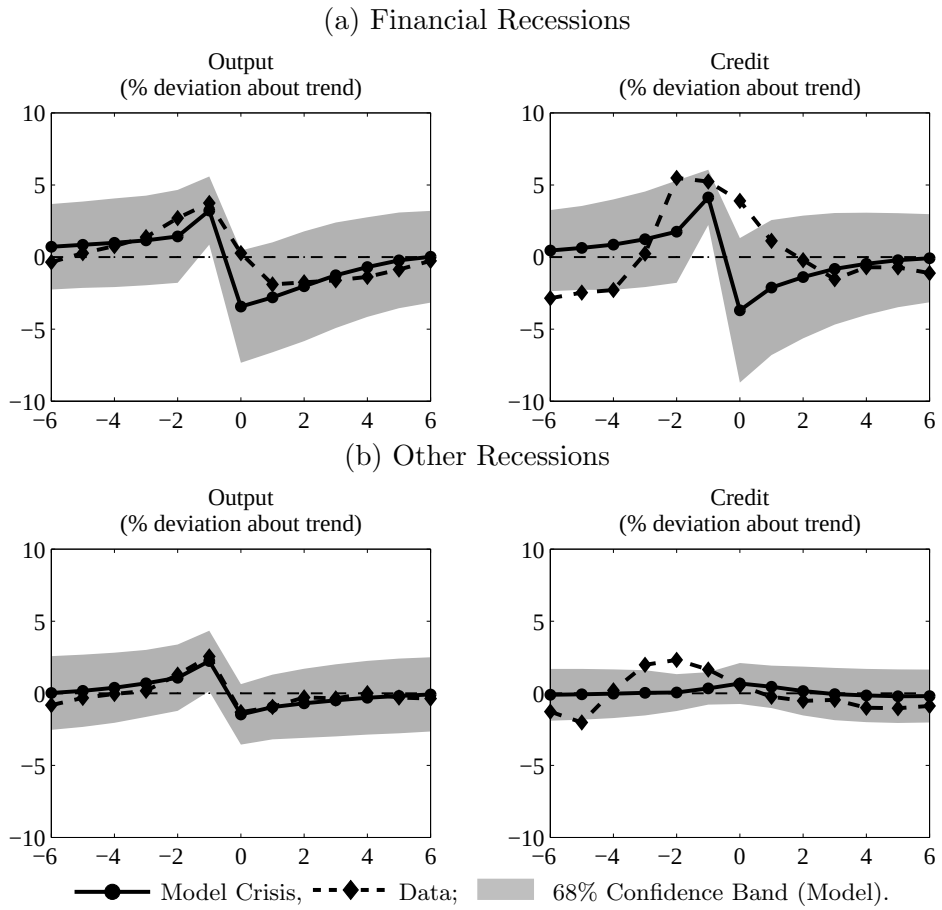
The comparison of the distributions of TFP between financial recessions and other recessions shows that there is no material difference. So, as far as the level of TFP is concerned, financial recessions are not different from other recessions. When both types of recessions break out, TFP is slightly below trend. In contrast, when we do the same comparison for bank assets, we find that bank assets are clearly distributed above their trend as financial recessions break out, but not as other recessions break out. These results point to the critical role of credit booms as an endogenous source of financial instability in our model.

The conditional distributions also show that most financial recessions burst when A_t is close to the absorption capacity frontier (*i.e.*, close to $\overline{A}_t$), as in this case even a small TFP shock may trigger a crisis; those are credit–boom–driven financial recessions (see the discussion of Figure 3 in the paper). In contrast, much fewer financial recessions burst when A_t is far above $\overline{A}_t$; those typically follow a large and sudden fall in $\overline{A}_t$, and are the shock–driven ones.

C.7 Recessions, Alternative HP parameter

Figure C.12 below reproduces Figure 9 of the paper, when one HP–filters the actual and simulated data with $\lambda = 100$, instead of $\lambda = 6.25$.

Figure C.12: Dynamics of output and credit gaps around recessions (HP-filtering with $\lambda = 100$)



Note: Average dynamics of the Hodrick-Prescott ($\lambda = 100$) cyclical component of (log) output and credit 6 periods before and after the start of a business cycle (period 0) with and without a banking crisis. To be consistent, we treat the simulated series of output and credit as we treat the actual data in Figure 1 of the paper.

C.8 Second Order Moments

Table 7 reports the second order moments for the main macroeconomic aggregates in the model, and compares them with actual data for the US. Both the data and model simulations are HPfiltered with a coefficient $\lambda = 6.25$ as recommended by Uhlig and Ravn (2002).

Table 7: Second Order Moments

	US Data			Model			RBC Model		
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
Output	2.17	1.00	–	1.80	1.00	–	1.51	1.00	–
Consumption	0.77	0.36	0.79	1.43	0.79	0.99	1.19	0.79	0.99
Investment	5.92	2.73	0.98	3.05	1.69	0.98	2.44	1.61	0.99
Hours	1.80	0.83	0.85	1.28	0.71	0.99	1.01	0.67	0.99

Note: (a): standard deviation, (b): relative standard deviation, (c): correlation with output. All data expressed in log–deviation from their HP–filter trend ($\lambda = 6.25$).

In the data, consumption is defined as the sum of consumption of non durable goods and services per capita. Investment is the sum of gross fixed private investment and consumption of durable goods per capita. Hours are total hours in the non farm business sector divided by total civilian population. Output is the sum of consumption and investment.

Consumption	=	`(PCNDA+PCESVA)/(CNP16OV*GDPA/GDPCA)`
Investment	=	`(GPDIA+PCDGA)/(CNP16OV*GDPA/GDPCA)`
Output	=	Consumption+Investment
Hours	=	`HOANBS/CNP16OV`

with

Mnemonic	Variable	Transformation
GDPA	Gross Domestic Product, Current Dollars	N/A
GDPCA	Gross domestic Product, Constant Dollars	N/A
PCESVA	Consumption of Services, Current Dollars	N/A
PCNDA	Consumption of non Durables, Current Dollars	N/A
PCDA	Consumption of Durables, Current Dollars	N/A
GPDIA	Gross Private Domestic Investment, Current Dollars	N/A
HOANBS	Total Hours Worked in Non Farm Business Sector	Average
CNP16OV	Civilian Population, 16 and over	End of Period

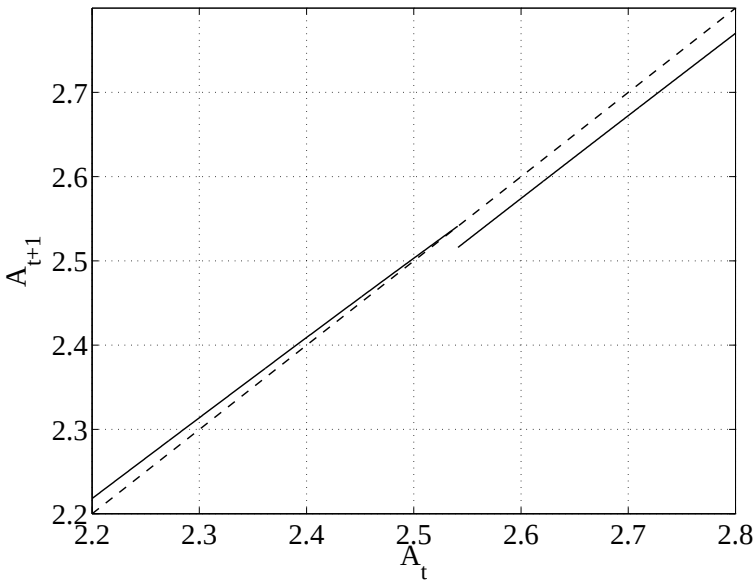
Note: All data are downloaded from `http://research.stlouisfed.org/fred2/`

D Variations of the Model

D.1 Deterministic Version of the Model

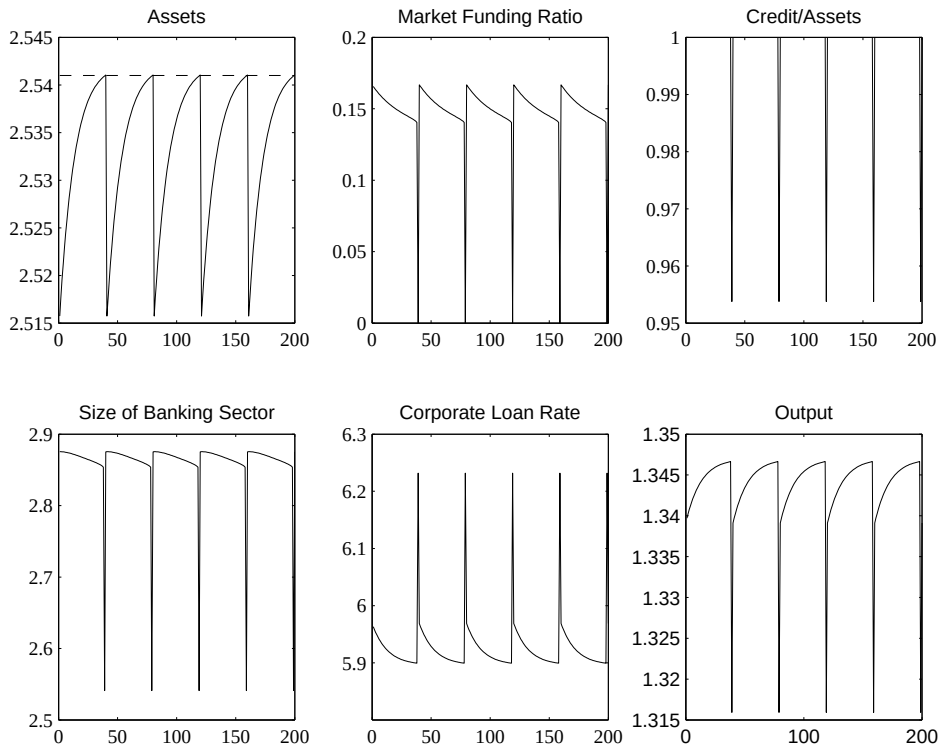
We now consider a deterministic version of the model (*i.e.*, with $z_t = z, \forall t$) to illustrate how endogenous cycles can emerge in the model.

Figure D.13: Endogenous deterministic cycles: asset accumulation decision rule



Whether or not the steady state is in a normal times regime depends on whether or not the value of A at the steady state is below $\overline{A}$. From the equations of the model in Appendix C of the paper, it is easy to see that in the steady state A is given by R (see relation (2)), that the deposit rate is equal to $r = 1/\beta$, and that R is ultimately pinned down by either relations (9a)–(12a) (for normal times) or relations (9b)–(12b) (crisis times). That is, R only depends on the three financial parameters θ, γ, and λ. As a way to illustrate the potential for endogenous cycles in the deterministic version of the model, we set $\lambda = 25$, $\gamma = 0.9394$ and select θ such that the model generates crises every 40 years ($\theta = 0.261375$). Simulations of the model indicate that endogenous cycles appear for values of $\theta \in [0.261251; 0.322846]$.

Figure D.14: Endogenous deterministic cycles: dynamics



Dynamics of aggregates, Dynamics of $\overline{a}_t$.

D.2 Model with Sub–prime Lending as Outside Option

In this section, we offer an alternative narrative of the model that resonates better with the 2008 US sub–prime crisis. We re–interpret banks' "originate–and–hold" business model (our baseline) as an "originate–and–distribute" one.

Consider interbank loans in the baseline model as collateralized loans, or non–recourse repo transactions, backed by the corporate loans. On the market, banks simultaneously (i) borrow from other banks, (ii) make loans to the representative firm, (iii) bundle and securitize those loans, and (iv) sell them off to the banks they borrowed from. Assume further that, as outside option, banks have the possibility to lend to low quality —"sub–prime"— firms indistinguishable *ex ante* from the representative firm; that sub–prime firms have a zero repayment probability;[4] and that banks get a unit return γ on those sub–prime loans.

Since sub–prime firms always default, sub–prime loans are worth nothing from the perspective of the lending banks. The moral hazard problem on the repo market comes from the fact that banks may have incentives to repo sub–prime loans in order to collect the commission fees, although such loans will not be paid back to the lending banks. To deter sub–prime lending, the lending banks limit the quantity of funds that banks can borrow, so that even the most inefficient banks with $p < \overline{p}_t$ —those that should be lending— have no interest in repo–ing sub–prime loans. They require that $\gamma(1 + \phi_t) \leqslant \rho_t$, which corresponds to the baseline incentive compatibility constraint (IC) in the paper (for $\theta = 1$). In this narrative of the model, it is the uncertainty about the quality of the underlying corporate loans (*i.e.*, whether or not they are sub–prime) that makes the interbank (repo) market prone to runs. Otherwise, the model is identical to our baseline.

Motivation for the constant return of diversion. In this version of the model, the unit return on sub–prime loans could be viewed as a commission fee, or as a gain from fraudulent activities, which the banks originating the sub–prime loan extract *ex ante*, to the detriment of the banks that purchase those loans. Both interpretations are consistent with our baseline assumption that γ is constant, and independent of the corporate loan rate, R_t, and therefore with the notion that the overall gain from diversion, $\gamma(1+\phi)a_t$, varies over time proportionally with the principal of the loan. In their overview of the securitization of sub–prime mortgage loans in the US, Ashcraft and Schuermann (2008) provide examples and a classification of a broad range of such gains. According to their classification, *ex ante* gains from fraud, which are independent of the loan rate, can arise from frictions between the originator of the loan

[4] These low quality firms need not be modeled explicitly as ultimately lending to those firms is an out–of–equilibrium strategy.

(the "borrowing bank" in our model) and the arranger of the loan (the "lending bank" in our model); they notably report one example of fraud, which accords reasonably well with our modelling of the moral hazard problem (p. 71):

"The borrowers, who include truck drivers, factory workers, a pastor and a hair stylist, say they were duped by acquaintances into signing stacks of documents and did not know they were applying for loans. Instead, they thought they were joining a risk-free investment group. Now, many of the loans are in default, the borrowers credit ratings are in ruins, and lenders are pursuing the organizers of the purported investment group in court. Companies stuck with the defaulting loans include Countrywide Financial Corp., the nations largest home lender, and Argent Mortgage Co., another big lender. A lawsuit filed by Countrywide accuses the organizers of acquiring homes and then fraudulently selling them for a quick profit [note: in our model this would be "$\gamma(1+\phi_t)a_t$"] *to the Virginia borrowers. Representatives of the borrowers put the total value of loans involved at about $80 million, which would make it one of the largest mortgage-fraud cases ever." (Hagerty and Hudson, 2006.)*

D.3 Model with Time Varying Return on Storage

Assume that the return on storage is a function of the corporate loan rate and, to fix ideas, consider the simple following *ad hoc* form:

$$\gamma_t = \overline{\gamma} R_t^{\varsigma}, \text{with } \varsigma \in [0,1]. \tag{5}$$

This specification is general enough for the purpose of our discussion, and encompasses our baseline model ($\varsigma = 0$). Replacing γ by γ_t in the expression of ϕ_t in Proposition 1 yields

$$\phi_t = \frac{\overline{p}_t R_t^{1-\varsigma} - \overline{\gamma}}{\overline{\gamma}\theta}$$

as borrowing limit and

$$\mu\left(\overline{p}_t\right) = \left(1 - \mu\left(\overline{p}_t\right)\right) \frac{\overline{p}_t R_t^{1-\varsigma} - \overline{\gamma}}{\overline{\gamma}\theta}, \tag{6}$$

as market clearing condition. Re-writing this condition as:

$$R_t = \tilde{\Psi}(\overline{p}_t) \equiv \left(\frac{\overline{\gamma}}{\overline{p}_t}\left(1 + \frac{\theta\ \mu\left(\overline{p}_t\right)}{1 - \mu\left(\overline{p}_t\right)}\right)\right)^{\frac{1}{1-\varsigma}}, \tag{7}$$

it is easy to see that, as long as $\varsigma < 1$, $\lim_{\overline{p}_t \downarrow 0} \tilde{\Psi}(\overline{p}_t) = \lim_{\overline{p}_t \uparrow 1} \tilde{\Psi}(\overline{p}_t) = +\infty$, and that $\tilde{\Psi}(\overline{p}_t)$ admits one unique, interior, and time invariant minimum $\underline{\overline{p}}$. Therefore, there exists a threshold $\overline{R} \equiv \tilde{\Psi}\left(\underline{\overline{p}}\right)$ for R_t below which there is no equilibrium with trade. In other words, the results of our baseline model hold even when γ_t varies with R_t, provided that $\varsigma < 1$ and γ_t is not *perfectly* correlated with R_t.

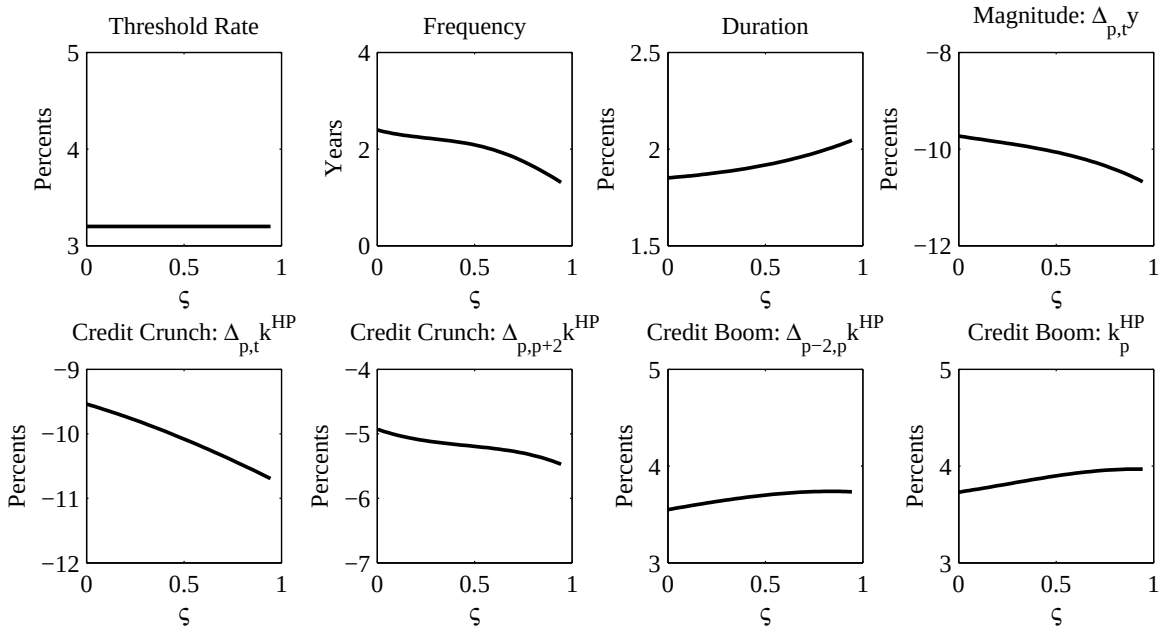
Moreover, it is easy to see from (6) that, in the special case where $\varsigma = 1$, then $\overline{p}_t$ is independent of R_t and time invariant in equilibrium. That is, borrowers' quality is constant over the business cycle. In that case, either the economy is perpetually in normal times, or it is perpetually in a crisis, which is not consistent with Fact #1.

To get a sense of how the elasticity of the return on storage to the corporate loan rate affects the quantitative properties of the model, we consider a range of values for ς, simulate the model, and compute the same statistics as in Table 1 of the paper.

For comparison purposes, and to isolate the effect of the elasticity, we keep $\overline{R}$ constant and equal to its baseline value ($\overline{R} = 1.032$), and adjust parameter $\overline{\gamma}$ accordingly, as we vary ς.

Figure D.15 shows that the quantitative properties of the model do not change in a material way, when the return on storage depends on (but is less than proportional to) the corporate loan rate.

Figure D.15: Sensitivity of the recession statistics to ς



This figure shows how the statistics on recessions (see Table 9) change as ς goes up; the case $\varsigma = 0$ corresponds to our baseline.

Motivation for the time–varying return of diversion. As in Section D.2, one can interpret the gains from diversion as private gains from sub–prime lending activities. In this version of the model, though, these gains may not reflect the same frictions as in Section D.2, because they are collected *ex post* and vary with the interest rate. Instead, this version of the model seems more consistent with frictions between the banks that service the loans (the "borrowing banks" in our model) and the arrangers of the loan (the "lending banks" in our model). Ashcraft and Schuermann (2008) indeed explain that servicers have a natural incentive to inflate their expenses —to the detriment of the arrangers, and that they are typically remunerated a flat fee plus a fee proportional to the cash flows of the loans. Hence, the overall return from inflating expenses may vary with the interest rate, but less than one–for–one; as in relation (5).

D.4 Model with Bank Leverage

It may be tempting to interpret the bank funding ratio as a leverage ratio. Leverage, however, is not determined in our model, since without any friction between the banks and the household, the deposit to equity ratio cannot be pinned down. The aim of this section is to illustrate how leverage can be determined and endogenized, and to present simulations of the model in that case. For simplicity, we keep a framework where the bank deposit to equity ratio does not have any impact on aggregate dynamics.

We make two additional assumptions. First, we assume that banks can collect deposits, which we define as the risk–free asset of the economy. Since deposits must be risk–free, there is a limit to the quantity of deposits that a given bank can collect. This limit is related to the value of the bank's assets in the worst possible state of the nature, *i.e.* γa_{t+1} for a bank born in t. (The worst possible state of the nature for a bank is one where the bank is inefficient and there is a crisis.) Second, we assume that banks are run by risk–neutral managers, whose objective is to maximize their banks' expected return on equity. That managers do not discount risk —or have a different discount factor than the shareholder's— is a standard assumption that creates a wedge between the managers and the shareholder's objectives. While in equilibrium the shareholder will be indifferent between equity and deposits, managers will in contrast be willing to raise as much deposits as possible. Let r^d_{t+1} be the risk–free (non–state contingent) gross return on deposits d_{t+1} and $r^e_{t+1}(p)$ be the *ex post* return on bank p's equity e_{t+1} at the end of period $t+1$ with, by definition, $a_{t+1} \equiv d_{t+1} + e_{t+1}$ and

$$r^e_{t+1}(p) \equiv \frac{r_{t+1}(p)a_{t+1} - r^d_{t+1}d_{t+1}}{e_{t+1}}, \tag{8}$$

where $r_{t+1}(p)$ is the optimal interest rate chosen by bank p. Since bank p defaults when $r^e_{t+1}(p) < 0$ the type $\widetilde{p}_{t+1}$ of the marginal bank that defaults at the end of period $t+1$ is given by

$$\widetilde{p}_{t+1} = r^{-1}_{t+1}\left(r^d_{t+1}\frac{d_{t+1}}{a_{t+1}}\right)\mathbb{I}_{\overline{p}_{t+1} \leqslant r^{-1}_{t+1}\left(r^d_{t+1}\frac{d_{t+1}}{a_{t+1}}\right)}, \tag{9}$$

where $\mathbb{I}_{\overline{p}_{t+1} \leqslant r^{-1}_{t+1}\left(r^d_{t+1}\frac{d_{t+1}}{a_{t+1}}\right)}$ is an indicator function that takes value one if $\overline{p}_{t+1} \leqslant r^{-1}_{t+1}\left(r^d_{t+1}\frac{d_{t+1}}{a_{t+1}}\right)$ and zero otherwise. The household's no–arbitrage condition between deposits and equity then requires that

$$\mathbb{E}_t(u'(c_{t+1})r^e_{t+1}) = r^d_{t+1}\mathbb{E}_t(u'(c_{t+1})), \tag{10}$$

where $r^e_{t+1} \equiv \int_0^1 r^e_{t+1}(p)\mathrm{d}\mu(p)$. This condition relies on r^d_{t+1} being a risk–free rate determined at the end of period t, yields r^d_{t+1} so that the household is indifferent between deposits and equity in equilibrium, and implies that $r^d_{t+1} < \mathbb{E}_t(r^e_{t+1})$. Risk–neutral managers choose the

deposit to equity ratio so as to maximize their respective banks' expected returns on equity[5]

$$\max_{d_{t+1}/e_{t+1}} \beta \mathbb{E}_t(r_{t+1}) + \beta \left(\mathbb{E}_t(r_{t+1}) - r^d_{t+1} \right) \frac{d_{t+1}}{e_{t+1}} \tag{11}$$

subject to the feasibility constraint that deposits must be risk–free,

$$d_{t+1} r^d_{t+1} \leqslant \gamma a_{t+1}. \tag{12}$$

Since (from (8) and (10)) $\mathbb{E}_t\left(r_{t+1}\right) > r^d_{t+1}$ it is clear that managers want to raise as much deposits as possible and that, in equilibrium, constraint (12) binds. Hence,

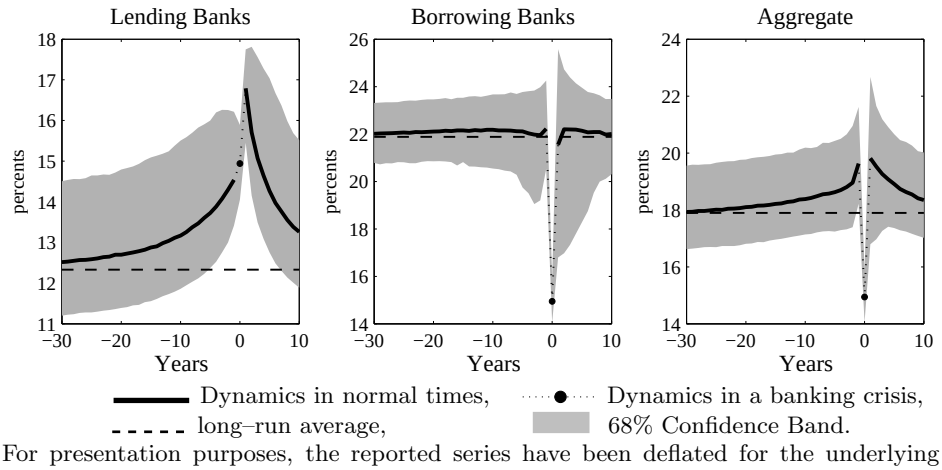
$$\frac{d_{t+1}}{e_{t+1}} = \frac{\gamma}{\max\left(r^d_{t+1} - \gamma, 0^+\right)}. \tag{13}$$

One can now easily derive borrowing (lending) banks' optimal leverage ℓ^b_{t+1} (ℓ^l_{t+1}) as

$$\ell^b_{t+1} = \frac{d_{t+1} + \phi_{t+1}(d_{t+1} + e_{t+1})}{e_{t+1}} \text{ and } \ell^l_{t+1} = \frac{d_{t+1}}{e_{t+1}}, \tag{14}$$

Bank leverage depends on both the market funding ratio and the deposit to equity ratio. It is easy to see from Proposition 1 of the paper and relation (13) above that these two ratios move in opposite directions. On the one hand the market funding ratio increases with ρ_{t+1}. On the other hand, the deposit to equity ratio decreases with r^d_{t+1}, as bank managers can afford raising more deposits when deposits become cheaper. Since ρ_{t+1} and r^d_{t+1} move together, it is not clear how leverage moves along the typical path.

Figure D.16: Typical path of bank leverage



For presentation purposes, the reported series have been deflated for the underlying exogenous growth trend.

[5]More precisely, bank managers choose d_{t+1}/e_{t+1} to maximize $\beta \mathbb{E}_t\left(r^e_{t+1}\right)$ subject to the identities $r_{t+1}a_{t+1} \equiv r^d_{t+1}d_{t+1} + r^e_{t+1}e_{t+1}$ and $a_{t+1} \equiv d_{t+1} + e_{t+1}$. If managers' incentives were aligned with the banks' shareholder, then they would instead maximize $\beta \mathbb{E}_t \left(\frac{u'(c_{t+1})}{u'(c_t)} r^e_{t+1} \right)$ and be indifferent between equity and deposits.

Figure D.16 reports the typical path of leverage; the rest of the dynamics is the same as in the baseline model (see Figures 5 and 6 in the paper). In the run–up to the typical financial recession, leverage increases for the lending banks, and stays constant for the borrowing banks. For the latter, the fall in the market funding ratio (see Figure 6 in the paper) almost exactly offsets the rise in leverage that is due to the increase in deposits. At the aggregate level, though, bank leverage goes up. During the crisis, market funding collapses and borrowing banks' leverage plummets, while that of the lending banks increase due to the decline in the deposit —risk free— rate (see relation (13)).

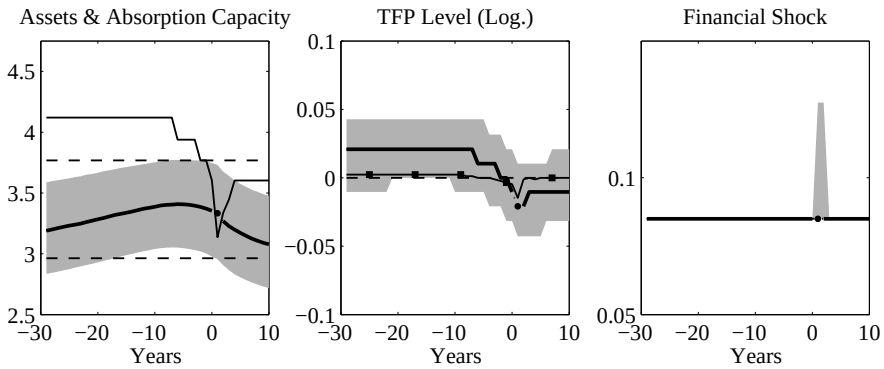
D.5 Model with Financial Shocks

We now consider a version of our baseline economy with financial shocks, in addition to the TFP shocks. We assume that the diversion parameter, θ, follows a stochastic process. High realizations of θ correspond to situation where diversion is more prevalent, hence strengthening the moral hazard problem and making the economy more prone to crises. θ is assumed to follow a three–state Markov chain. The middle state corresponds to our baseline calibration, the two extreme values correspond to 50% deviations from this value: $\theta \in \{0.0465, 0.0930, 0.1395\}$. We assume that the economy spends most of the time in the average state, and transits infrequently to either the high or low diversion state. The transition matrix takes the following form:

$$P_\theta = \begin{pmatrix} 0.500 & 0.500 & 0.000 \\ 0.025 & 0.900 & 0.025 \\ 0.000 & 0.500 & 0.500 \end{pmatrix}$$

Figure D.17 reports the typical path for asset holdings, the (log) technology shock and the financial shock, along with their 68% confidence interval (shaded area). Figure D.17 reports

Figure D.17: Typical path (two–shock model)



Dynamics in normal times, Dynamics during a crisis, Dynamics of $\overline{A}_t$, Average across simulations, 68% Confidence band around typical path, Underlying TFP innovations (ε_t). For presentation purposes, the reported series have been deflated for the underlying exogenous growth trend.

the typical path to financial recessions in this version of the model. It shows that the introduction of the financial shock does not affect in a material way the typical path of either assets or TFP. This result does not mean that financial shocks do not play any role in this version of the model, though. Indeed, we find that some of the crises are caused by a high realization of the financial shock, as witnessed by the widening in the confidence interval around the crisis (right panel). Rather, we interpret this result as suggesting that credit–boom–driven crises remain prevalent in our model, even when one introduces an additional, exogenous financial shock.

D.6 Model with Lower Labor Supply Elasticity

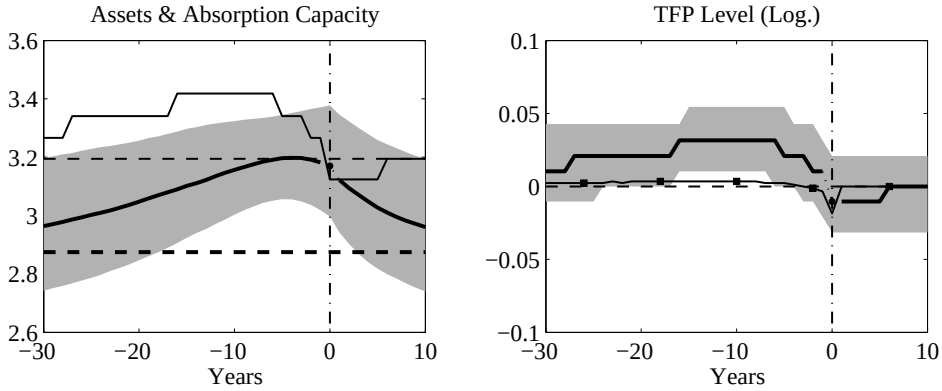
We consider a version of the model with a lower labor Frish elasticity equal to 0.5 ($\upsilon = 2$) instead of 2 ($\upsilon = 0.5$). The financial block of the model is re–calibrated accordingly, to match *(i)* the average interest rate spread between the corporate loan rate and the risk free rate, *(ii)* the average real corporate loan rate, and *(iii)* the average frequency of crises. We obtain $\gamma = 0.96$, $\lambda = 30$, and $\theta = 0.09$. The parameters are quite close to our baseline calibration, except for the distribution of the ps, which is more concentrated towards the top (there are more efficient banks). Based on this calibration, the model generates an average interbank loan rate of 1.03% and an implied threshold for the real corporate loan rate of 3.5%.

Table 8: Calibration (changes from Baseline)

Parameter		Values
Inverse of Frish elasticity	υ	2.000
Labor disutility	ϑ	0.946
Bank distribution; $\mu(p) = p^{\lambda}$	λ	30
Diversion cost	θ	0.090
Storage technology	γ	0.946

Figure D.18 reports the typical path to financial recessions in this version of the model. The results are very similar to our baseline. In particular, the TFP shock that triggers the typical financial recession is 1.43–σ_z large (against 1.45–σ_z in the baseline). Since hours worked are less pro–cyclical than in the baseline, financial recessions are not as big: as Table 9 shows, output falls by 4.82%, against 9.69% in the baseline (Table 4 in the paper) and 6.84% in the data (Table 1 in the paper); the credit crunch is also less severe. In this sense, the reduction in the labor Frish elasticity helps the model fit better the dynamics of output and credit during financial recessions. On the other hand, though, with the model further under–estimate the size of the credit boom: credit only grows by 2.90% in the two years that precede the financial recessions, against 3.55% in the baseline and 4.55% in the data. This version of the model also predicts shorter financial recessions (1.74 years) than the baseline model (1.84 years) and than what is observed in the data (2.32 years).

Figure D.18: Typical path (with lower labor Frish elasticity)



Dynamics in normal times, Dynamics during a crisis, Dynamics of $\overline{A}_t$, Average across simulations, 68% Confidence band around typical path, Underlying TFP innovations (ε_t). For presentation purposes, the reported series have been deflated for the underlying exogenous growth trend.

Table 9: Statistics on recessions in the model with $\upsilon = 2$

	Financial	Other	All	Severe	Mild
Frequency (%)	2.38	8.91	11.29	3.76	3.76
Duration (years)	1.74	1.29	1.39	1.79	1.07
Magnitude ($\Delta_{p,t}y$, %)	-4.82	-1.78	-2.42	-4.32	-1.12
Credit crunch					
$\Delta_{p,t}k^{HP}$ (%)	-7.67	0.20	-1.46	-4.26	0.15
$\Delta_{p,p+2}k^{HP}$ (%)	-3.99	0.07	-0.78	-2.34	0.10
Credit boom					
$\Delta_{p-2,p}k^{HP}$ (%)	2.90	0.12	0.71	1.73	0.08
k_p^{HP} (%)	3.05	0.07	0.69	1.78	0.02

Note: $\Delta_{p,t}x$ (resp. $\Delta_{p,p+2}x$, $\Delta_{p-2,p}x$) denotes the percentage change of variable x from peak to trough (resp. from peak to peak+2 years, and peak-2 years to peak), where x denotes either output (y) or the Hodrick Prescott ($\lambda = 6.25$) cyclical component of credit (k^{HP}). Statistics based on a 500,000 time period simulation.

D.7 Model with Wealth Effects in Labor Supply Decisions

We consider a version of the model with wealth effects in the labor supply decisions, and assume the following utility function:

$$u(c_t, h_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \vartheta \frac{h_t^{1+\upsilon}}{1+\upsilon},$$

which is commonly used in the DSGE literature. This utility function being inconsistent with a balanced growth path, we abstract from growth ($\psi = 1$). From a computational point of view, the presence of wealth effects prevents us from deriving an analytical expression for the absorption capacity, $\overline{A}_t$. This makes the solution method less straightforward and less transparent, as $\overline{A}_t$ must then be determined numerically, alongside the decision rules. To solve this model, we use a point–wise approximation, instead of a polynomial one.

As the baseline model, the parameters of the banking sector are calibrated jointly so that *(i)* the spread between the real corporate loan rate and the implicit real risk free rate equals 1.7%, *(ii)* the real corporate loan rate equals 4.4%, and *(iii)* a financial recession occurs on average every 42 years (Fact #1).

Table 10: Calibration (changes from baseline)

Parameter		Values
Bank distribution; $\mu(p) = p^{\lambda}$	λ	35
Diversion cost	θ	0.080
Storage technology	γ	0.969

Figure D.19: Typical path (with wealth effects in labor)

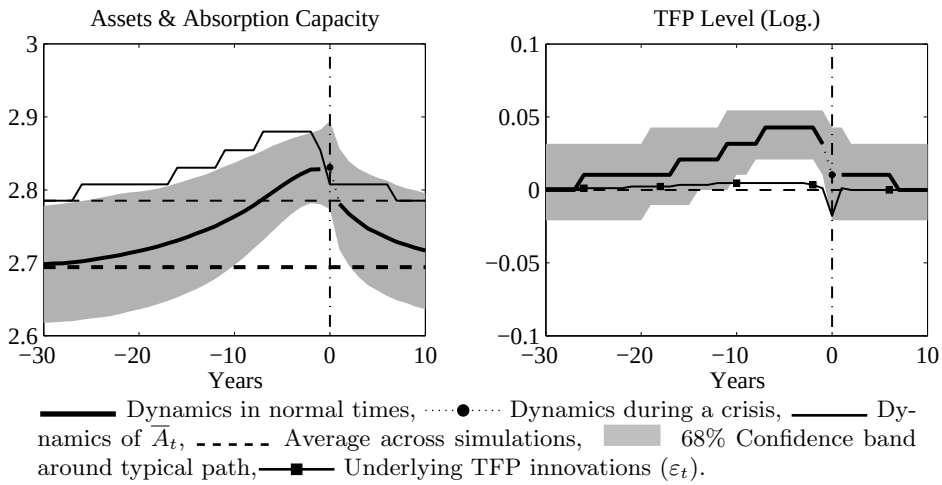


Figure D.19 reports the typical path to financial recessions in this version of the model. The typical financial recession follows a credit boom as in the baseline model, but the run–up

phase is shorter and features a continuous increase in TFP until the recession begins. The recession is then triggered by -1.37–σ_z shock comparable to the -1.45–σ_z shock in the baseline model.

Table 11 reports the corresponding statistics on recessions. The model is consistent with the three stylized facts described in Section 2 of the paper. Importantly, output losses during financial recessions are smaller in this version of the model (-5.35%) than in the baseline model (-9.69%; see Table 4 in the paper), and closer to those observed in the data (-6.84%; see Table 1 in the paper).

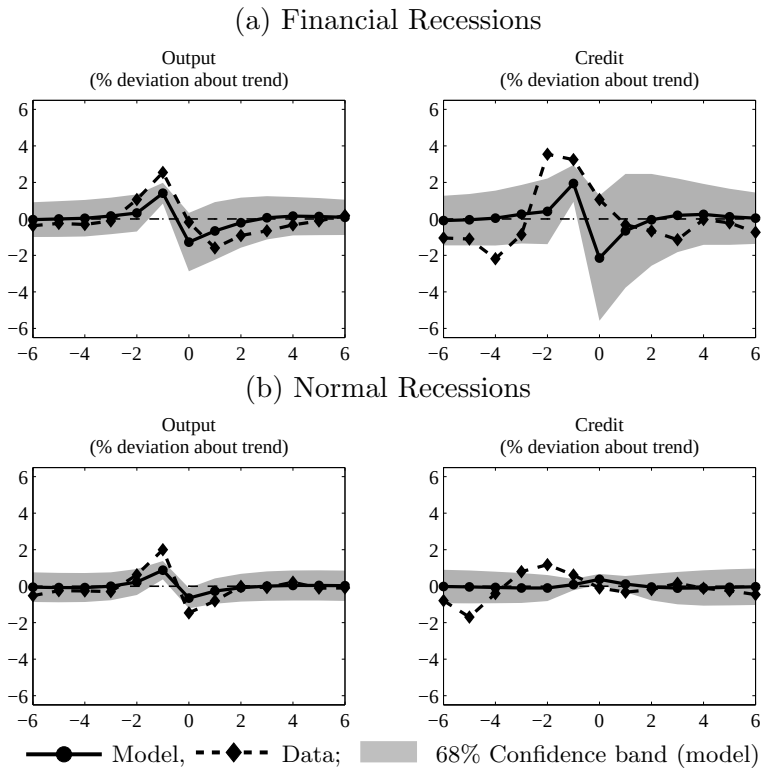
Table 11: Statistics on recessions in the model

	Financial	Other	All	Severe	Mild
N. events	11,622	44,950	56,572	18,857	18,857
Frequency (%)	2.32	8.99	11.31	3.77	3.77
Duration (years)	1.84	1.50	1.57	2.16	1.07
Magnitude ($\Delta_{p,t}y$, %)	-5.35	-2.17	-2.83	-4.76	-1.52
Credit crunch					
$\Delta_{p,t}k^{HP}$ (%)	-6.90	0.23	-1.23	-4.15	0.20
$\Delta_{p,p+2}k^{HP}$ (%)	-2.52	0.03	-0.49	-1.43	-0.07
Credit boom					
$\Delta_{p-2,p}k^{HP}$ (%)	1.85	0.18	0.53	1.22	0.17
k_p^{HP} (%)	2.04	0.08	0.48	1.27	0.10

Note: $\Delta_{p,t}x$ (resp. $\Delta_{p,p+2}x$, $\Delta_{p-2,p}x$) denotes the percentage change of variable x from peak to trough (resp. from peak to peak+2 years, and peak-2 years to peak), where x denotes either output (y) or the Hodrick Prescott ($\lambda = 6.25$) cyclical component of credit (k^{HP}). Statistics based on a 500,000 time period simulation.

Compared with the baseline, the credit cycle is also overall smoother in the presence of wealth effects in labor supply decisions. These properties are illustrated in Figure D.20, which compares the dynamics of (HP–filtered) output and credit around recessions, in the model and in the data.

Figure D.20: Dynamics of output and credit gaps around recessions



Note: Average dynamics of the Hodrick-Prescott ($\lambda = 6.25$) cyclical component of (log) output and credit 6 periods before and after the start of a business cycle (period 0) with and without a banking crisis. For the sake of consistency, we treat the simulated series of output and credit as we treat the actual data in Figure 1 of the paper.

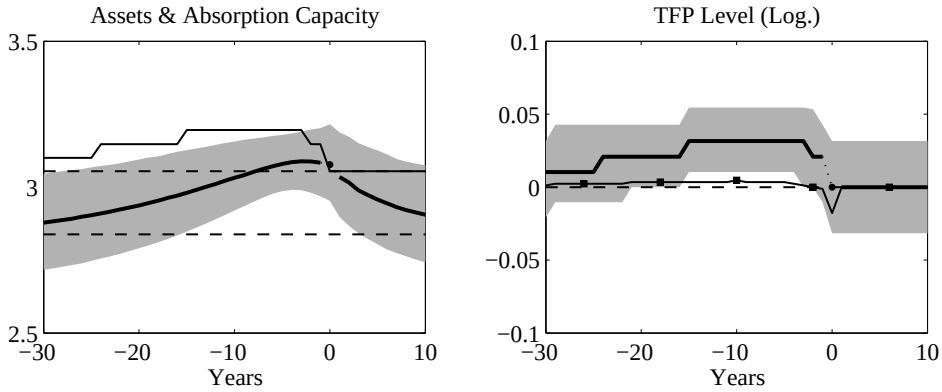
D.8 Model with Inelastic Labor supply

We consider a version of the model with a totally inelastic labor supply ($\vartheta = 0$): labor supply is exogenous and set equal to one. The financial block of the model is re–calibrated accordingly, to match *(i)* the average interest rate spread between the corporate loan rate and the risk free rate, *(ii)* the average real corporate loan rate, and *(iii)* the average frequency of crises. We obtain $\gamma = 0.96$, $\lambda = 28$, and $\theta = 0.08$. The parameters are quite close to our baseline calibration, except for the distribution of the ps, which is more concentrated towards the top (there are more efficient banks). Based on this calibration, the model generates an average interbank loan rate of 1.14% and an implied threshold for the real corporate loan rate of 3.34%. Figure D.21 reports the typical path to financial recession in this version of the model. The main difference with our baseline model is that TFP does not fall below trend during the recession: the mere mean reversion to its trend is sufficient to trigger the typical financial recession (and a crisis).

Table 12: Calibration (changes from baseline)

Parameter		Values
Bank distribution; $\mu(p) = p^{\lambda}$	λ	27
Diversion cost	θ	0.075
Storage technology	γ	0.961

Figure D.21: Typical path (with inelastic labor supply)



Dynamics in normal times, Dynamics during a crisis, Dynamics of $\overline{A}_t$, Average across simulations, 68% Confidence band around typical path, Underlying TFP innovations (ε_t). For presentation purposes, the reported series have been deflated for the underlying exogenous growth trend.

D.9 Model with Constant Saving Rate

We consider a model *á la* Solow, where the households saves a constant fraction s of her income. The equations of the model with a constant saving rate s are the following:

$$
\begin{aligned}
&1. \quad y_t = z_t k_t^{\alpha} h_t^{1-\alpha} + (\gamma + \delta - 1)(A_t - k_t) \\
&2. \quad R_t = \alpha k_t^{\frac{-\upsilon(1-\alpha)}{\upsilon+\alpha}} z_t^{\frac{1+\upsilon}{\upsilon+\alpha}} \left(\frac{1-\alpha}{\vartheta}\right)^{\frac{1-\alpha}{\upsilon+\alpha}} + 1 - \delta \\
&3. \quad i_t = s y_t \\
&4. \quad h_t = \left(\frac{(1-\alpha) z_t}{\vartheta}\right)^{\frac{1}{\upsilon+\alpha}} k_t^{\frac{\alpha}{\upsilon+\alpha}} \\
&5. \quad \overline{A}_t \equiv ((1-\alpha)/\vartheta)^{\frac{1}{\upsilon}} \left(\alpha / \left(\overline{R} + \delta - 1\right)\right)^{\frac{\upsilon+\alpha}{\upsilon(1-\alpha)}} z_t^{\frac{1+\upsilon}{\upsilon(1-\alpha)}} \\
&6. \quad i_t = \psi A_{t+1} - (1-\delta) A_t \\
&7. \quad y_t = c_t + i_t \\
&8. \quad \chi_t = (R_t - r_t) A_t - (R_t - \gamma)(A_t - k_t)
\end{aligned}
$$

If $A_t \leqslant \overline{A}_t$ (normal times)

$$
\begin{aligned}
&9a. \quad k_t = A_t \\
&10a. \quad \frac{r_t}{R_t} = \int_{\overline{p}_t}^{1} p \frac{\mathrm{d}\mu(p)}{1 - \mu(\overline{p}_t)} \\
&11a. \quad \overline{p}_t = \frac{\rho_t}{R_t} \\
&12a. \quad R_t = \frac{\rho_t}{\mu^{-1}\left(\frac{\rho_t - \gamma}{\rho_t - (1-\theta)\gamma}\right)}, \text{ with } \rho_t > \overline{\rho}
\end{aligned}
$$

If $A_t > \overline{A}_t$ (crisis times)

$$
\begin{aligned}
&9b. \quad k_t = A_t - \mu(\gamma / R_t) A_t \\
&10b. \quad \frac{r_t}{R_t} = \frac{\gamma}{R_t} \mu(\gamma / R_t) + \int_{\gamma/R_t}^{1} p \, \mathrm{d}\mu(p) \\
&11b. \quad \overline{p}_t = \frac{\gamma}{R_t} \\
&12b. \quad \rho_t = \gamma
\end{aligned}
$$

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